Linear 1st Order Differential Eqs. with constant coeff: When $r=\lambda$.

Q: Solve the differential Eq.

$$y' + 6y = 5e^{-6x}.$$

A: (i) Solve homogenous equation.

Trying $y=e^{\lambda x}$ gives: $(y'+6y=0)$

$$\lambda + 6 = 0$$

$$\lambda = -6$$

$$\therefore \quad y_{CF} = Ce^{-6x}.$$

(ii) Solve non-homogenous equation.

$$y' + 6y = 5e^{-6x}.$$

The RHS is $p(x) = 5e^{-6x}$. But $y_{CF} = Ce^{-6x}$ i.e. both like e^{-6x}. We can't have y_{CF} and y_{PI} the same. So

Try $y_{PI} = x\alpha e^{-6x}$.

So $y'_{PI} = \alpha e^{-6x} + -6x\alpha e^{-6x}$.

Substitute: $\alpha e^{-6x} - \cancel{6x\alpha e^{-6x}} + \cancel{6x\alpha e^{-6x}} = 5e^{-6x}$

i.e. $\alpha e^{-6x} = 5e^{-6x}$

$\therefore \quad \alpha = 5.$

So $y_{PI} = 5xe^{-6x}.$

(iii) General solution:

$$y = y_{CF} + y_{PI}$$

$$y(x) = Ce^{-6x} + 5xe^{-6x}$$