Abstract

This paper studies a heterogeneous agents model that combines frictions in the labor market with incomplete asset markets and nominal rigidities in price setting. Workers experience job terminations that can either send them into short term unemployment or into longer term unemployment. An increase in job uncertainty depresses aggregate demand which is transmitted to the supply side and produces a significant drop in job finding rates because firms cut back on vacancy postings. The amplification mechanism is small when asset markets are complete, prices are flexible or unemployment is predominantly short term. Applied to the Great Recession, the model can account for the sharp rise in the level of unemployment and for much of the shift and movement along the Beveridge curve observed during this recession. Job uncertainty also emerges as a plausible candidate for the shock that sent the US economy into a liquidity trap and we show that the zero lower bound on nominal interest rates can amplify the recession very significantly in this environment.

Keywords: job uncertainty, unemployment, incomplete markets, the zero lower bound

JEL Classification: E21, E24, E31, E32, E52

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1 Introduction

The U.S. economy has witnessed very depressed labor market conditions during the Great Recession. At the onset of the recession, the job loss rate increased temporarily while the job finding rate quickly fell to barely half its pre-recession level and has so far failed to recover. As a result, the civilian unemployment rate went up from 4.7 percent in December 2007 to 9.9 percent by June 2009 while the mean duration of unemployment surged from 16.5 weeks to 29.7 weeks over the same period and then reached hitherto unprecedented levels close to 40 weeks in late 2012. Understanding the sources and consequences of this severe deterioration in labor market outcomes is obviously a very important issue. This paper develops a theory in which labor market weaknesses interact with frictions in goods and financial markets to produce a powerful amplification mechanism of labor market shocks. We claim that this mechanism is relevant for understanding the Great Recession.

The model that we build is aimed at accounting for the persistent drop in the job finding rate and the ramifications thereof. A key feature of the model is that labor market uncertainties impact on the level of aggregate demand. We study a model in which an absence of unemployment insurance contracts gives risk averse households a motive for self-insuring against unemployment. In this setting increasing risk of job loss and uncertain job finding prospects during unemployment trigger a precautionary savings motive for employed workers. This precautionary savings motive can produce a decline in aggregate demand in response to increased job loss risk which goes significantly beyond the decrease in the income of workers that experience a job loss, see also Carroll and Dunn (1997) and Carroll, Dynan and Krane (2004). An important additional feature of our theory is that we allow for differences across unemployed workers in their prospects for finding jobs. This aspect is motivated with reference to the increase in the average duration of unemployment observed in the U.S. during the Great Recession, see also Kroft et al (2013), which we argue is inconsistent with the idea that the pool of unemployed workers face identical job finding prospects, see also Hornstein (2012). We assume that some proportion of the unemployed are less efficient searchers than ‘normal’ households, see also Krusell and Smith (1999) and Krusell et al (2009). We refer to this phenomenon as mismatch and discuss how it interacts with the precautionary savings motive to produce a large decline in aggregate demand when it occurs at the same time as an increase in

\[1\] The idiosyncratic uncertainty shock that we highlight is very different in nature to aggregate uncertainty shocks due to changes in the second moments of productivity and policy shocks highlighted by e.g. Baker et al (2012) and Stock and Watson (2012).
the risk of a job loss, a feature that we will argue is relevant for the Great Recession.²

We model these features by combining a model with matching frictions in the labor market with an incomplete financial markets model in which households cannot purchase unemployment insurance contracts. We add three further important ingredients. First, firms are faced with nominal rigidities in price setting. This feature is central to our theory because it allows changes in aggregate demand to be transmitted to the supply side. Secondly, real wages are assumed to be rigid so that wage cuts do not insulate new job hires from deteriorating economic conditions.³ Third, monetary policy is described by a Taylor rule for the short-term nominal interest rate. The model is calibrated to match statistics for the U.S. economy prior to the Great Recession assuming moderate degrees of risk aversion and of nominal rigidities. We impose a zero borrowing limit which implies that the consumption losses due to unemployment shocks are key determinants of the precautionary savings motive.⁴

We simulate the model in response to the short burst in the rate of inflow to unemployment observed in the U.S. at the onset of the Great Recession and a mismatch shock which determines the share of job losers that become low search efficiency unemployed workers.⁵ The latter is calibrated so that the model matches the share of workers who have been unemployed for 6 months or more and we find a significant increase in mismatch from mid-2009 onwards just as the increase in the rate of inflows to unemployment started to level out. In response to these shocks, the model implies a rise in the unemployment rate and a drop in vacancy postings that are very similar to the empirical counterparts observed during the Great Recession. Thus, the model also reproduces the much-discussed movements along and the outward shift of the Beveridge curve. We contrast our results with those of the early 1990’s recession and find that the labor market shocks were much smaller in that recession but that the amplification mechanism of our model was relevant even in this recession.

