THE DARK CORNERS OF THE LABOR MARKET[*]

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Abstract

Standard models predict that episodes of high unemployment are followed by recoveries. This paper shows, by contrast, that a large shock may set the economy on a path towards very high unemployment, with no recovery in sight. First, I estimate a reduced-form model of flows in the U.S. labor market, allowing for the possibility of multiple steady states. Next, I estimate a non-linear search and matching model, in which multiplicity of steady states may arise due to skill losses upon unemployment, following Pissarides (1992). Both exercises suggest a stable steady state with around 5 percent unemployment and an unstable one with around 10 percent unemployment. The search and matching model can explain observed job finding rates remarkably well, due to its strong endogenous persistence mechanism.

Key Words: unemployment, multiple steady states, non-linear models

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The main lesson of the crisis is that we were much closer to those dark corners than we thought—and the corners were even darker than we had thought too. – Olivier Blanchard (2014), in "Where Danger Lurks".

1 Introduction

A large body of literature has developed models of cyclical swings in the labor market, often within the search and matching paradigm of Diamond, Mortensen and Pissarides (DMP). Most of these models predict that, following a one-time shock, unemployment gradually reverts back to a unique steady-state level (see Figure 1, left panel). Episodes of sustained high unemployment may occur in these models, but only if the economy is repeatedly hit by adverse shocks. In models with multiple steady states, by contrast, a single shock may set the labor market on a path towards a "Dark Corner": a region of economic states with high unemployment and no tendency to revert back (see Figure 1, right panel).[1]

This paper shows that models with multiple steady states –while seldom used for quantitative purposes– can provide a superior account of the dynamics of the U.S. labor market. I reach this conclusion after estimating (i) a reduced-form model of stocks and flows in the labor market and (ii) a search and matching model of the business cycle. Both models allow for multiplicity of steady states but may also deliver a single steady state, depending on estimation outcomes. Multiple steady states can emerge if either one of the flow rates is affected by the unemployment rate.[2] Multiplicity can also arise due to non-linearities in the model relations. I show that allowing for both potential sources of multiplicity can substantially improve the statistical performance of the model.

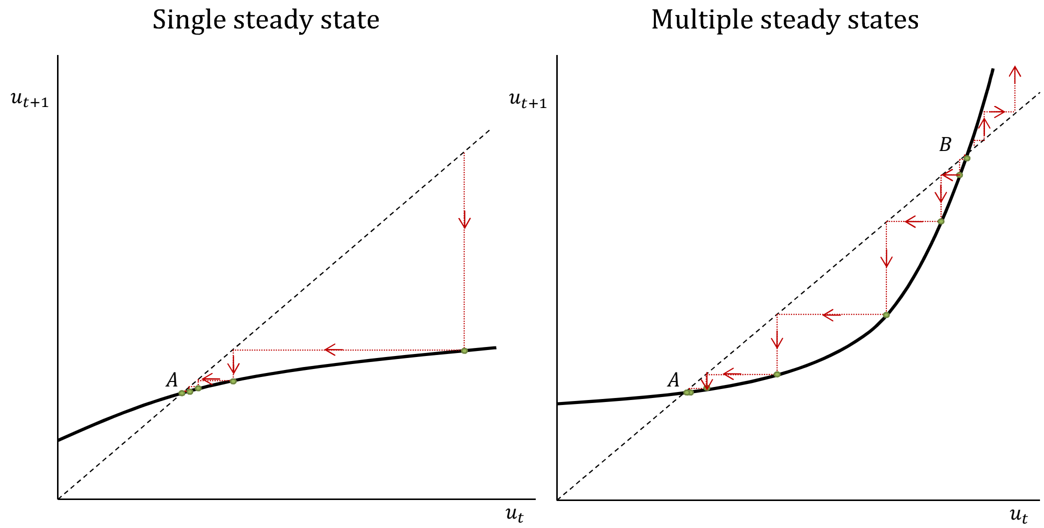
I first estimate a reduced-form model of U.S. flow rates in and out of unemployment, using a simple methodology based on forecasting regressions. My results indicate a stable steady state with about five percent unemployment, and an unstable steady state with around ten percent unemployment. This suggests that, following the Great Recession of 2008, the U.S. economy was nearly drawn into a Dark Corner with high long-run unemployment.

The second set of evidence is based on an estimated search and matching model in the

[1] For examples of models with multiple steady states, see e.g. Diamond (1982), Pissarides (1992), and Kaplan and Menzio (2014).

[2] A simple way to see this is to consider the transition identity $u_{t+1} = u_t\,(1 - ue_t) + (1 - u_t)\,eu_t$, where u_t is the unemployment rate in period t, ue_t is the unemployment outflow rate, and eu_t is the unemployment inflow rate. If either of the two transition rates depends linearly on u_t, the right-hand side becomes quadratic in u_t, giving rise to two solutions for a steady state level $\overline{u} = u_{t+1} = u_t$.

Figure 1: Model illustrations.



Notes: black solid lines illustrate the relation between unemployment today (u_t) and unemployment tomorrow (u_{t+1}). Points A and B indicate steady states. Red arrows illustrate transition dynamics. In the right panel, the area to the right of point B represents the "Dark Corner".

tradition of DMP, but extended to allow for a loss of human capital upon unemployment, following Pissarides (1992). In this model, skill losses associated with higher unemployment discourage hiring, which further pushes up unemployment. Depending on parameter values, this mechanism may give rise to multiple steady states. The economy is further hit by stochastic shocks to the rate of job loss, which are taken directly from the data. I find that, based on these shocks alone, the model can match observed job finding and unemployment rates remarkably well. This quantitative success is due to the presence of a strong endogenous persistence mechanism created by the incidence of skill losses, coupled with a moderate non-linearity in the firms' vacancy posting condition.

Like the reduced-form model, the estimated search and matching model has a stable steady state with around five percent unemployment and an unstable one with around ten percent unemployment. Perhaps surprisingly, multiple steady states arise despite the fact that the estimated degree of skill loss upon unemployment is only moderate. Specifically, I estimate that when a worker goes through an unemployment spell, she suffers a one-time productivity loss equivalent to two weeks of output.[3] At face value, the model may therefore seem a

[3] An extensive literature shows that job displacement has large and persistent effects on earnings, see e.g. Davis and von Wachter (2011). Jarosch (2014) presents evidence that a loss of human capital is an important driver behind these earnings losses. Huckfeldt (2013) analyzes the cost of job loss via occupational switching.

minimal departure from a basic DMP model, but the dynamics are nonetheless dramatically altered.

Considering the aftermath of the Great Recession, the multiple-steady-state models can account particularly well for the slow recovery of the labor market. The right panel of Figure 1 clarifies this point. Suppose that the economy starts from the stable steady state with low unemployment (point A in the figure, about 5 percent in the data). Next, a one-time wave of job losses brings unemployment just below the second, unstable steady state (point B in the figure, about 10 percent in the data). Ultimately, unemployment will revert back to its initial level, following the step-wise path illustrated by the red line. Initially, however, the speed of this transition is slow. By contrast, in the single-steady-state model (Figure 1, left panel), the speed of transition is particularly high when the economy is far away from steady-state point A. The latter is difficult to reconcile with the fact that, following the Great Recession, the job finding rate declined to an unprecedented level and stayed very low for a sustained period of time.

The presence of non-linearities further generates countercyclical fluctuations in uncertainty about aggregate unemployment. According to the model, unemployment uncertainty rose particularly sharply during the Great Recession, which is in line with suggestive evidence from the Survey of Professional Forecasters.

The analysis relates to an empirical literature which analyzes labor market transition rates, see e.g. Hall (2005b), Shimer (2005), Elsby, Solon, and Michaels (2006), and Fujita and Ramey (2009). Barnichon and Nekarda (2012) develop a reduced-form forecasting model of unemployment and emphasize the benefits of conditioning separately on unemployment in- and outflows. This is important since the two flow rates have distinctly different time series properties and therefore each contain valuable information on the state of the economy, which would be lost when conditioning on only the current level of unemployment. I follow this "flow approach", but focus on estimating steady-state rates of unemployment rather than constructing near-term forecasts. Finally, the finding that there may be multiple steady states connects this paper to an empirical literature investigating the possibility of "hysteresis" in unemployment, see Ball (2009) for an overview.

On the theoretical side, I integrate the endogenous persistence mechanism of Pissarides (1992) into a business cycle model, and estimate the model while fully accounting for non-linearities.[4] The latter connects the paper to Petrosky-Nadeau, Kuehn, and Zhang (2013), who study the importance of non-linearities in generating episodes of very high unemployment in a DMP model with a single steady state. The relevance of endogenous persistence is emphasized in Fujita and Ramey (2007) and Mitman and Rabinovich (2014) who study, respectively, DMP models with sunk costs to vacancy creation and endogenous unemployment benefit extensions. I show that the *interaction* of non-linearities and endogenous persistence can produce a close fit to the data and give rise to multiplicity of steady states.

The analysis further relates to a strand of literature which studies labor market models with "non-standard" properties. Blanchard and Summers (1987) and Saint-Paul (1995) study equilibrium multiplicity due to government policies and labor market institutions. More recently, Kaplan and Menzio (2014) and Schaal and Taschereau-Dumouchel (2016) present models in which multiplicity derives from externalities in consumer behavior. They calibrate their models and confront them with the data using numerical simulations.[5] In this paper, I go one step further and estimate a model directly using labor flow data. A practical advantage of my model is that has a unique equilibrium, even when there are multiple steady states, avoiding issues related to sunspots and equilibrium selection, which would complicate estimation.

Finally, there is a close link between the search and matching model and the reduced-form model. Essentially, the forecasting equations of the latter are the reduced-form equivalents of the core Euler equation for vacancies in the search and matching model. Without skill losses, the unemployment rate is not a state variable for the firms' vacancy posting decision, whereas with skill losses the unemployment rate becomes a key state variable. In the reduced-form analysis, I find that omitting the unemployment rate in the forecasting regression produces strong autocorrelation in forecast errors, suggesting that a key piece of information on the aggregate state is missing. Including the unemployment rate in the forecasting regression, however, absorbs this autocorrelation and improves forecast accuracy. The reduced-form analysis thus provides a way of confronting the Euler equations of DMP-style models with the data. Since Hall (1978), researchers have used this type of reduced-form approach to

[4] Esteban-Pretel and Faraglia (2008) and Laureys (2014) also integrate skill losses in DMP-style models. They, however, calibrate their models and solve them by linearization. Further, they stay away from parameterizations with multiple steady states. Here, instead, I estimate the model, using a global solution method. The latter is crucial to allow for the possibility of multiple steady states.

[5] Other recent contributions proposing "non-standard" models include Eeckhout and Lindenlaub (2015), Azariadis, Kaas, and Wen (2015), Fajgelbaum, Schaal, and Taschereau-Dumouchel (2015), Beaudry, Galizia, and Portier (2015) and Sniekers (2015).

scrutinize a large variety of theories, including models of investment, asset pricing models, and New-Keynesian models. Somewhat surprisingly, the DMP model has not received the same kind of attention, even though its core can be conveniently summarized by a single Euler equation.

The remainder of this paper is organized as follows. Section 2 presents the reduced-form empirical evidence. Section 3 describes the search and matching model and presents the estimation results and their implications for business cycles and government policy. Section 4 concludes.

2 Reduced-form model

This section presents a method to estimate steady states without making structural assumptions. The method is based on multi-step forecasts and it allows for non-linearities, and therefore closely connects to the local projection method proposed by Jordà (2009). However, whereas the local projection method uses multi-step forecasts to estimate impulse response functions to macroeconomic shocks, I use them to estimate steady states.

Subsection 2.1 presents the general methodology. In subsection 2.2, the method is tailored to the U.S. labor market. Section 2.3 presents results.

2.1 Methodology

Consider a Dynamic and Stochastic General Equilibrium model with variables that are observed in discrete time. Let $\mathcal{S}_t \in \mathbb{R}^m$ be the vector of state variables of the model in period t, which may or may not be unobserved by the researcher. The DSGE model implies a joint distribution of the model variables at any future period $t+k$, conditional upon the current state $\mathcal{S}_t$. In this subsection, I lay out how estimating the moments of this conditional distribution in the data can be used to reveal the dynamic properties of the underlying model, and in particular the presence of "Dark Corners". Towards this end, I introduce the following notation. Let $\mathbf{x}_t \in \mathbb{R}^n$ be a vector containing a subset of all observable model variables, selected by the researcher. Furthermore, let $F_k(\mathcal{S}_t)$ be the function that maps the state $\mathcal{S}_t$ into the direct-multistep conditional point forecast of $\mathbf{x}_t$, i.e. $F_k(\mathcal{S}_t) = \mathbb{E}[\mathbf{x}_{t+k}| \mathcal{S}_t]$, where $\mathbb{E}$ is the expectations operator and $k \geq 0$ is the forecast horizon. Note that $F_0(\mathcal{S}_t)$ is simply the mapping from the current state to the current outcome, i.e. $F_0(\mathcal{S}_t) = \mathbb{E}[\mathbf{x}_t| \mathcal{S}_t] = \mathbf{x}_t$.

By evaluating these forecasting functions, at various points in the state space, the model's dynamic properties can be revealed. To appreciate why this is the case, note that the forecasting functions reveal the direction in which model variables are expected to move. This direction in turn depends on the state of the economy relative to the steady state. Moving from one side of the steady state to another side, e.g. due to an exogenous shock, typically flips the direction, see e.g. Figure 1. More formally, the sign of $F_k(\mathcal{S}_t) - F_0(\mathcal{S}_t)$ is helpful in detecting potential steady-state points. Additionally, the magnitude of $F_k(\mathcal{S}_t) - F_0(\mathcal{S}_t)$, i.e. the speed of transition, is contains useful information. As illustrated in Figure 1, the speed of convergence (or divergence) around steady state points is typically low.

A practical issue which confronts the researcher is that the state of the economy, $\mathcal{S}_t$, may not be entirely observable. This may seem problematic since the optimal forecasts condition on the state vector. To overcome this issue, I exploit that the economic state reveals itself through endogenous variables which are functions of the state. That is, I condition the forecast for $\mathbf{x}_{t+k}$ on $\mathbf{x}_t$ rather than on $\mathcal{S}_t$ and verify ex post that $\mathbf{x}_t$ contains a sufficient number of variables to reveal $\mathcal{S}_t$.[6] This verification step is implemented by checking for autocorrelation in the forecast residuals. Positive autocorrelation indicates that $\mathbf{x}_t$ omits some important piece of information about $\mathcal{S}_t$ which induces persistent forecast errors. If this is the case, the forecasting function is misspecified and additional variables should be included in $\mathbf{x}_t$. Additionally, the model fit as measured by for example the R^2 may be useful in distinguishing between various candidate specifications. A second practical issue is that the form of the forecasting function is unknown. I will use polynomials to approximate the forecasting functions flexibly. Again, evaluating the degree of residual autocorrelation and the model fit is helpful in assessing assumptions on the functional form of the forecasting function.

