2nd Order ODE example - IVP with $p(x)$ = constant

Q: Solve $y'' - 9y = 5$ with $y(0) = 2$, $y'(0) = 3$.

(i) Solve Homogenous Eq: $y'' - 9y = 0$

Auxillary Eq: $\lambda^2 - 9 = 0$

$\lambda^2 = 9 \Rightarrow \lambda = \pm 3$

i.e. $\lambda_1 = 3$, $\lambda_2 = -3$.

so $y_{CF} = Ae^{3x} + Be^{-3x}$.

(ii) Solve None-Homogenous $P(x) = 5$

so try constant, $y_{PI} = \alpha$

$y'_{PI} = 0$

$y''_{PI} = 0$

Substitute and compare coefficients:

$-9\alpha = 5$

$\Rightarrow \alpha = -5/9$

so $y_{PI} = -5/9$

(iii) General solution $y = y_{CF} + y_{PI}$.

i.e. $y(x) = Ae^{3x} + Be^{-3x} - 5/9$.

Now apply $y(0) = 2$, $y'(0) = 3$.

First $y'(x) = 3Ae^{3x} - 3Be^{-3x}$.

$\therefore$ $2 = A + B - 5/9 \Rightarrow A + B = \frac{23}{9}$ (1)

$3 = 3A - 3B \Rightarrow A - B = 1$ (2)

(2) $\Rightarrow A = 1 + B$.

(1) $\Rightarrow 1 + B + B = \frac{23}{9} \Rightarrow 2B = \frac{14}{9} \Rightarrow B = \frac{7}{9}$

$\Rightarrow A = 1 + \frac{7}{9} \Rightarrow A = \frac{16}{9}$

Finally: $y(x) = \frac{16}{9}e^{3x} + \frac{7}{9}e^{-3x} - \frac{5}{9}$