The large effects of the labor market shocks occur due to their impact on aggregate demand.

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²Our notion of mismatch is different from the one proposed by e.g. Shimer (2007) who defines mismatch in terms of imbalances between vacancies and unemployment across (local) labor markets.

³We adopt this last friction since there seems to have been little downward real wage flexibility in the US during the Great Recession, see also Shimer (2012).

⁴Following Hurd and Rohwedder (2011) we calibrate the consumption decline upon job loss to 11 percent.

⁵We take no view on the underlying sources of the job separation shock but the financial crisis of 2007/08 would be a plausible candidate.
which, because of nominal rigidities, is transmitted to the supply side and produces a decline in vacancy posting. Fewer vacancies, in turn, lead to a decrease in the job finding rate which stimulates even further precautionary savings setting in motion an amplification mechanism. We dissect the amplification mechanism by comparing with the results that arise when neutralizing either the transmission from the demand side to the supply side by assuming flexible prices or the precautionary savings motive (against idiosyncratic shocks) by assuming insurance within large families. In either case, labor market shocks have only limited impact on unemployment and the counterfactual analyses fail to reproduce the Beveridge curve. When prices are flexible, firms exploit labor market slackness to hire labor more easily and the drop in demand due to labor market uncertainty has little impact on equilibrium quantities. When households can insure against idiosyncratic shocks, employed workers have little incentive to save for precautionary reasons and the labor market shocks therefore have little impact on aggregate demand. Thus, it is the combination of frictions in goods and financial markets that produces amplification.

Mismatch shocks are central to our ability to account for the Great Recession. When we eliminate this shock we find an outward shift of the Beveridge curve but little movement along it. While we refrain from modeling the roots of this shock, one might speculate about structural interpretations. An appealing interpretation rests on disparity between the supply and demand for jobs across industries, occupations and skill groups. Such imbalances imply that workers who lose their jobs find it difficult to find jobs because of a lack of demand for workers in the ‘local’ labor market segment. Şahin et al (2012) evaluate the empirical relevance of this source of mismatch during the Great Recession and construct a mismatch index on the basis of the fraction of hires that are lost due to immobility of workers across local labor market segments. They find that the direct contribution of this measure of mismatch to the increase in unemployment of up to 1.5 percentage points. Kroft et al (2013) go one step further and allow also for duration dependence and for non-participation. They find that negative duration dependence and transition to and from non-participation are both important for understanding the increase in the long-term unemployment during the Great Recession. A contributing factor to the observed decrease in search efficiency may also derive from financial constraints which make unemployed workers in negative equity (or with large potential capital losses on their housing stock) unwilling to move in their pursuit of new job opportunities, see e.g. Sterk (2011). Finally, as in Pissarides (1992), long unemployment spells may lead to human capital depreciation making the supply of suitable jobs more limited.\footnote{Such a theory would be more consistent with a three state model in which job losers initially enter the high}
Our analysis has implications for monetary policy design. ‘Standard’ New Keynesian models typically rest on a representative agent setting with insurance against unemployment within large diversified households. In such settings, monetary policy should be aimed at addressing the inefficiencies imposed by nominal rigidities which typically can be implemented by letting interest rates react very aggressively to deviations of inflation from its target. We find that similar results hold in our incomplete markets model because a very aggressive monetary response neutralizes the aforementioned amplification mechanism and therefore stabilizes the economy. We further consider the impact of imposing a zero lower bound (ZLB) on short term nominal interest rates. We find that the labor market shocks during the Great Recession were sufficiently large to take the economy into a liquidity trap. When the ZLB binds, not only does job uncertainty hold back goods demand for precautionary savings reasons, but the lack of interest rate response produces a further drop in output required to clear the savings market. Quantitatively, we find that the model with a ZLB produces an even larger recession than observed in the U.S., which we interpret as an indication that policy interventions may have softened the recession.