Finally, let me discuss the notion of a "steady state" in some more detail. In the framework laid out above, one could label states at which $F_k(\mathcal{S}_t) = F_0(\mathcal{S}_t)$, at some horizon k, as "expectational steady states", as these are states of the world at which expected value $\mathbb{E}[\mathbf{x}_{t+k}|\,\mathcal{S}_t]$ coincides with the current value $\mathbf{x}_t$. This notion differs somewhat from that of a "deterministic steady state", which is typically defined as a steady state in a counterfactual version of the model without uncertainty. Without uncertainty or non-linearities, the two

[6] The advantage of the method is thus that the researcher can learn about steady states while remaining agnostic about the endogenous state variables of the underlying model. As a result, however, the method does not inform one about whether, dynamically, the model has a unique saddle-path stable equilibrium or not.

steady state concepts coincide precisely. In models with shocks and non-linearities, however, there is a discrepancy due to Jensen's inequality. In this study, I am not directly interested in deterministic steady states, as they pertain to counterfactual model versions. Nonetheless, one advantage of the search and matching model, to be developed in Section 3, is that I can compute both deterministic steady state and expectational steady states. It will then become clear that in my application the discrepancy between the two types of steady states is rather small.

2.2 Application to the U.S. labor market

I now apply the method to estimate a reduced-form model of stocks and flows in the U.S. labor market. Specifically, I estimate a model of the job loss rate, the job finding rate and the unemployment rate.

Consider a labor market in which workers flow stochastically between employment and unemployment. Time is discrete and indexed by t. Job losses materialize at the beginning of each period and the rate at which employed workers lose their jobs is denoted by $\rho_{x,t}$. After job losses occur, a labor market opens and a matching process between job searchers and firms takes place. The pool of job searchers consists of those workers who just lost their jobs and those who have been unemployed for some time.[7] Let the rate at which job searchers find jobs be denoted by $\rho_{f,t}$. Those who find a job during period t become employed within the same period. Hence, job losers may immediately find a new job without becoming unemployed. It follows that the unemployment rate, u_t, evolves according to the following transition identity:

$$u_t = \rho_{x,t} \left(1 - \rho_{f,t}\right) \left(1 - u_{t-1}\right) + \left(1 - \rho_{f,t}\right) u_{t-1}, \tag{1}$$

where the first and second terms on the right-hand side are, respectively, the number of new and continuing job seekers, both expressed as a fraction of the labor force.[8]

The job finding rate and the job loss rate are determined as functions of the aggregate state of the economy. In a fully structural model, these functions would be the equilibrium

[7] I abstract from flows in and out of the labor force and on-the-job search.

[8] While I estimate a model for the average job finding rate, I do not rule out that transition rates are heterogeneous across workers. Various recent papers emphasize the importance of heterogeneity in job finding rates, see e.g. Ravn and Sterk (2012), Kroft, Lange, Notowidigdo, and Katz (2014) and Hall and Schulhofer-Wohl (2015). In that case, the composition of the pool of job searchers may become a key state variable driving the average job finding rate. My method allows for this possibility by admitting the presence of unobserved states.

outcome of agents' decisions, resource constraints, market clearing conditions, and so forth. Here, I treat the underlying model structure as unknown and use a reduced-form approach instead. The two transition rates $\rho_{f,t}$ and $\rho_{x,t}$ are uniquely pinned down as functions of the state vector $\mathcal{S}_t$.[9] Given these variables and u_{t-1}, u_t follows mechanically from Equation (1).

Model specifications. The next step is to decide which variables to include in the outcome vector $\mathbf{x}_t$. At the very minimum, I include $\rho_{f,t}$ and $\rho_{x,t}$. Recall that these variables are not necessarily the state variables of the "true" underlying model, but could instead be "jump" variables that are pinned down as functions of the state variables. Given steady-state values for $\overline{\rho}_x$ and $\overline{\rho}_f$, one can compute $\overline{u}$ using the steady-state solution of Equation (1), which is given by $\overline{u} = \overline{\rho}_x \left(1 - \overline{\rho}_f\right) / \left(\overline{\rho}_x \left(1 - \overline{\rho}_f\right) + \overline{\rho}_f\right)$.[10] Doing so guarantees that the joint steady-state solution is consistent in an accounting sense. It further allows one to compute the steady-state rate of unemployment without including u_t in $\mathbf{x}_t$. That said, it may be the case that including u_t in $\mathbf{x}_t$ is required to fully identify the state of the economy. Indeed, in the structural model presented in the next section the unemployment rate is itself a key state variable for the job finding rate. If so, omitting u_t in the forecasting regression may induce misspecification. I will therefore consider models with and without u_t as a regressor. In those models that include u_t, I will impose the steady-state version of Equation (1) rather than estimating a separate forecasting equation for u_{t+k}.

Following the procedure described in the previous section, I have considered a battery of candidate specifications. To avoid unnecessary repetition, I discuss in the main text only three specifications, which turn out to summarize the main results. The Appendix presents results for various alternative models with additional lags, additional macro variables, and additional higher-order terms. These alternatives, however, either produce autocorrelation in the forecast errors, or produce results similar to the three baseline specification.

The first specification, labeled Model (I), assumes that the job finding rate forecast, $\mathbb{E}_t \rho_{f,t+k}$, can be expressed as a linear function of only $\rho_{f,t}$ and $\rho_{x,t}$. In model (II), $\mathbb{E}_t \rho_{f,t+k}$ is estimated as a linear function of $\rho_{f,t}$, $\rho_{x,t}$ and u_t, whereas in model (III) $\mathbb{E}_t \rho_{f,t+k}$ is linear in

[9] This is consistent with the possibility of multiple short-run equilibria. In that case, outcomes would be determined by a sunspot variable, which would be part of $\mathcal{S}_t$.

[10] It should be emphasized that steady state solutions for $\overline{\rho}_f$, $\overline{\rho}_x$ and $\overline{u}$ may not all be between zero and one. If so, then the solution is not practically relevant. Such a finding, however, could indicate a corner solution (i.e. a solution in which one or more variables has value zero or one). The stability properties of the various steady states determine whether this is the case. I will address this possibility in more detail when discussing the empirical findings.

$\rho_{f,t}$, $\rho_{x,t}$, u_t as well as u_t^2. All specifications also include a constant term. For the rate of job loss, $\rho_{x,t}$, all three models assume an AR(1) process, i.e. $\mathbb{E}_t \rho_{x,t+k}$ is a linear function of only $\rho_{x,t}$ and a constant. The Appendix considers various alternative specifications, but it turns out that, in contrast to the job finding rate, the job loss is well described by a simple AR(1) process.

Note that model (I) excludes the unemployment rate from the state vector and rules out multiple steady states by construction. Model (II) includes the unemployment rate in a linear fashion and it is straightforward to verify that this model allows for at most two interior steady states. Model (III) adds u_t^2, which allows for more than two interior steady state solutions.

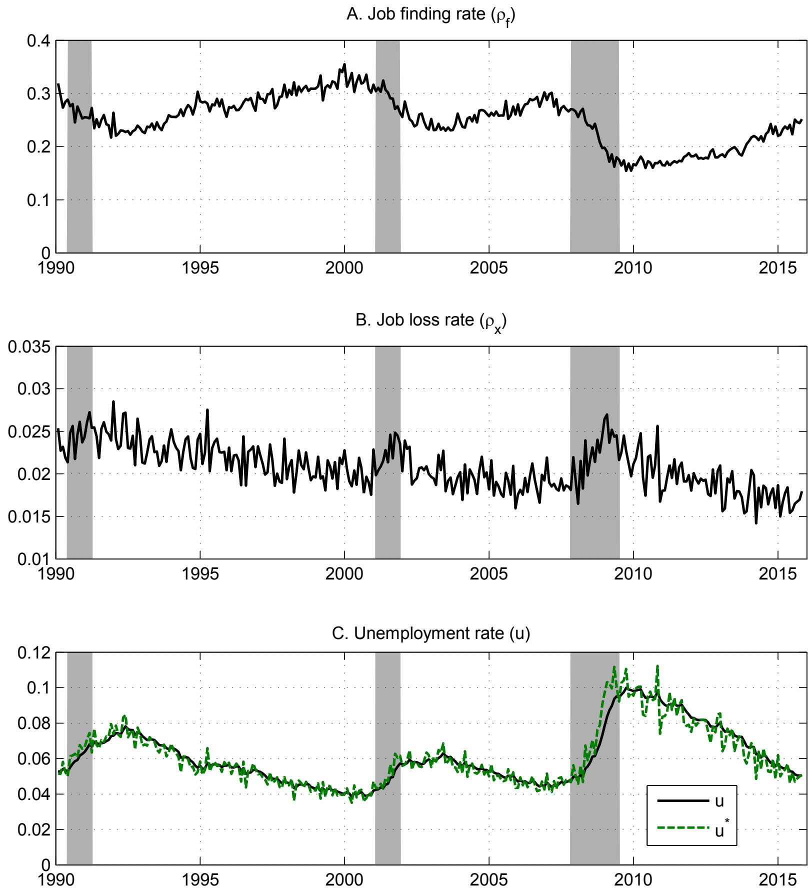
Data. The labor market data are taken from the Current Population Survey (CPS). I obtain monthly observations for u_t and $\rho_{f,t}$. The job finding rate, $\rho_{f,t}$, is measured as the unemployment-to-employment transition rate as reported in the CPS section on gross labor market flows. These data are available from January 1990 onwards and I end the sample in November 2015. The job loss rate is constructed to be consistent with the transition equation (1), i.e. as $\rho_{x,t} = \frac{u_t - (1-\rho_{f,t})u_{t-1}}{(1-\rho_{f,t})(1-u_{t-1})}$.[11]

The two upper panels of Figure 2 plot the two transition rates from CPS gross flow data. The lower panel plots the unemployment rate, as well as an approximation defined as $u_t^* \equiv \frac{\rho_{x,t}(1-\rho_{f,t})}{\rho_{x,t}(1-\rho_{f,t})+\rho_{f,t}}$, which is the unemployment rate that would prevail if the current transition rates, $\rho_{x,t}$ and $\rho_{f,t}$, would be permanently frozen at their current levels (see Hall (2005b)).

Figure 2 highlights two important and well-known observations that motivate my choice to estimate steady-state rates of unemployment based on forecasting equations for the transition rates (the "flow approach"). The first observation is that the time series properties of the job finding rate and the job loss rate are quite different. It is therefore likely that the two series convey distinct information about the state of the aggregate economy. The job finding rate, plotted in the upper panel, is subject to slow-moving fluctuations. Particularly striking is the slow recovery of the job finding rate after the sharp decline in during 2008. The job loss rate, plotted in the middle panel, displays much less persistence. The increase in $\rho_{x,t}$ around 2008 is also less persistent than the decline in $\rho_{f,t}$.

[11] Similar results are obtained for a data going back to 1970 (or even 1948). To this end, I construct transition rates based on unemployment duration data in the CPS, rather than the gross flow data. The details of this are presented in the Appendix.

Figure 2: Raw data.



Notes: Monthly data over the period January 1990 until November 2015. u_t is the civilian unemployment rate. $\rho_{f,t}$, is measured as the unemployment to employment rate in the CPS. For the computation of $\rho_{x,t}$ and u_t^*, see text. Shaded areas denote NBER recessions.

The second well-known observation is that there is a very tight link between the unemployment rate and the two transition rates. The bottom panel of Figure 2 shows that the unemployment rate approximation u_t^*, which is a function of only $\rho_{f,t}$ and $\rho_{x,t}$, closely matches the actual unemployment rate u_t for most of the sample period. Thus, the key to understanding of unemployment rate dynamics lies in the time series behavior of the two transition rates. Of course, there may be a two-way interaction between the transition rates and the unemployment rate. In fact, it is exactly such interaction that may give rise to multiple steady states.

Estimation method. Figure 2 suggests that measured transition rates are noisy. This is not very surprising, since CPS data are based on a survey among about 60,000 respondents, out of which only a small fraction experiences a change in employment status in a given month. The presence of i.i.d. noise can induce coefficient bias when estimating the forecast equations. To avoid such bias, I use an Instrumental Variables estimator, implemented through a standard Two Stage Least Squares procedure. As instruments, I use lags of the three variables, $\rho_{f,t-1}$, $\rho_{x,t-1}$, and u_{t-1}. For completeness, the appendix reports results obtained using Ordinary Least Squares. While there some small quantitative differences, the main findings are not affected.

2.3 Findings

This subsection presents the outcomes of the estimated reduced-form model. I first consider diagnostics statistics for the three models. Based on these, I select a baseline specification. Next, I present the estimated steady-state values for the unemployment rate and discuss the implications for unemployment dynamics. Finally, I discuss the extent to which the U.S. labor market reached a "Dark Corner" during the Great Recession.

Model selection. I first check for misspecification of the model. The left panel of Figure 3 plots the correlation between the (in-sample) forecast residuals in month t and in month $t+k+1$, for the three models.[12] Each of the two statistics is computed for a range of forecast horizons, between $k = 1$ and $k = 24$ months. Underlying each forecast horizon is a separately estimated direct multi-step forecasting equation for $\rho_{f,t+k}$. The figure shows that Models (I) and (II) produce positively autocorrelated residuals for a wide range of forecasting horizons and are thus invalidated. Model (III), by contrast, does not produce substantial residual autocorrelation at any forecast horizon.[13] The level of unemployment thus emerges as a key piece of information about the state of the economy, once non-linearities are accounted for. Omitting this piece of information leads to misspecification, and induces persistence in the forecast errors, i.e. residual autocorrelation.

