Exponentials & Logarithms

Example.

Exam Q5 2009

2 hrs after experiment, bacterium population in sample is recorded as 1200 and is found to be 3600 after further 2 hrs. Assume exponential growth find.

a) number of bacterium when exp started.

b) the growth constant of population.

c) the time when population is 20,000.

A b) at $t = 2$, $P(2) = 1200$

at $t = 4$, $P(4) = 3600$

assume $P = P_0 e^{\lambda t}$

we know $1200 = P_0 e^{2\lambda}$ (1)

and $3600 = P_0 e^{4\lambda}$ (2)

$(1) \Rightarrow P_0 = \frac{1200}{e^{2\lambda}}$

$\Rightarrow 3600 = \frac{1200}{e^{2\lambda}} e^{4\lambda}$

$$\Rightarrow \frac{3600}{1200} = e^{2\lambda}$$

$$\Rightarrow 3 = e^{2\lambda}$$

$$\Rightarrow \ln(3) = \ln(e^{2\lambda})$$

$$\ln(3) = 2\lambda \ln(e)$$

$$\ln(e) = 1$$

$$\Rightarrow \ln(3) = 2\lambda.$$

$$\Rightarrow \lambda = \frac{\ln(3)}{2}$$ answer part (b).

A(a) so $1200 = P_0 e^{2\ln(3)/2}$

$$\therefore 1200 = P_0 e^{\ln(3)}.$$

$e^{\ln(a)} = a \Rightarrow 1200 = P_0 \cdot 3$

$$\Rightarrow P_0 = \frac{1200}{3}$$

$P_0 = 400$ answer part (a)

A(c) find t, when $P = 20000$

$$P = P_0 e^{\lambda t}$$

i.e.

$$20000 = 400 e^{\frac{\ln(3)}{2} t}$$

$$\Rightarrow \frac{20000}{400} = e^{\frac{\ln(3)}{2}t}$$

$$\Rightarrow 50 = e^{\frac{\ln(3)}{2}t}$$

Take logarithm both sides.

$$\ln(50) = \ln\left[e^{\frac{\ln(3)}{2}t}\right]$$

$$\ln(50) = \frac{\ln(3)}{2}t$$

$$\Rightarrow \boxed{\frac{2 \times \ln(50)}{\ln(3)} = t}$$

Final answer.

Given $\ln(3) \approx 1.099$, $\ln(20) \approx 2.996$

$\ln(50) \approx 3.912$

$$\Rightarrow t \approx \frac{2 \times 3.912}{1.099}$$ (silly answer!)