Linear 2^{nd} order Differential Eqs. with const. coeff. when $r = \lambda_1 = \lambda_2$

Q: Solve differential Eq:

$$y'' + 12y' + 36y = 3e^{-6x}.$$

A:-(i) Same method as 1^{st} order. First Solve homogenous eq.

$$y'' + 12y' + 36y = 0$$

(try $y = e^{\lambda x}$)

$$\lambda^2 + 12\lambda + 36 = 0$$

$$(\lambda + 6)(\lambda + 6) = 0$$

$\therefore$ repeated real solution:

$$\lambda_1 = -6 \ , \ \lambda_2 = -6.$$

second order eq. So $y_{CF} = Ae^{\lambda_1 x} + Be^{\lambda_2 x}$

but since $\lambda_1 = \lambda_2$ we multiply one by x. (term of y_{CF}) ie.

$$y_{CF} = Ae^{-6x} + Bxe^{-6x}.$$

So y_{CF} has two components which are linearly in dependent.

(ii) Now solve non-homogenous Eq.

$$y'' + 12y' + 36y = 3e^{-6x}.$$

RHS has $p(x) = 3e^{-6x}$ so try $y_{PI} = \alpha e^{-6x}$,

but y_{CF} already has ~~y~~ terms e^{-6x} and xe^{-6x}.

Need another linearly independant term

So try $y_{PI} = x^2 \alpha e^{-6x}$

$$y'_{PI} = 2x\alpha e^{-6x} - 6x^2\alpha e^{-6x}$$

$$y''_{PI} = 2\alpha e^{-6x} - 24x\alpha e^{-6x} + 36x^2\alpha e^{-6x}.$$

Substitute: gives $2\alpha e^{-6x} = 3e^{-6x}$

$$\alpha = 3/2.$$

So $y_{PI} = \frac{3}{2}x^2 e^{-6x}.$

(iii) Finally, General solution

$$y = y_{CF} + y_{PI}$$

$$y(x) = Ae^{-6x} + Bxe^{-6x} + \frac{3}{2}x^2 e^{-6x}.$$