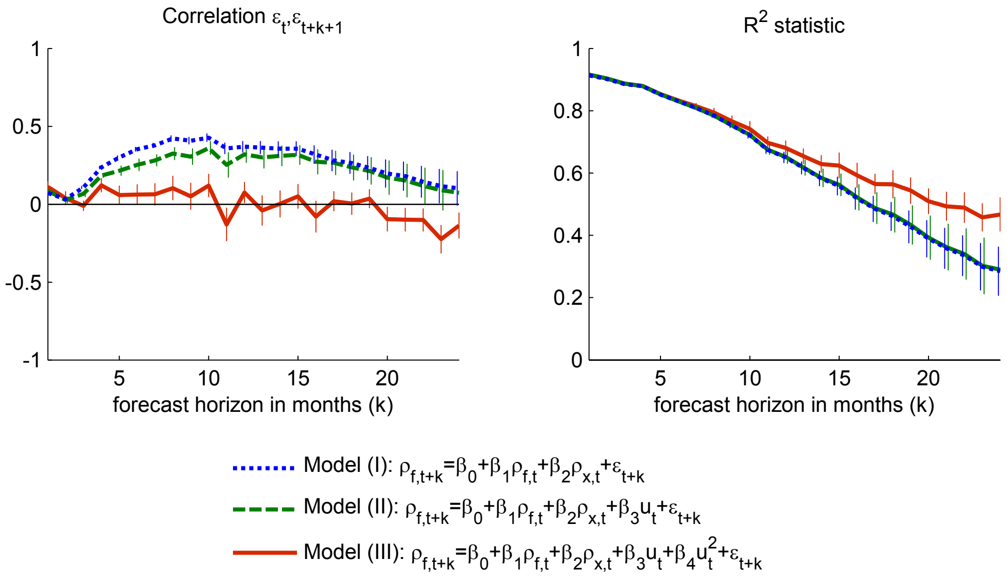
Allowing for non-linearities also improves the forecast accuracy of the model. This is shown in the right panel of Figure 3, which plots the R^2 statistic for the three specifications, again for forecast horizons between 1 and 24 months.[14] Especially at longer horizons, model

[12] The residuals are constructed as $\varepsilon_{t+k} = \mathbb{E}_t\rho_{f,t+k} - \beta\mathbf{z}_t$, where $\mathbf{z}_t$ is the vector of observables and β is the vector of estimated coefficients. Due to overlapping forecast horizons, residuals of closer time periods are generally correlated and hence not useful to diagnose misspecification.

[13] At a range of longer horizons, all models produce negative correlations. This, however is less concerning, since omitted variables typically do not produce negative autocorrelation in the residuals.

[14] The R^2 statistic is computed based on the residuals $\widehat{\varepsilon}_{t+k} = \mathbb{E}_t\rho_{f,t+k} - \beta\widehat{\mathbf{z}}_t$, where $\widehat{\mathbf{z}}_t$ is vector of the fitted

Figure 3: Diagnostic statistics.



Notes: Vertical bars denote bands of two standard deviations, obtained via a bootstrap procedure and centered around the point estimates.

(III) produces a better fit than the other two specifications.

Model (III) is the only model out of the three which survives the autocorrelation test and also delivers the best fit. In what follows, I will therefore use Model (III) as the baseline specification. As mentioned above, the Appendix shows that adding further higher-order terms or additional macro variables has little impact on the results. The baseline specification thus appears to extract enough information about the aggregate state from the observables, as already suggested by the lack of autocorrelation in the residuals.

Estimation results. I now evaluate what the estimated forecasting functions reveal about the possible presence of a "Dark Corner". Figure 4 visualizes the results by plotting u_t^* against u_{t+k}^*. Underlying each value of u_t^* on the horizontal axis is a pair of values for $\rho_{f,t}$ and $\rho_{x,t}$. For the sake of parsimony, I vary only $\rho_{f,t}$ and set $\rho_{x,t}$ to its estimated unconditional mean. Given u_t^*, $\rho_{f,t}$ and $\rho_{x,t}$, I then compute the corresponding values for u_{t+k}^* using the estimated forecasting functions, using the point forecasts for $\rho_{f,t+k}$ and for $\rho_{x,t+k}$.[15]

The four panels in Figure 4 plot these curves for four different forecast horizons equal to, respectively, $k = 6$, 12, 18 and 24 months. Regardless of the forecast horizon, the baseline

value from the first stage regression. This avoids a mechanical reduction in measured fit due to noise in the observations.

[15] Recall that $\rho_{x,t}$ is estimated to follow a linear AR(1) process. Given that I set $\rho_{x,t}$ to the estimated unconditional mean, the forecast for $\rho_{x,t+k}$ coincides with $\rho_{x,t}$.

model, Model (III), features two intersections with the 45-degree line, indicating at least two steady states. One of these is located around 5.5 percent and the other one around 9.5 percent. The shape of the curves imply that the steady state with low unemployment is stable, whereas the one with high unemployment is not. That is, the estimation results indicate the presence of a Dark Corner starting at about 10 percent unemployment. Beyond this level, unemployment is not forecasted to come down but rather to further increase. It follows that there must be a third steady state with even higher, possibly extreme unemployment. However, the data have little to say about the precise location of this third steady state: outside the range of values for unemployment observed in the data, confidence bands become very wide.

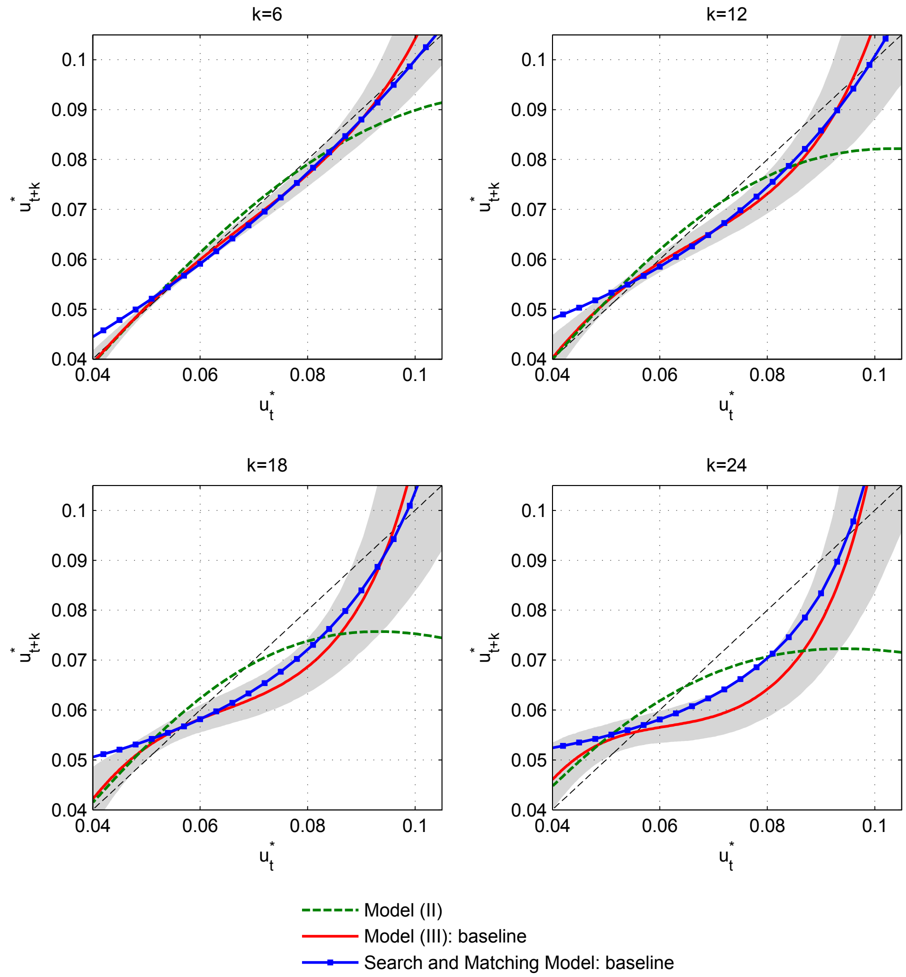
For purely illustrative purposes, Figure 4 also plots the corresponding curve for Model (II), even though this model was invalidated due to residual autocorrelation. This model has a single steady state over the range of unemployment rates observed in the data, which is stable and located between six and seven percent unemployment. The illustration clarifies that including unemployment in the forecasting equation does not mechanically leads one to conclude that multiple steady states are relevant for the labor market. Indeed, Model (II) does include the unemployment rate in the forecast and does have two steady-state solutions. However, the unstable solution features a negative unemployment rate and is therefore not relevant. Figure 4 also plots the corresponding curves for the estimated Search and Matching model. I will discuss these in Section 3.

Dynamics and Dark Corner. A more detailed view on the implied dynamics is provided by Figure 5, which presents a phase diagram for the estimated baseline model. In the literature, phase diagrams are often used to study the deterministic dynamics of theoretical models but they can be equally helpful to visualize dynamic patterns in the data. I simplify the exposition by studying a two-dimensional diagram with u on the horizontal axis and ρ_f on the vertical axis, setting ρ_x equal to its sample average as before.

The green solid line (the "u-nullcline") traces out combinations of ρ_f and u for which, according to Equation (1), u remains constant (i.e. setting $\Delta u = u_{t+1} - u_t = 0$). Similarly, the blue line (the "ρ_f-nullcline") traces out pairs of ρ_f and u, for which ρ_f in expectation stays constant according to its forecasting equation (i.e. setting $\Delta\rho_f = \mathbb{E}_t\rho_{f,t+k} - \rho_{f,t} = 0$). The grey dots represent observed data points over the sample period.

The two nullclines intersect exactly at the two steady-state points and divide the diagram into five segments with different forecasted directions of motion, indicated by black horizontal

Figure 4: Estimation results of the reduced-form model at different forecast horizons (k).

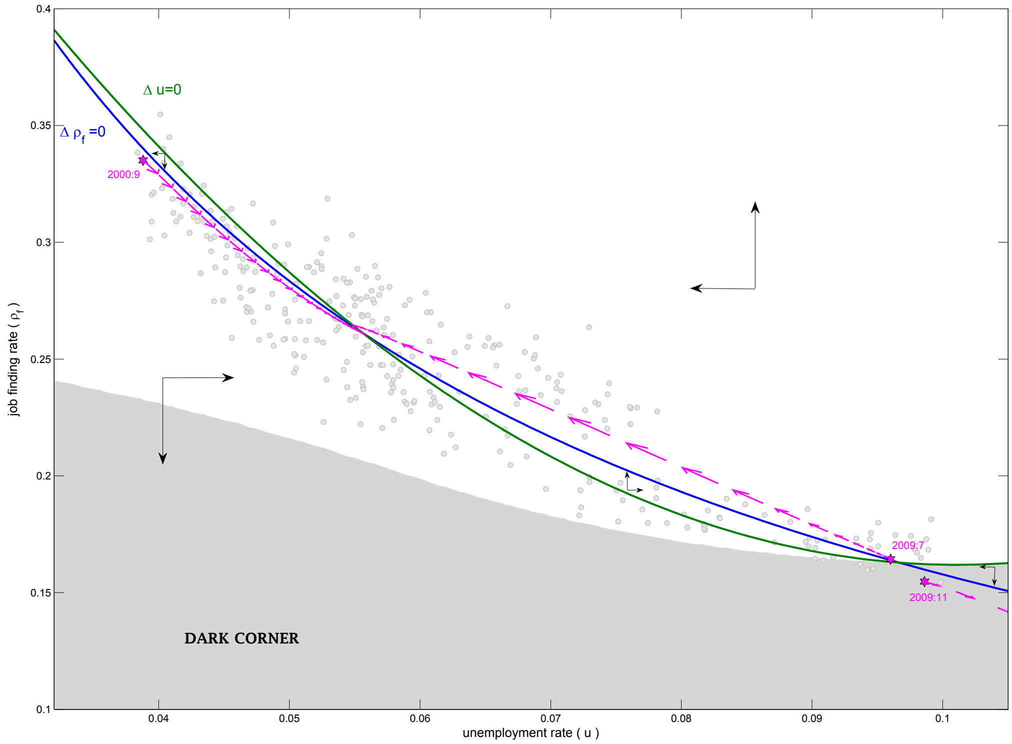


Notes: The figure plots $u_t^* \equiv \frac{\rho_{x,t}(1-\rho_{f,t})}{\rho_{x,t}(1-\rho_{f,t})+\rho_{f,t}}$ against u_{t+k}^*, where intersections with the 45-degree line indicate steady states. Here, u_t^*, is computed by constructing for a range of values for $\rho_{f,t}$, setting $\rho_{x,t}$ to its sample average $\overline{\rho}_x$. Next, u_{t+k}^* is computed for each value of u_t^* by using the forecasting model to evaluate $\rho_{f,t+k}$, again setting $\rho_{x,t+k} = \overline{\rho}_x$. The shaded areas plot 90 percent confidence bands for the model (III). These bands are uniform and have been computed based on a bootstrap method. The forecast horizon, k, is denoted in months.

and vertical arrows. The diagram confirms that the steady-state with around 5 percent unemployment is stable, whereas the steady-state with around 10 percent is unstable. Further, for most values of ρ_f and u, the two variables are forecasted to move in opposite directions, which is in line with the very strong negative co-movement between the two variables in the data. There are empirically relevant states in which the two variables co-move positively but as these zones are small so the economy tends to spend little time in these regions.

To further illustrate the dynamics implied by the estimated model, Figure 5 also plots

Figure 5: Phase diagram for the estimated reduced-form model.



Notes: phase diagram for the reduced-form model with $k = 36$, setting $\rho_{x,t}$ to its sample average. Grey dots denote observed data points. The blue and green solid lines denote the two nullclines. Pink arrows denote three example forecast paths, initialized at data points observed in September 2000, July 2009, and November 2009. The size of the arrows indicate the speed of transition. Black arrows denote forecasted directions of u_t and $\rho_{f,t}$. The shaded "Dark Corner" area denotes the set of observations for which the forecasted paths move away from the low-unemployment steady state. To convert the k-period ahead forecasts for $\rho_{f,t}$ into one-period ahead forecasts the following simple average is taken: $\frac{k-1}{k}\rho_{f,t} + \frac{1}{k}E_t\rho_{f,t+k}$.

three examples of forecasted paths. The starting values for u and ρ_f are chosen to correspond to actual data points, being September 2000, July 2009, and November 2009. The subsequent forecasted paths are computed by jointly iterating on Equation (1) and the estimated forecast equation. The paths are illustrated by pink arrows, varying in size to indicate the speed of transition: larger arrows correspond to faster transitions. The paths starting in September 2000 ultimately leads to the stable low-unemployment steady state, with a gradually declining speed of transition. The path starting in July 2009 leads to the same stable steady state. However, as the economy is initially close the unstable steady state with high unemployment, the recovery is initially slow. Gradually, the speed of recovery accelerates, reaching a maximum once the unemployment has declined to about 7.5 percent. Subsequently, the speed of recovery declines, as the economy reaches the low-unemployment steady state. Finally, consider the path starting in November 2009. Along this path, unemployment does not come down to the stable steady state, but instead diverges upward, possibly to an extreme level.

The notion of a "Dark Corner" is illustrated by the grey shaded area in Figure 5. This area represents the collection of starting values for which unemployment is not forecasted to revert back to the stable low-unemployment steady state. According to the model, the U.S. labor market entered the Dark Corner in September 2009 and in November 2009. Two remarks are appropriate here. First, measured job finding rates are somewhat noisy and the precise location of the Dark Corner is subject to estimation uncertainty. Second, and perhaps more importantly, entering the Dark Corner does not imply that unemployment will not come down. A benign shock may be sufficient to move the economy out of the Dark Corner. Such a shock may be fairly small, since in the proximity of the unstable steady state, deterministic dynamics are slow and hence the effects of shocks are relatively important. With these nuances in mind, the estimation results do suggest that during the Great Recession the U.S. economy was nearly drawn into a Dark Corner with high long-run unemployment.

Figure 5 also helps to understand intuitively what patterns in the data identify the Dark Corner. Specifically, one can observe two clusters of data points. One is centered around the stable steady state with around five percent unemployment, whereas the other one is centered around the unstable steady state at around ten percent unemployment. In between those clusters, there are relatively few data points. Also, unemployment never falls much below four percent. This suggests that in response to moderate shocks, the attraction force of the stable steady state quickly pulls back unemployment to around 5 percent. By contrast, the effects of large shocks that increase unemployment beyond, say, nine percent do not appear to be undone as quickly. This suggests that as unemployment reaches sufficiently high levels, the attraction force of the stable steady state is counterbalanced by a second attraction force which pulls unemployment towards the Dark Corner. Thus, the estimated Dark Corner does not derive just from the fact that the Great Recession triggered a large increase in unemployment, but also from the fact that episodes of either very low or moderately high unemployment have been relatively short lived.

3 Search and matching model

The estimates of the previous section suggest that structural models with multiple steady-state rates of unemployment may provide a better description of fluctuations in the U.S.

labor market than standard models with a single steady state. This section presents an explicit structural model of fluctuations in the labor market. Depending on the structural parameters, the model may feature either one or more steady states. I estimate these parameters using the same data as in the previous subsection and compute the steady state(s) of the estimated structural model. Thus, the structural estimation in this section complements the reduced-form approach of the previous section.

The model is based on a discrete-time version of the search and matching model of Pissarides (1985) and Pissarides (2000), extended in two dimensions. First, I introduce aggregate uncertainty. Specifically, I introduce aggregate shocks to the rate of job loss, which are taken directly from the data. The model is then evaluated on the basis of whether, given observed job loss rates, it can replicate observed job finding rates and unemployment rates. Secondly, I introduce a loss of skill upon unemployment, following Pissarides (1992), which gives rise to the possibility of multiple steady states.

The skill-loss mechanism is certainly not the only way to generate multiplicity of steady states. For example, Kaplan and Menzio (2014) present a model in which externalities in shopping behavior and aggregate demand generate multiplicity of equilibria as well as steady states. For my purpose, the model with skill losses has two specific advantages. First, it nests a standard DMP model with a unique steady state, which is obtained by setting a single parameter, which controls the degree of skill losses, to zero. The estimated skill loss parameter therefore allows one to gauge relatively easily how strongly –in an economic sense– model assumptions need to be altered in order to obtain steady-state multiplicity. Second, the skill loss mechanism preserves uniqueness of the (short-run) equilibrium. That is, there is no scope for sunspot equilibria, which facilitates estimation of the model as no assumptions on equilibrium selection need to be made. This allows me to focus exclusively on my main goal, which is to investigate the empirical relevance of steady-state multiplicity, rather than multiplicity of equilibria in general.

The remainder of this section is organized as follows. The model is described in Section 3.1. Section 3.2 discusses the properties of the equilibrium and explains under what conditions multiple steady states may arise. Section 3.3 discusses the estimation procedure, the estimated parameter values and the fit of the model. In Section 3.4 I present the key implications of the estimated model. Specifically, I quantify (i) the implied steady state values, (ii) the states of the world in which the economy is in a Dark Corner and (iii) endogenous fluctuations in unemployment uncertainty. Finally, I discuss the importance of endogenous persistence and linearities in generating the result, as well some policy implications.

3.1 Model

The economy is populated by a unit measure of risk-neutral workers who own the firms.

Workers. The transition structure and timing of the labor market are the same as in the reduced-form model. Employed workers lose their job with a probability $\rho_{x,t}$ at the very beginning of a period. This probability is exogenous, but subject to stochastic shocks, which are revealed when job losses occur. Subsequently, a labor market opens up to firms and to workers searching for a job. The pool of job searchers consists of those workers who just lost their jobs and those who were previously unemployed. The labor market is subject to search and matching frictions and only a fraction $\rho_{f,t} \in [0,1]$ of the job searchers meets with a firm. In the equilibrium, all workers who meet a firm become employed, so $\rho_{f,t}$ is also the job finding rate. It follows that the aggregate unemployment rate, u_t, evolves as in Equation (1). After the labor market closes, production and consumption take place. Unemployed workers obtain a fixed amount of resources b from home production, whereas employed workers receive wage income. Note that some job losers immediately find a new job, whereas others become unemployed.

As in Pissarides (1992), workers who become unemployed lose some skills. In particular, the productivity of any worker who is hired from unemployment is reduced by a certain, time-invariant amount χ in the initial period of re-employment. After being employed for one period, a worker regains her old productivity level. One can think of the productivity loss as the cost required to re-train a worker to become suitable for employment. The fraction of job searchers with reduced skills is denoted by p_t and equals the ratio of the number of previously unemployed workers to the total number of job searchers:

$$p_t = \frac{u_{t-1}}{u_{t-1} + \rho_{x,t}\left(1 - u_{t-1}\right)}. \tag{2}$$

Firms. On the supply side of the economy, there is a unit measure of identical firms who maximize the expected present value of net profits, operating a constant returns-to-scale technology to which labor is the only input. In order to hire new workers, firms post a number of vacancies, denoted v_t, which come at a cost $\kappa > 0$ per unit. I assume that firms can only observe the skill status of new hires only one period after the worker has worked at the firm for at least one period, which renders the search for new hires is entirely random. When choosing the optimal number of vacancies, firms take as given the stochastically fluctuating rate of job separations, $\rho_{x,t}$, the fraction of new hires with reduced skills, p_t, and the rate at which vacancies are filled, denoted q_t. The value of a firm, V, can be expressed recursively as:

$$
\begin{aligned}
&V(n_{t-1}, \mathcal{S}_t) = \max_{h_t, n_t, v_t} \overline{A} n_t - \chi p_t h_t - w_t n_t - \frac{\kappa}{q_t} h_t + \beta \mathbb{E}_t V(n_t, \mathcal{S}_{t+1}), \\
&\text{subject to} \\
&n_t = \left(1 - \rho_{x,t}\right) n_{t-1} + h_t, \\
&h_t = q_t v_t, \\
&h_t \geq 0,
\end{aligned}
$$

where n_t denotes the number of workers in the firm, $\mathcal{S}_t$ is the state of the aggregate economy, h_t is the number of new hires and w_t is the wage of a worker. I assume that firms have all the bargaining power.[16] As a result, firms pay worker their reservation wage, which equals the home production flow, i.e. $w_t = b$. Effectively, real wages are therefore sticky, as in for example Hall (2005a). The output of the firm is given by $\overline{A} n_t - \chi p_t h_t$, where $\overline{A} > b$ is a productivity parameter and the term $\chi p_t h_t$ is the total reduction in output due to new hires with reduced skills. The costs faced by the firms consist of two components. First, $w_t n_t$ is the baseline wage bill. Second, $\frac{\kappa}{q_t} h_t$ is the total cost of posting vacancies.

The first constraint in the firms' decision problem is the transition equation for the number of workers in the firm. The second constraint relates the number of vacancies to the number of new hires. The third constraint states that the number of new hires cannot be negative, preventing the firms from generating revenues by firing workers. In line with the empirical results, the rate of job loss is assumed to follow an AR(1) process:

$$
\rho_{x,t} = (1 - \lambda_x) \overline{\rho}_x + \lambda_x \rho_{x,t-1} + \varepsilon_{x,t},
$$

where bars denote steady-state levels, $\lambda_x \in [0, 1)$ is a persistence parameters and $\varepsilon_{x,t}$ is a normally distributed shock innovation with mean zero and standard deviation σ_x.

[16] This assumption considerably simplifies the solution and estimation of the model but is not necessary to obtain steady-state multiplicity.

Matching technology. Let the number of job searchers at the beginning of period t be denoted by $s_t \equiv u_{t-1} + \rho_{x,t}(1 - u_{t-1})$. Job searchers and vacancies are matched according to a Cobb-Douglas matching function, $m_t = s_t^{\alpha} v_t^{1-\alpha}$, where m_t is the number of new matches and $\alpha \in (0,1)$ is the elasticity of the matching function with respect to the number of searchers.[17] From the matching function it follows that the vacancy yield, $q_t = \frac{m_t}{v_t}$, and the job finding rate, $\rho_{f,t} = \frac{m_t}{s_t}$, are related as:

$$q_t = \rho_{f,t}^{\frac{\alpha}{\alpha-1}}. \tag{3}$$

The evolution of the aggregate employment rate is identical to the evolution of firm-level employment due to symmetry across firms.

Equilibrium. As shown in the Appendix, the firms' first-order optimality conditions deliver an Euler Equation for vacancy posting, which can be expressed as:

$$\chi p_t - \xi_t + \kappa \rho_{f,t}^{\frac{\alpha}{1-\alpha}} = \overline{A} - b + \beta \mathbb{E}_t \left(1 - \rho_{x,t+1}\right) \left(\chi p_{t+1} + \kappa \rho_{f,t+1}^{\frac{\alpha}{1-\alpha}} - \xi_{t+1}\right), \tag{4}$$

where ξ_t is the Lagrange multiplier on the constrained restricting hiring to be non-negative, which satisfies the Kuhn-Tucker conditions $\rho_{f,t} \geq 0$, $\xi_t \geq 0$ and $\xi_t \rho_{f,t} = 0$ at any point in time. The above equation is useful to characterize the equilibrium:

Definition. *An equilibrium is characterized by policy functions for the job finding rate, $\rho_f(\mathcal{S}_t)$, for the unemployment rate $u(\mathcal{S}_t)$, for the fraction of reduced-skill hires, $p(\mathcal{S}_t)$, and for the Lagrange multiplier $\xi(\mathcal{S}_t)$, which satisfy the unemployment transition equation (1), the equation for the fraction or reduced-skill hires (2), the vacancy Euler equation (4), the Kuhn-Tucker conditions $\rho_{f,t} \geq 0$, $\xi_t \geq 0$ and $\xi_t \rho_{f,t} = 0$, as well as the exogenous law of motion for $\rho_{x,t}$. The state of the aggregate economy can be summarized as $\mathcal{S}_t = \left\{\rho_{x,t}, u_{t-1}\right\}$.*

Note that u_{t-1} is a state variable only because it enters the the definition of p_t, Equation (2), which in turn enters the vacancy posting condition (4). In the absence of skill losses ($\chi = 0$), Equation (2) can be dropped from the model, eliminating p_t as a variable. Then, the equilibrium policy function for $\rho_{f,t}$ can be solved from a dynamic system containing only Equation (4) and the transition law for $\rho_{x,t}$. In this system, $\rho_{x,t}$ is the only state variable.

[17] In the literature, it is common to introduce a scaling's parameter in front of the matching function. However, in my application this parameter is isomorphic to the vacancy cost κ. I therefore normalize the scaling's parameter immediately to one.

Given a simulation for $\rho_{f,t}$ and $\rho_{x,t}$, and an initial level of unemployment, the path of the unemployment rate can be computed separately using Equation (1).

Relation to the reduced-form model. There is a close connection between the search and matching model and the reduced-form models of the previous section, due to the fact that the central Euler equation of the DMP model, Equation (4), is essentially a one-period ahead forecasting equation for the job finding rate, $\rho_{f,t+1}$. To see this connection more explicitly, let us set $\kappa = \alpha = \frac{1}{2}$ (this simplifies the analysis but is not important) and linearize Equation (4) around some interior deterministic steady state. After rearranging, the linearized equation can be written as:

$$\mathbb{E}_t\widehat{\rho}_{f,t+1} = \chi\widehat{p}_t - \beta\left(1-\overline{\rho}_x\right)\chi\mathbb{E}_t\widehat{p}_{t+1} + \widehat{\rho}_{f,t} - \beta\left(\chi p + \rho_f\right)\lambda_x\widehat{\rho}_{x,t},$$

where variables without time subscripts denote values in the deterministic steady state and hats denote deviations from the steady state. Without skill losses ($\chi = 0$), the equation reduces to a one-period-ahead forecasting function for the job finding rate which is a linear function of only $\rho_{f,t}$ and $\rho_{x,t}$, as in reduced-form Model (I). Importantly, the unemployment rate u_t is not a state variable and hence has no additional forecasting power. With skill losses, the unemployment rate becomes a state variable useful for forecasting the job finding rate, which enters nonlinearly via $p_{t+1} = \frac{u_t}{u_t+\rho_{x,t+1}(1-u_t)}$.

Section 2 presented reduced-form forecasting models for the job finding rate and showed that including the unemployment rate, in a non-linear way, helps forecasting the job finding rate. The above analysis makes clear that we can re-interpret these reduced-form regressions as speaking to the Euler equations of the DMP models, favouring the version with skill losses. The relation between reduced-form regressions and Euler equations is similar in spirit to Hall (1978), who also scrutinized Euler equations using reduced-form forecasting regressions.

3.2 Equilibrium properties

This subsection discusses the equilibrium properties of the model. I first show that the equilibrium is unique and then discuss how the existence of multiple steady states depends on parameter values.

Uniqueness of the equilibrium. It is straightforward to show that the equilibrium of the model is unique. To this end, let us express the Euler equation for vacancies, Equation (4), as:

$$\kappa\rho_{f,t}^{\frac{\alpha}{1-\alpha}} - \xi_t = \sum_{k=0}^{\infty} \Lambda_{t,t+k}\left(\overline{A} - b\right) - \chi p_t, \tag{5}$$

where $\Lambda_{t,t+k} \equiv \beta^k \mathbb{E}_t \prod_{i=1}^{k}\left(1 - \rho_{x,t+i}\right)$ is the rate at which firms discount future profits, accounting for the survival probability of the match. The above condition equates the marginal costs of hiring to the marginal benefits. On the left-hand side, $\kappa\rho_{f,t}^{\frac{\alpha}{1-\alpha}} = \kappa/q_t$ represents expected vacancy cost associated with hiring an additional worker, and ξ_t is the Lagrange multiplier on the constraint that hiring cannot be negative. On the right-hand side, $\sum_{k=0}^{\infty} \Lambda_{t,+k}\left(\overline{A} - b\right) - \chi p_t$ is the expected net present value of profits that a worker will generate, taking into account that a fraction p_t of new hires have suffered a skill loss.

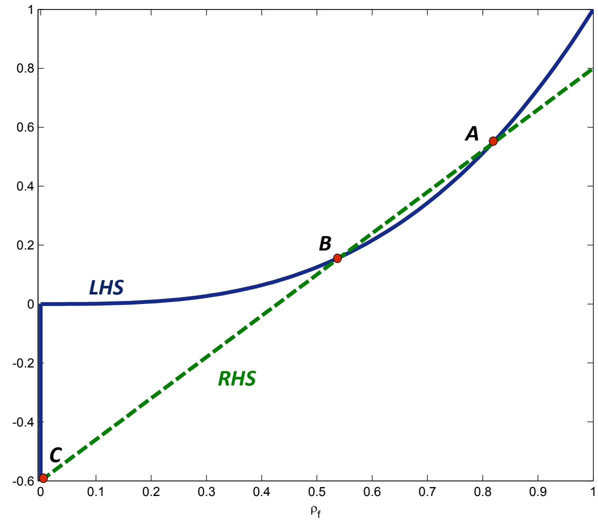
The first step in demonstrating uniqueness of the equilibrium is to note that the right-hand side of Equation (5) is exclusively pinned down by the two state variables of the model: the exogenous rate of job loss $\rho_{x,t}$ and the previous unemployment rate u_{t-1}. Specifically, the current level $\rho_{x,t}$ pins down the present value $\sum_{k=0}^{\infty} \Lambda_{t,+k}\left(\overline{A} - b\right)$, whereas from Equation (2) it can be seen that p_t is a function of only the two state variables, $\rho_{x,t}$ and u_{t-1}.

It is now useful to distinguish between two cases. First, suppose that the right-hand side is strictly positive. It follows from Equation (1) and the Kuhn-Tucker conditions that $\xi_t = 0$ and that the job finding rate is pinned down uniquely as $\rho_{f,t} = \left(\sum_{k=0}^{\infty} \Lambda_{t,+k}\left(\overline{A} - b\right)/\kappa - \chi p_t/\kappa\right)^{\frac{1-\alpha}{\alpha}} > 0$. Given $\rho_{f,t}$ and the state variables, Equation (1) can then be used to solve for u_t. Next, consider the complementary case in which the right-hand side of Equation (4) is negative. From the Kuhn-Tucker conditions it follows that in this case $\xi_t = \sum_{k=0}^{\infty} \Lambda_{t,t+k}\left(\overline{A} - b\right) - \chi p_t \geq 0$ and $\rho_{f,t} = 0$. Again, u_t can then be found using Equation (1). From the fact that in both cases we can solve uniquely for $\rho_{f,t}$ and u_t, given the state variables $\rho_{x,t}$ and u_{t-1}, it follows that the equilibrium is unique.

Multiplicity of steady states. While the equilibrium is always unique, there can be multiple steady states. I now illustrate how the existence of multiple steady states depends on parameter values, focusing on cases that are most relevant in light of the estimation results. A more complete and formal discussion of multiplicity of steady states in a model with skill losses can be found in Pissarides (1992).

Consider the deterministic steady state of the model, i.e. the steady state of a version

Figure 6: Multiplicity of steady states: illustration.



in which we shut off the aggregate shocks ($\sigma_x = 0$). Let upper bars denote steady-state values. It is straightforward to verify that in any steady state it holds that $\overline{p} = 1 - \overline{\rho}_f$. Let $LHS = \kappa \rho_f^{\frac{\alpha}{1-\alpha}} - \xi$ and $RHS = \sum_{k=0}^{\infty} \Lambda_{+k} \left(\overline{A} - b\right) - \chi \left(1 - \overline{\rho}_f\right)$ be, respectively, defined as the left- and right-hand side of Equation (5) in the steady state.

Figure 6 illustrates LHS and RHS as a function of $\overline{\rho}_f$, for a case in which $\alpha > \frac{1}{2}$ and $\chi > 0$. For positive values of $\overline{\rho}_f$, LHS is a convex and monotonically increasing function. For $\overline{\rho}_f = 0$, the left-hand side is not uniquely determined. When $\overline{\rho}_f$ equals zero, the only restriction is that that $\overline{\xi} \geq 0$. The right-hand side of the equation is a linearly increasing function of $\overline{\rho}_f$. In the illustration, there are three steady-state points. Steady-state A has the highest job finding rate (and thus the lowest unemployment rate). Steady state B has a lower but positive job finding rate, whereas steady state C features a zero job finding rate. It follows that steady states B and C are interior steady states with $\overline{u} \in (0, 1)$ whereas steady state C features hundred percent unemployment ($\overline{u} = 1$).

To better understand the properties of the steady state, note that the effective costs of hiring an additional worker consist of two components. The first is related to costs of posting vacancies whereas the other derives from reduced productivity due to skill losses. In steady state A, the vacancy filling rate is relatively low and therefore the vacancy component of hiring costs is relatively high. On the other hand, the unemployment rate is relatively low in this steady state, which implies that a relatively low fraction of new hires has reduced skills. In steady states B the opposite is true: the vacancy component of hiring costs are relatively low, but because unemployment is high, the implicit cost of reduced productivity due to skill losses is high (both relative to steady state A). On net, however, the hiring cost is the same in both steady states. In steady state C, the vacancy cost of hiring reduces to zero, but all workers need to be re-trained. In the illustration, the latter cost is high enough for firms not to post any vacancies.

Without skill losses, hiring costs have only one component (vacancies) and there can only be one interior steady state. When χ equals zero, RHS no longer depends on $\overline{\rho}_f$. Given that, for $\overline{\rho}_f > 0$, the left-hand side is monotonically increasing in $\overline{\rho}_f$, there can then be maximally one solution.

A special case with maximally one interior steady state (even with skill losses) arises when the matching function elasticity, α, equals exactly one half. In that case, LHS becomes linear in $\rho_{f,t}$, ruling out multiple interior intersections.

3.3 Estimation

This subsection estimates the parameters of the search and matching model and infers from this the implied steady states. To illuminate the mechanism behind the results, a version without skill losses is also estimated.

Estimation procedure. The model period is set to one month, in line with the frequency of the data. Two parameter values are set a priori. The subjective discount factor, β, is set to imply an annual real interest rate of 4 percent. The productivity of a worker, $\overline{A}$, is normalized to one.

The remaining parameters are estimated. The parameters of the exogenous process for the job loss rate, $\rho_{x,t}$, are obtained by estimating an AR(1) process based on its observed counterpart in the data. As mentioned above, transition rates are rather noisy and I therefore smooth them using a 3 month moving average filter. To take out any slow moving trend, the series is put through the Hodrick-Prescott filter with a smoothing coefficient equal to $81 \cdot 10^5$. This value corresponds to one used by Shimer (2005) for quarterly data, but is converted to the appropriate monthly value using the adjustment factor recommended by Ravn and Uhlig (2002). The estimated persistence parameter is computed based on the autocorrelation of the filtered series at a one-year horizon. Given this parameter value, the shock innovations are backed out using the AR(1) equation. I set $\overline{\sigma}_x$ equal to the standard deviation of these shock innovations over the sample. The estimated values of $\overline{\rho}_x$, λ_x and σ_x are, respectively, 0.021, 0.896 and $6.95e^{-4}$.

The parameters α, χ, κ and b, are estimated using an indirect inference procedure. To this end, I construct a grid for these parameters. For each set of values on the grid, I simulate the model, feeding in job loss shocks to replicate exactly the job loss rates observed in the data over the period January 1990 until November 2015. Each simulation is initialized using the unemployment rate and job loss rate observed in January 1990. Next, I compare the simulated job finding rate series, labeled $\widehat{\rho}_{f,t}$ to the time series for the actual job finding rate in the data. I select the parameter values that minimize Root-Mean-Squared-Error criterion $RMSE \equiv \sqrt{\frac{1}{T}\sum_{t=1}^{T}\left(\rho_{f,t} - \widehat{\rho}_{f,t}\right)^2}$. To illustrate the importance of skill losses, I also estimate a restricted version of the model without skill losses, setting $\chi = 0$. Throughout the estimation, α is restricted to be within the 0.5-0.7 range recommended by Petrongolo and Pissarides (2001), based on an extensive survey of empirical studies.[18,19]

Parameter estimates. The top part of Table 1 presents the parameter estimates. First consider the baseline model. The matching function elasticity, α, is estimated to be around 0.65, in line with conventional estimates. The estimate for b is a fairly high, see also Hagedorn and Manovskii (2008), and implies that an employed, fully skilled worker generates a profit flow of about one percent for the firm. The skill loss parameter χ is estimated to be almost precisely 0.5, which is equivalent to two weeks of the output generated by an employed worker. This value may seem small at face value.[20] However, given that a firm makes only a small profit from employing a worker, the skill loss has nonetheless a large effect on the firms'

[18] For the baseline model, this restriction turns out to be irrelevant. In the model without skill losses, however, α would otherwise be driven to an implausibly low value of 0.14, which would complicate a comparison to the baseline. At the end of Section 3.4, I will discuss the role of α in more detail.

[19] In each step of the estimation procedure, the model needs to be solved non-linearly and simulated given a set of parameter values. Towards this end, I exploit two features of Equation (5). First, one can express the first term on the right-hand side as $\Lambda\left(\rho_{x,t}\right)\left(\overline{A} - b\right)$, where $\Lambda\left(\rho_{x,t}\right)$ is a function defined as $\Lambda\left(\rho_{x,t}\right) \equiv \beta^k \mathbb{E}_t \prod_{i=1}^{k}\left(1 - \rho_{x,t+i}\right)$, given an initial value of $\rho_{x,t}$. Since $\Lambda\left(\rho_{x,t}\right)$ does not depend on any of the parameter values to be estimated, it can be computed before searching over the parameter values. With this function at hand, it is straightforward to solve and simulate the model. Given u_{t-1} and $\rho_{x,t}$, we can compute p_t and $\Lambda\left(\rho_{x,t}\right)$, and hence the entire right-hand side of Equation (5). It is then straightforward to solve for $\rho_{f,t}$, ξ_t and u_t.

[20] To get a better sense the magnitude, a comparison to the empirical findings in Davis and von Wachter (2011) may be useful. They estimate that the present value cost of job displacements, to a *worker*, varies between 1.4 and 2.8 *years* of predisplacements earning. These numbers do not directly speak to my estimate as χ is the portion of the cost of job loss that fall upon the subsequent *employer* rather than the worker. As far as I am aware, there is little direct empirical evidence on this number. That said, given that job loss appears to create very large externalities on the worker, it does not seem implausible that there is also some externality on the subsequent employer.

Table 1: Estimation results search and matching models.

		baseline	no skill losses
I. estimated parameter values			
matching function elasticity	α	0.651	0.500
skill loss	χ	0.499	0
vacancy cost	κ	1.035	1.000
flow from unemployment	b	0.989	0.994
II. model fit			
$RMSE$		0.489	0.975
R^2		0.764	0.099
1y autocorrelation $\rho_{f,t}$ relative to data		0.995	0.145
III. steady states			
unemployment rate s.s. A (stable)	$\overline{u}^A$	0.0548	0.058
unemployment rate s.s. B (unstable)	$\overline{u}^B$	0.1016	–
unemployment rate s.s. C (stable)	$\overline{u}^C$	1	–

inventive to post vacancies.

The middle part of Table 1 reports the $RMSE$ of the job finding rate in the model, relative to the actual data. To facilitate the interpretation of the magnitude, the number has been scaled by the standard deviation of the observed job finding rate over the sample. In the baseline model, the $RMSE$ is about one half of a standard deviation. An alternative measure of fit is obtained by running a linear regression of the form $\widehat{\rho}_{f,t}= \gamma_0+\gamma_1\rho_{f,t}+\epsilon_t$, i.e. a regression of job finding rate observed in the data on a constant and the model-predicted job finding rate over the sample. The R^2 of this regression is about 0.764. To examine the degree of persistence in the model vis-à-vis the data, I compute the autocorrelation of the job finding rate at a one year horizon. In the model, the autocorrelation coefficient is 0.834, which is only one percent lower than its counterpart in the data (0.842).

The right column of Table 1 reports estimated parameter values for a version of the model without skill losses ($\chi = 0$). The values of κ and b are similar to the baseline. The $RMSE$, however, is twice as high as in the baseline model. The R^2 for this model version is 0.099. Thus, without skill losses the model can explain less than ten percent of the observed fluctuations in the job finding rate. This model further fails to account for the degree of persistence that is observed in the data. The one-year autocorrelation in the job finding rate is only 0.122, which is about 85 percent lower than in the data.

To further illustrate the fit of the models, Figure 7 plots the simulated time and actual time paths of $\rho_{x,t}$, $\rho_{f,t}$, and u_t. By construction, the time path of $\rho_{x,t}$ is the same as in the data. The job finding rate series predicted by the model with skill losses is strikingly similar to its counterpart in the data. During certain periods there is a discrepancy in the levels of the two series, but even then the dynamics are close. For the years after 2010, the model with skill losses matches almost perfectly the job finding rate in the data. The low job finding rate over this period is ultimately driven by a spike in the job loss rate during 2008 and 2009. By 2011, however, the rate of job loss has returned to its pre-crisis level. The fact that the job finding rate remains persistently low, highlights the strong endogenous propagation mechanism of the model. By contrast, the model without skill losses produces hardly any fluctuations in the job finding rate. As a result, this model fails to account for the large and persistent increase in unemployment following the Great Recession. The mild increase in unemployment that the model does produce, is largely a direct effect of the job loss shocks.

The quantitative success of the baseline model derives from its strong endogenous persistence mechanism. Via this mechanism, a short-lived wave of job losses creates long-lasting effects on unemployment. This happens as the job losses increase unemployment, and therefore the incidence of skill losses. This discourages firms to post vacancies and, as a result, the labor market remains depressed for a sustained period.[21]

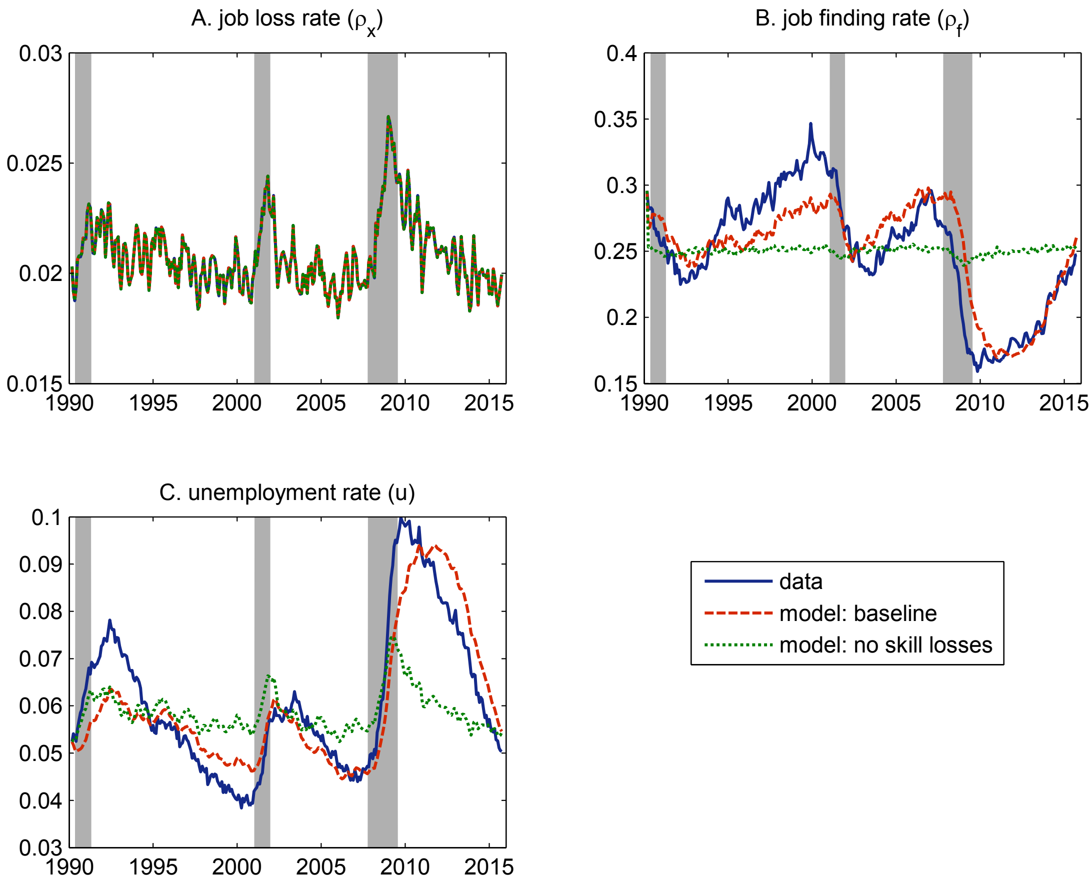
3.4 Implications of the estimated model

This subsection presents the key results implied by the model, estimated over the period 1990-2015. The Appendix shows that similar results are obtained for the period 1970-1990.

Multiple steady states. With the parameter estimates at hand, one can compute the steady states of the model. The deterministic-steady-state rates of unemployment are presented in the three bottom rows of Table 1. The baseline model has three steady states, labeled A, B and C. Steady state A features an unemployment rate of about 5.5 percent. By inspecting the dynamics of the model around this steady state, it can be verified that this steady state is stable. Steady state B is unstable, and features an unemployment rate of about 10.2 percent. These results are much in line with the estimates from the reduced-form

[21] These results are consistent with the reduced-form evidence presented in Fujita and Ramey (2009), who document that job loss rates leads unemployment and job finding rates, suggesting that standard accounting decompositions of unemployment fluctuations may understate the economic importance of fluctuations in the job loss rate.

Figure 7: Search and matching models versus data



Notes: shaded areas denote NBER recessions.

model.

The baseline model further features an exterior steady state with 100 percent unemployment. Clearly, this is an extreme prediction. It would be straightforward, however, to extend the model to generate a more reasonable high-unemployment steady state. To this end, one would need to introduce some mechanism which pushes down hiring costs when the unemployment rate becomes very high. For example, it might be reasonable to assume that wages become more flexible when unemployment reaches very high levels. In this paper, however, I refrain from such extensions given that in the data series for the U.S. there is no information on where the third steady state would be located. For other countries which did experience sustained periods of extreme unemployment, such as Spain or Greece, it might be possible to identify the location of a third steady state with very high unemployment.

In the restricted model without skill losses, there is by construction only one steady state.

This steady state is estimated to be located at 5.9 percent and is stable. As noted above, however, this model fails to explain observed job finding rates and unemployment rates by a very wide margin.

Figure 4 illustrates the "expectational steady states" of the skill-loss model, as indicated by the intersections with the 45-degree lines. The figure makes clear that the skill-loss model closely lines up with the baseline reduced-form model. Further, the expectational steady states are very similar to the deterministic steady states shown in Table 1: there is a stable steady state with around 5 percent unemployment and an unstable one around 10 percent unemployment.

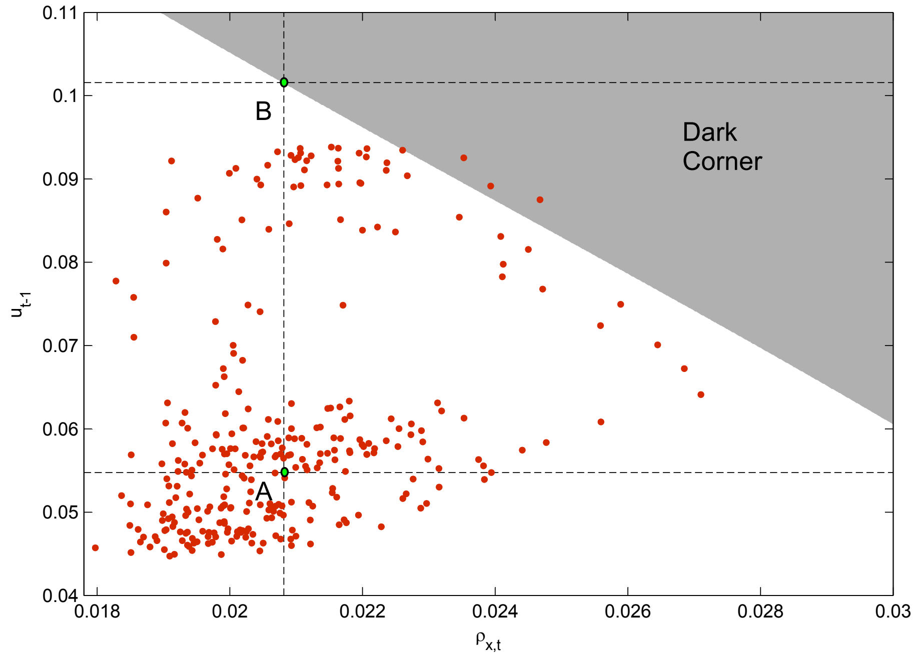
Dark Corner. Figure 8 illustrates how close the labor market came to a Dark Corner, according to the estimated baseline model. On the axes of the figure are the two state variables, $\rho_{x,t}$ and u_{t-1}. The white shaded area captures states for which, in the absence of further shocks, the economy will converge to the low-unemployment steady state A. The grey area illustrates the Dark Corner, i.e. states for which the economy diverges towards the high-unemployment steady state C. This happens for a sufficiently high job loss rate and/or unemployment rate.

The red dots represent data points from the estimated model. Three of these are just within the Dark Corner region of the state space. Despite moving briefly into this region, the economy escaped a transition towards very high unemployment due to the occurrence of benign shocks. Note also that there is a cluster of data points just outside the Dark Corner region. In this part of the state space, unemployment is expected to come down, but the transition is slow.

Time-varying uncertainty. Due to its non-linearities, the model generates fluctuations in unemployment uncertainty. Specifically, forecast uncertainty about unemployment moves countercyclically over the business cycle. To illustrate this point, Figure 9 plots the interquartile range (75th minus 25th percentile) of unemployment rate forecasts one year ahead. In the baseline model, forecast uncertainty is countercyclical, increasing particularly sharply during the Great Recession. Without skill losses, the model generates almost no fluctuations in forecast uncertainty.

The drivers behind the countercyclical uncertainty in the baseline model can be understood as follows. During times of moderate unemployment, the tendency of the economy to

Figure 8: Dark corner in the estimated model.



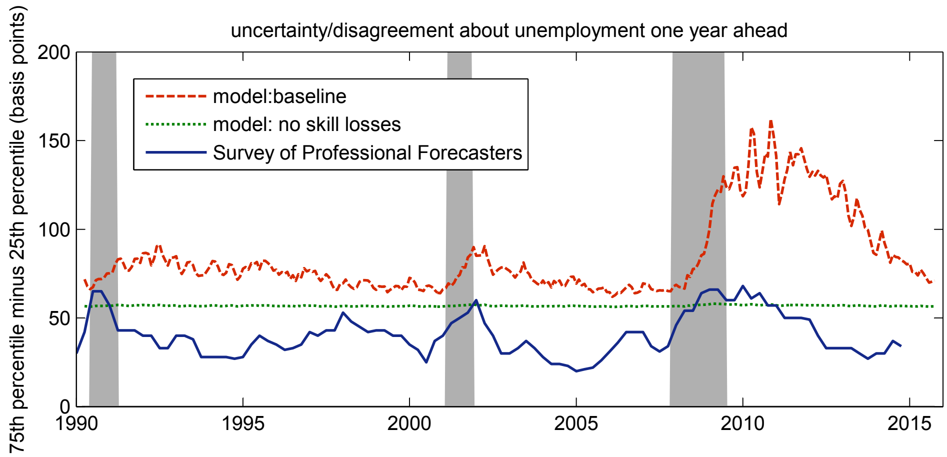
Notes: Illustration of the deterministic dynamics in the estimated baseline model, i.e. setting all future shocks to zero. The white area denotes states for which unemployment will converge to the low-unemployment steady state, *A*. The grey shaded area indicates states for which unemployment will diverge towards the high-unemployment steady state, *C*.

revert back to the low-unemployment steady state helps to forecast unemployment. As unemployment increases, however, there is an increased probability that the economy is drawn into the Dark Corner region. If this happens, unemployment will have a tendency to increase. At the border of the Dark Corner region, unemployment can thus take two opposite directions, depending on the particular realization of shocks. This gives rise to particularly high forecast uncertainty. In the model without skill losses, by contrast, there is no unstable steady state and forecast uncertainty is roughly constant over time.

Figure 9 also plots the dispersion of one-year unemployment rate forecasts observed in the Survey of Professional Forecasters. While this series measures the dispersion across *forecasters* rather than across *forecasts*, it nonetheless provides suggestive evidence that uncertainty about aggregate unemployment increases during recessions, as predicted by the model.[22]

[22] Bachmann, Elstner, and Sims (2013) provide firm-level evidence that ex ante forecast disagreement is correlated with dispersion in ex post forecast errors. Jurado, Ludvigson, and Ng (2015) present evidence that macro forecast uncertainty is countercyclical.

Figure 9: Unemployment uncertainty.



Notes: The red dashed line shows the difference between the 75th and 25th percentiles of unemployment rate forecasts one year ahead, in the baseline model. The green dotted line shows the same variable for the model without skill losses. In both models, unemployment forecasts were generated by Monte Carlo simulations, each initialized at data points from the estimated model. The blue solid line plots the difference between the 75th and 25th percentiles of unemployment rate forecast one year ahead, across forecasters in the Survey of Professional Forecasters. Shaded areas denote NBER recessions.

The interaction between non-linearities and endogenous persistence. The emergence of multiple steady states is due to the interaction of a strong endogenous persistence mechanism and a moderate non-linearity. To appreciate this point, recall that without skill losses, the model has no endogenous persistence and, by construction, multiple interior steady states cannot arise. Similarly, when the matching function elasticity, α, is set exactly to one half, the left-hand side of the vacancy posting condition, equation (5) becomes linear and multiple steady states are ruled out as well. Indeed, χ has been estimated to be larger than zero, and α to be larger than one half.

Above it has been shown that a model without endogenous persistence ($\chi = 0$) fails to account for the data by a wide margin. To investigate the quantitative importance of the non-linearity, I re-estimated the model (with skill losses) setting α equal to exactly one half. While this knife-edge configuration rules out multiplicity of steady states by construction, it does allow for endogenous persistence. For this version, I estimate $\chi = 0.85$ and a steady-state rate of unemployment of 5.98 percent. As anticipated, the fit of this model is worse than the baseline. In particular, the scaled $RMSE$ is 0.58, versus 0.49 in the baseline. This reduction in model fit stems mainly from the recovery period after the Great Recession, which ended in

July 2009, according to the NBER. For the post-recession period, the $RMSE$ is 0.46 in the model with α equal to one half, versus only 0.21 in the baseline model. I conclude that, while the introduction of endogenous persistence alone can create a strong improvement in model fit, allowing in addition for some degree of non-linearity in the vacancy posting condition is important to account for the slow recovery after the Great Recession.

Policy. While the purpose of this study is to provide a positive rather than a normative analysis, it is natural to ask what kind of government policy may undo the macro effects of skill losses and prevent a Dark Corner. In the model, a straightforward way to achieve this is to introduce a time-varying subsidy on new hires, financed by a lump-sum tax on households. Let the subsidy be denoted by τ_t. In the presence of this subsidy, Equation (5) becomes:

$$\kappa\rho_{f,t}^{\frac{\alpha}{1-\alpha}} - \xi_t = \sum_{k=0}^{\infty} \Lambda_{t,t+k} \left(\overline{A} - b\right) - \chi p_t + \tau_t.$$

It is now clear that when setting $\tau_t = \chi p_t$, the model effects of skill losses is neutralized and the model becomes observationally equivalent to the basic DMP model. Recall that p_t is an increasing function of the unemployment rate. The policy thus involves a *countercyclical* subsidy.

4 Concluding remarks

The main finding of this paper is that models with multiple steady-state rates of unemployment can provide an empirically compelling description of U.S. labor market dynamics over the business cycle. I have shown this by estimating a reduced-form model as well as a search and matching model, both allowing for the possibility of multiple steady states. Although I am not aware of any prior research that tries to estimate steady-state rates of unemployment in similar ways, various authors have proposed alternative models in which multiple steady states can arise.[23] Distinguishing empirically between such models is important, given that government policies can have dramatic impacts if they can prevent the economy from slipping into a Dark Corner with high long-run unemployment.

[23] See for example Rochetau (1999), Den Haan (2007) and Ellison, Keller, Roberts, and Stevens (2014), in addition to references mentioned in the introduction of this paper.

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Appendix

A. Reduced-form model: robustness exercises

This appendix presents the following results for the reduced-form framework of Section 2.1:

1. Additional cubic term;
2. AR(1) specification;
3. Alternative data source (duration-based CPS data);
4. Longer data sample (1960-2014);
5. Additional macro variables / unobserved states;
6. Alternative estimator (OLS);
7. Additional lags;
8. Specification for the job loss rate.

1. Additional cubic term. I add a cubic term to the specification baseline reduced-form model. In particular,in Model (IV), $\mathbb{E}_t\rho_{f,t+k}$ is a linear function of $\rho_{f,t}$, $\rho_{x,t}$, u_t, u_t^2, u_t^3 and a constant. Figure A1 shows that the model diagnostics for model (IV) are very similar to those for the baseline, Model (III). Thus, the addition of the cubic term has little effect. For parsimony, I select model (III) as the baseline.

Figure A1: Diagnostics statistics: additional higher-order terms and AR(1)

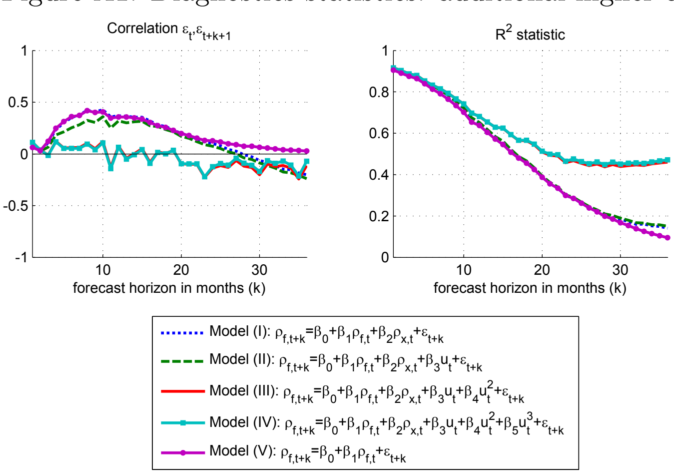
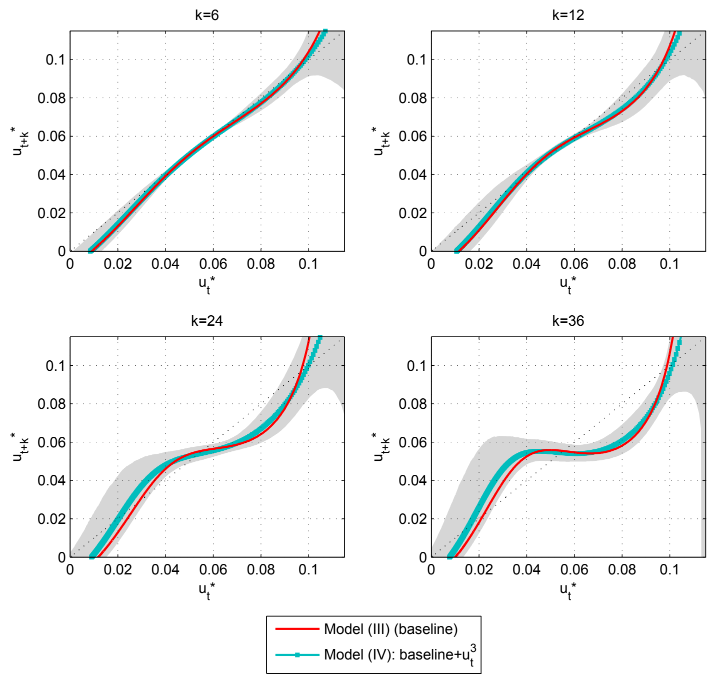


Figure A2: Steady states: additional cubic term



2. AR(1) specification. Next, I consider a model that is even simpler than models (I)-(IV). Specifically, in Model (V) $\mathbb{E}_t\rho_{f,t+k}$ is a linear function of only $\rho_{f,t}$ and a constant, which corresponds to a simple AR(1) process. Figure A2 also presents model diagnostics for this specifications are similar to those of models (I) and (II): there is substantial autocorrelation, invalidating the AR(1) specification.

3. Alternative data source (duration-based CPS data) Next, I re-estimate the model based on duration-based data from the CPS rather than the gross flow data. Following Shimer (2005), the number of workers who have been out of work for less than a month is used in combination with unemployment data to compute the job finding rate. To account for the trend present in this series, I take out a linear trend, estimated over the period 1948-2007. Next, I re-construct the level by adding the average the average of the period January 1990-2007. Finally, I compute the job loss rate as for the gross flow data. That is, the job loss rate is constructed to be consistent with the unemployment rate, the (de-trended) job finding rate, and the transition identity for unemployment, Equation (1).

Figures A3 and A4 show, respectively, the diagnostics statistics and the implied steady-state curves. As for the gross-flow data, Model (III) emerges as the only specification without substantial positive autocorrelation in the residuals. The implied steady-state curves are also very similar.

Figure A3: Diagnostics statistics: duration-based CPS data (1990-2015)

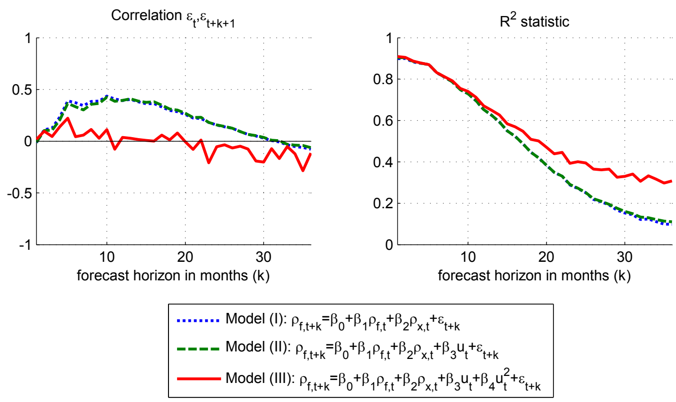
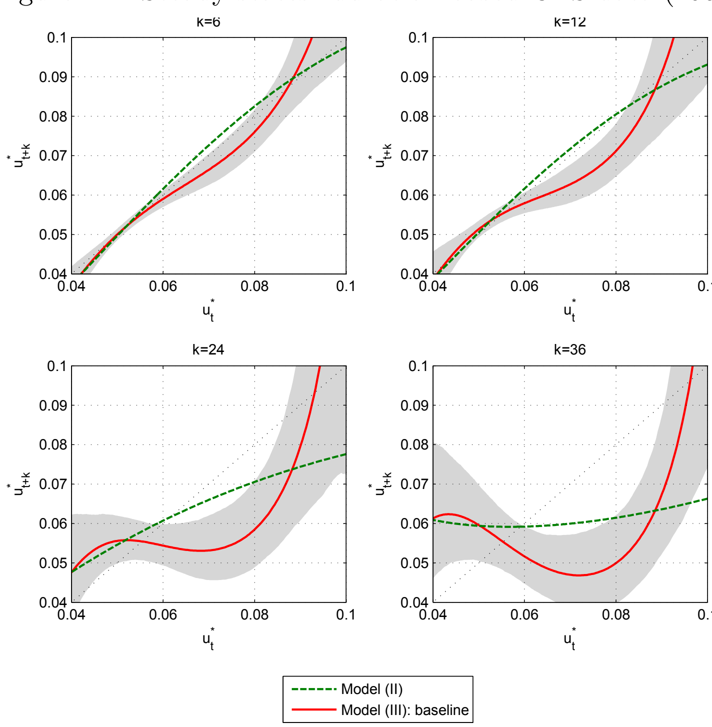


Figure A4: Steady states: duration-based CPS data (1990-2015)

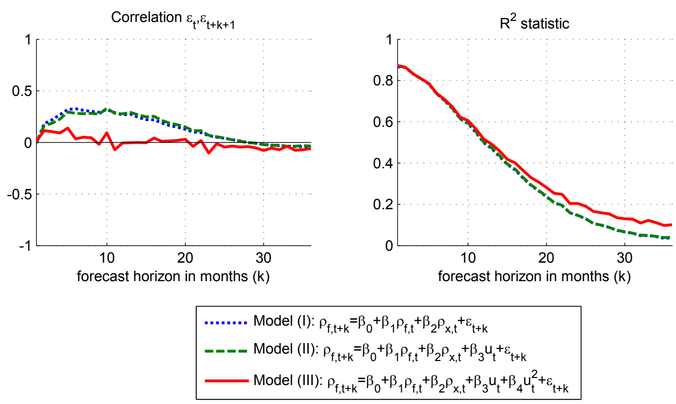


4. Longer data sample (1970-2015). Using the duration-based data, I repeat the exercise over a longer sample starting in 1970 (the gross flow data start only in 1990), see Figures A5 and A6.[24] Again, Model (III) is the only model which survives the autocorrelation test. The

[24]The duration-based can be constructed back to 1948. However, since in the period 1948-1970 the unemployment rate never exceeds 8 percent, the pre-1970 data points have little to say about the possibility of a second steady state. I therefore omit these data points. I have verified that result presented below is not

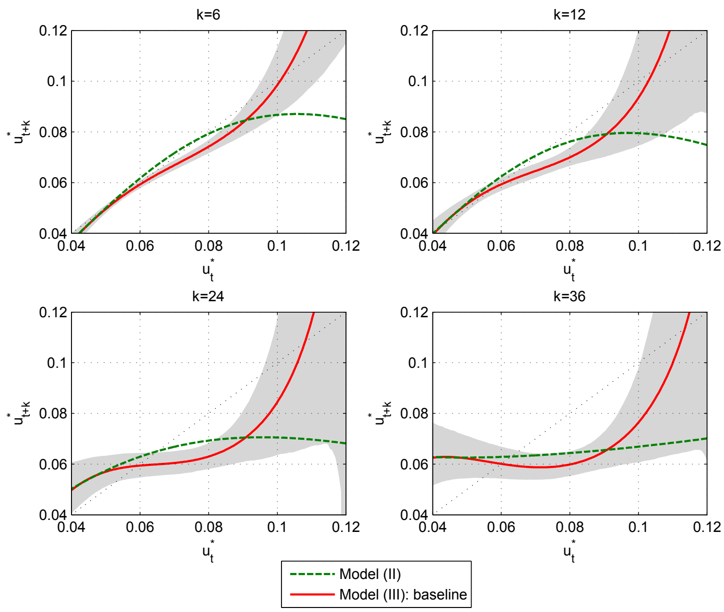
improvement in the R^2 statistic is smaller than in the post-1990 sample, suggesting that most of the forecast improvement comes from the Great Recession period, which receives a smaller weight in the longer sample. The implied steady-state points are again similar to the baseline, although the estimated unstable steady state levels of unemployment are somewhat higher. That said, the estimates also seem less precise, as the confidence bands are substantially larger, especially for the 24 and 36 month ahead forecast specification.

Figure A5: Diagnostics statistics: duration-based CPS data (1970-2015)



substantially altered by adding the 1948-1970 data.

Figure A6: Steady states: duration-based CPS data (1970-2015)



5. Additional macro variables The baseline specification appears to extract enough information about the aggregate state, as judged by the lack of residual autocorrelation. Nonetheless, I check if results change if additional macro variables are added to the framework. By adding a variable, the framework allows for an additional unobserved state. I add, in turn, three key macro variables: Industrial Production a percentage change from the year before (IP), the Federal Funds Rate (FFR), and annual Consumer Price Inflation (CPI). Results are again based on the CPS gross flow data. Figures A7 presents the model diagnostics statistics and shows that these are very similar to those for the baseline model without additional variables. Figure A8 presents the steady-state curves and shows that these are also similar to those of the baseline: they all deliver a stable steady state around 5 percent unemployment and an unstable one close to 10 percent.

Figure A7: Diagnostics statistics: additional macro variables

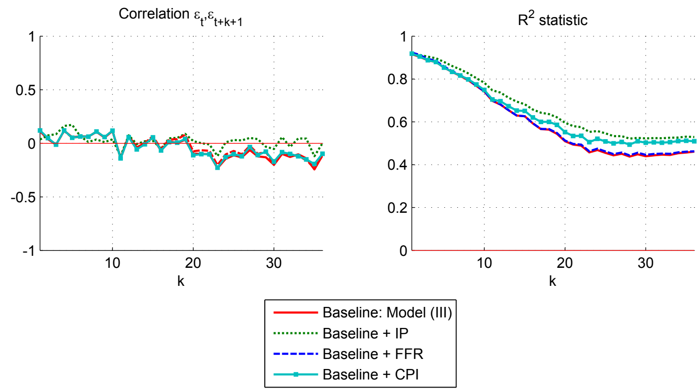
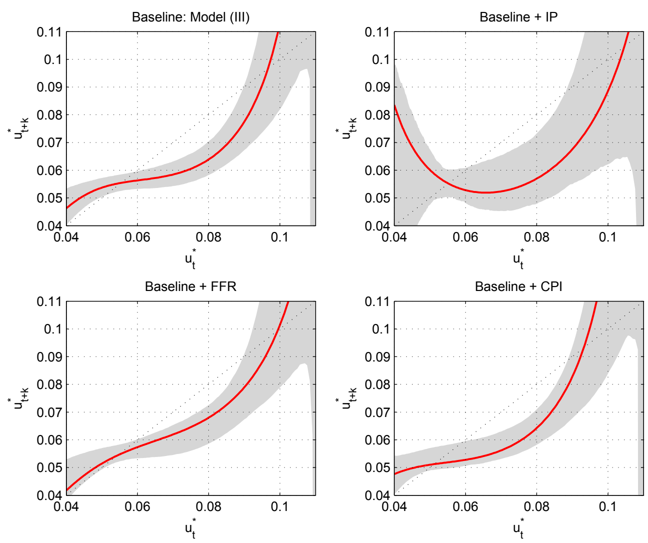


Figure A8: Steady states: additional macro variables



6. Alternative estimator (OLS) To account for the likely present I use an IV estimator to estimate the model. For completeness, I also report the results obtained using OLS, see Figure A9 and A10. Figure A9 shows that while model (III) produces less autocorrelation in the residuals than models (I) and (II), there is still some autocorrelation left. Again, Model (III) provides a substantial improvement in terms of the R^2 statistic. Figure A10 shows that model (III) delivers steady-state estimates very similar to the baseline IV estimates.

Figure A9: Diagnostics statistics: OLS estimator

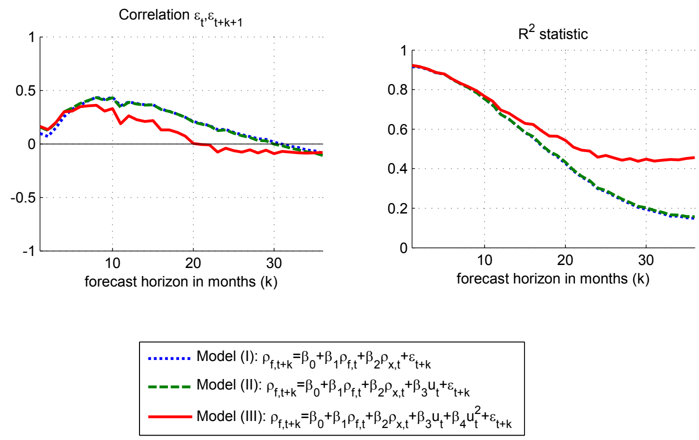
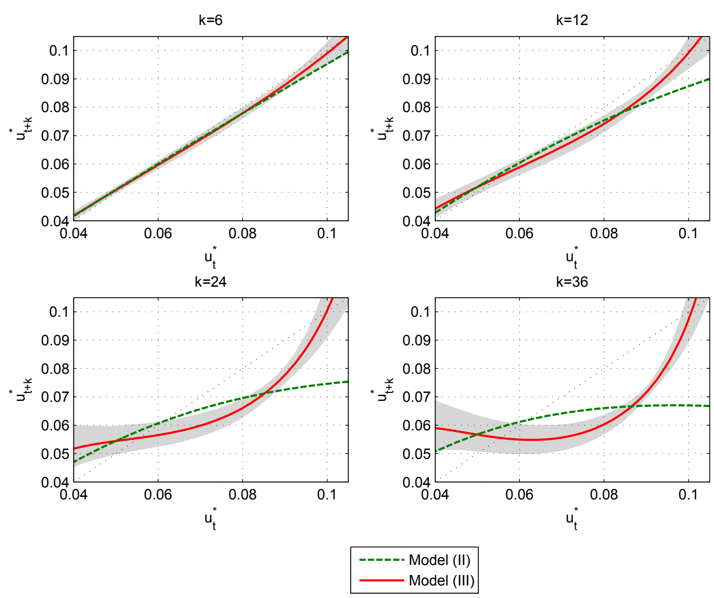


Figure A10: Steady states: OLS estimator



7. Additional lags Next, I consider models with additional lags. I add two lags for all regressors to the forecasting equation and estimate the model using OLS.[25] Results for 1 additional lag (not reported) are very similar. Figures A11 and A12 present the results.

[25] Using the IV method would be more involved, since that method already uses lagged values as instruments.

Figure A11 shows that, in contrast to the baseline, both models (II) and (III) produce little or no residual autocorrelation, whereas model (I) does produce autocorrelation. Model (III) produces a somewhat higher R^2 than Model (II) and a substantially higher R^2 than Model (I).

Figure A12 shows, however, that both models (II) and (III) generate multiple steady states similar to the baseline, although the unstable steady state is estimated to has somewhat higher unemployment for Model (II). Overall, the results appear to be robust to adding additional lags.

Figure A11: Diagnostics statistics: additional lags

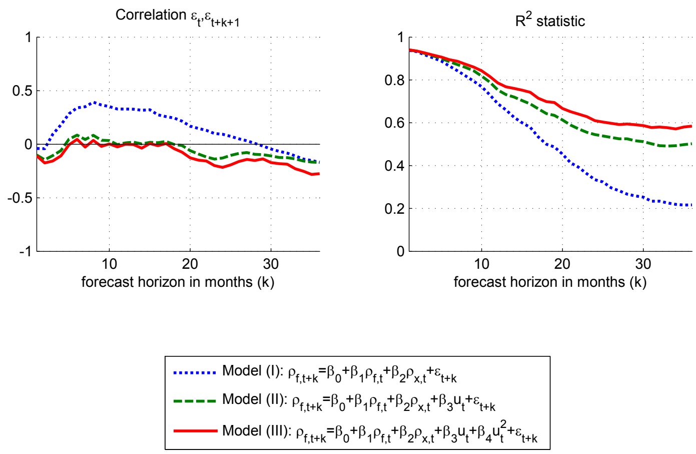
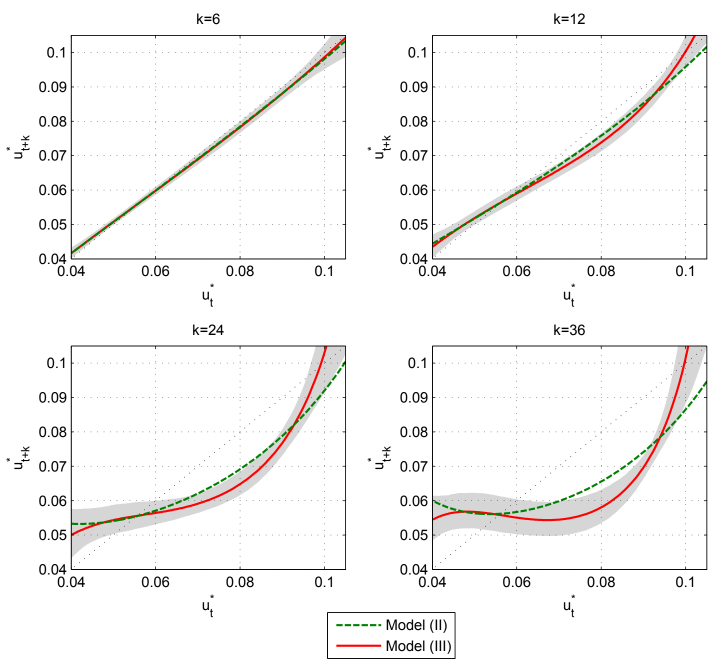
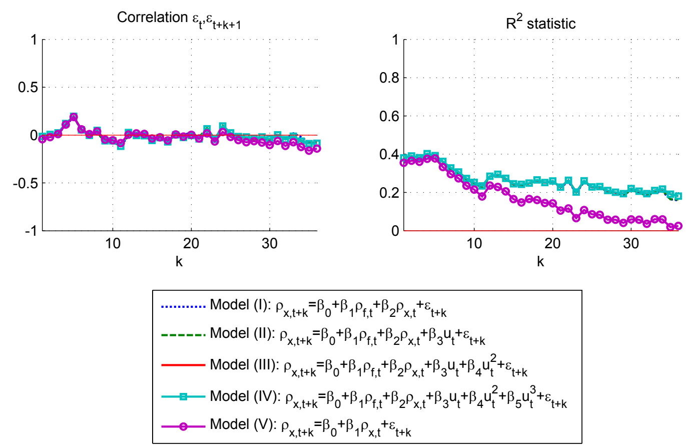


Figure A12: Steady states: additional lags



8. Specification for the job loss rate Finally, I consider the specification for the job loss rate. As in the baseline, I use CPS gross flow data and the IV estimator. Figure A13 presents the results for five specifications. The first four are the same as those for the job finding rate, except that the job loss rate is now the depend variable in the forecasting regression. Model (V) corresponds to a simple AR(1). The left panel of Figure A13 makes clear that none of the specifications features substantial autocorrelation in the residuals. Thus, a simple AR(1) survives the autocorrelation test. While adding additional variables improves the R^2 statistic somewhat at longer horizons, I conclude that the job loss rate is well described by an AR(1), which has the benefit of parsimony.

Figure A13: Diagnostics statistics: job loss rate



B. Euler equation

First, we derive explicitly the firms' Euler equation for vacancies. The firms' problem can be written as:

$$V(n_{t-1},\mathcal{S}_t) = \max_{n_t,h_t} \left(\overline{A}-w_t\right)n_t - \chi p_t h_t - \kappa\frac{h_t}{q_t} + \beta\mathbb{E}_t V(n_t,\mathcal{S}_{t+1})$$

subject to

$$n_t = \left(1-\rho_{x,t}\right)n_{t-1}+h_t,$$

$$h_t \geq 0.$$

The first-order conditions for n_t and h_t are:

$$\begin{aligned} \overline{A}-w_t-\mu_t+\beta\mathbb{E}_t\left(1-\rho_{x,t+1}\right)\mu_{t+1} &= 0 \\ \chi p_t - \frac{\kappa}{q_t}+\mu_t+\xi_t &= 0 \\ \xi_t h_t &= 0 \end{aligned}$$

where μ_t and ξ_t are, respectively, the Lagrange multiplier on the employment transition equation and the non-negativity constraint on hires. The third condition is the complementary slackness condition. The first two equations can be combined to obtain:

$$\chi p_t + \frac{\kappa}{q_t} - \xi_t = \overline{A} - w_t + \beta \mathbb{E}_t \left(1 - \rho_{x,t+1}\right) \left(\chi p_{t+1} + \frac{\kappa}{q_{t+1}} - \xi_{t+1}\right).$$

C. Search and matching model: 1970-1990

The baseline search-and-matching model is estimated over the period 1990-2015. Below I present estimation results for the period 1970-1990. Because data set used for the baseline model (transition rates based on gross flows) data start in 1990, I instead use duration-based transition rates (see also Appendix B). The top panel of Table 2 presents the estimated parameters for the model with and without skill losses. In the baseline version with skill losses, the estimated matching function elasticity, α, is roughly 0.65, which is extremely similar to the estimate for the $1990-2015$ period (see Table 1). The estimated degree of skill losses, χ, is 0.72, which is somewhat higher than estimate for the period 1990-2015.

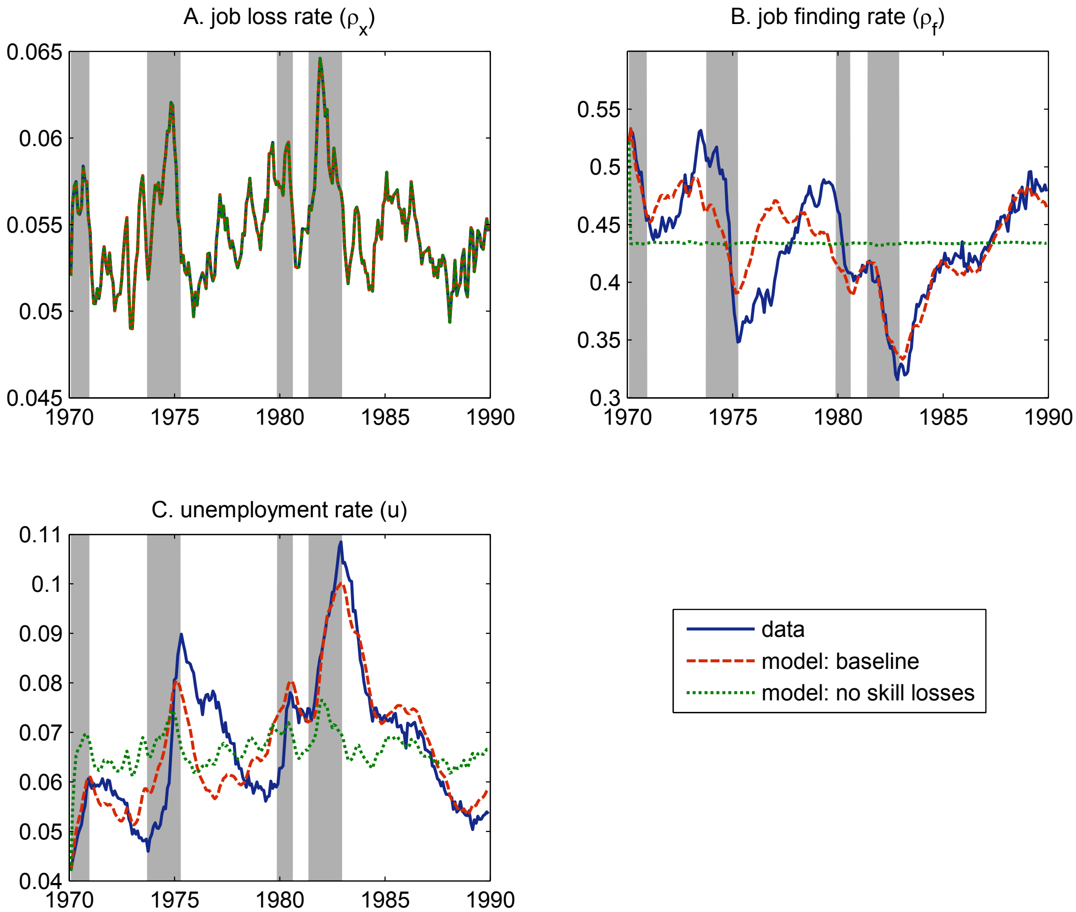
The middle panel of Table 2 reveals that, as for the post 1990 period, the introduction of skill losses dramatically improves the fit of the model. The bottom panel presents the estimated steady-state rates of unemployment. With skill losses, there is a stable steady state of unemployment at around 6.5 percent and an unstable steady state at 11.5 percent unemployment. These estimates are similar to the ones obtained for the post 1990 period, albeit somewhat higher. Without skill losses, there is only one steady state, located at 6.7 percent unemployment. Figure 10 plots the simulated time paths versus the estimated data. Again, the baseline model fits the data remarkably well, in particular for the 1980s.

Overall, the results are much in line with the results for the $1990-2015$ period, as presented in the main text.

Table 2: Estimation results search and matching models: 1970-1990.

		baseline	**no skill losses**
I. estimated parameter values			
matching function elasticity	α	0.645	0.888
re-training cost	χ	0.720	0
vacancy cost	κ	0.896	1.280
flow from unemployment	b	0.965	1.000
steady state job loss rate	$\overline{\rho}_x$	0.045	0.045
persistence job loss rate	λ_x	0.842	0.842
st.dev. job loss shocks	σ_x	$1.53e^{-3}$	$1.53e^{-3}$
II. model fit			
$RMSE$		0.582	0.992
R^2		0.660	0.012
1y autocorrelation $\rho_{f,t}$ relative to data		1.331	0.128
III. steady states			
unemployment rate s.s. A (stable)	$\overline{u}^A$	0.0646	0.0666
unemployment rate s.s. B (unstable)	$\overline{u}^B$	0.1146	–
unemployment rate s.s. C (stable)	$\overline{u}^C$	1	–

Figure 10: Search and matching models versus data: 1970-1990



Notes: shaded areas denote NBER recessions.