Our theory is closely related to a number of other contributions to the literature. Gomes, Greenwood and Rebelo (2001) and Krusell, Mukoyama, and Şahin (2010) investigate the impact of self-insurance in incomplete markets settings with frictional labor markets. Krusell and Smith (1999) and Krusell et al (2009) study a self-insurance model with risk of both short-term and long-term unemployment.\footnote{A key difference between their analysis and ours is that agents in their economy are able approximately to self-insure against unemployment shocks while we calibrate towards empirical estimates of the impact of unemployment transitions on consumption.} We add goods market frictions and find this aspect to be important. Challe and Ragot (2012) and Bayer et al (2013) study the impact of precautionary savings in an incomplete markets setting. The latter of these papers also introduces nominal rigidities and allows for time-varying variance of idiosyncratic earnings shocks while we model uncertainty through unemployment risk and the impact of mismatch.\footnote{Basu and Bundick (2012), Rendahl (2012), and Schaal (2012) also investigate the impact of uncertainty or news shocks in models with labor market frictions. Caggese and Perez (2012) look at precautionary behavior in a model that combines labor market and financial market frictions.} Guerrieri and Lorenzoni (2012) also examine an incomplete markets setting with nominal rigidities focusing upon the impact of tightening borrowing constraints. Leduc and Liu (2013) present time-series evidence that increases in ‘uncertainty’ search efficiency state and thereafter may experience a transition to the low search efficiency state. This model has very similar implications to the one we analyze.
impacts negatively on aggregate demand and argue that labor market frictions and nominal rigidities are essential for accounting for this relationship. Most similar to our analysis are the recent contributions of Gornemann, Kuester and Nakajima (2012) and McKay and Reis (2012) who both study incomplete markets models with labor and goods market frictions but focus upon very different questions from us. Gornemann, Kuester and Nakajima (2012) examine the distributional effects of monetary policy when agents face unemployment risk while McKay and Reis (2012) focus upon the impact of automatic fiscal stabilizers. Unlike us, none of these contributions examine the consequences of allowing for longer-term unemployment.

The remainder of this paper is structured as follows. Section 2 reviews the labor market impact of the Great Recession. In Section 3 we present the model. Section 4 examines the quantitative properties of the model and provides an analysis of the Great Recession. Section 5 extends the analysis to the ZLB and provides some robustness analysis. Section 6 summarizes and concludes.

2 The Great Recession and the Labor Market

The financial crisis produced one of the longest and deepest recessions in U.S. history. According to the NBER, the contraction lasted 18 months (December 2007 - June 2009), the longest since the Great Depression. The Great Recession also triggered a major deterioration of labor market conditions. Unemployment rose from 4.7 percent in July 2007 to 10 percent by October 2009, and has subsequently remained stubbornly high, see Figure 1.

The flows in and out of unemployment provide a useful way to gain some further insight into the determinants of the change in unemployment. We measure the average instantaneous job finding rate, $\lambda_t$, and the average job separation rate, $\rho_t$ as:

$$\lambda_t = \frac{m_t}{u_{t-1}}$$

$$\rho_t = \frac{e_t}{n_{t-1}}$$

where $u_t$ is the level of unemployment, $n_t$ the stock of employment, $m_t$ the flow of workers from unemployment to employment, and $e_t$ the number of (permanent) job separations. All data were obtained from the Current Population Study (CPS) apart from $e_t$ which we got from the Bureau of Labor Statistics. The appendix contains precise data sources.

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Figure 2 illustrates $\lambda_t$ and $\rho_t$. The initial rise in unemployment was triggered by a temporary increase in the inflow rate into unemployment but its persistence derives from a stubborn decline in the outflow rate (the job finding rate), a pattern that is not unusual for U.S. recessions. Thus, any theory meant to address the labor market slump during the Great Recession has to say something about why the outflow rate has persisted at such low levels since the onset of the recession.

An important observation is that while the peak of the unemployment rate during the Great Recession does not stand out as particularly high relative to previous recessions, as also stressed by Kroft et al (2013), the impact on the duration of unemployment is very different from previous recessions, see Figure 3. The duration of unemployment usually increases during recessions but only to moderate levels. For example, in the early 1980’s recession, the mean duration of unemployment went up from 13 weeks in 1981 to 21 weeks by the summer of 1983. In contrast, during the Great Recession the duration of unemployment sky rocketed from 17.2 weeks in July 2007 to 40 weeks in late 2012, almost twice its previous peak value in the post-WWII sample, see also Rothstein (2011) and Wiczer (2013).

The long period of high unemployment provides one mechanical reason for the increase in longer term unemployment during the Great Recession but by itself it cannot account for the magnitude of the rise in the average spell of unemployment. To appreciate this, consider the following computation of the mean duration. Assume that all unemployed workers face identical job prospects so that the mean unemployment duration can be approximated by the inverse of the instantaneous job finding rate, $1/\lambda_t$. Figure 3 shows $1/\lambda_t$ along with CPS estimates of the mean duration of unemployment. According to the actual CPS estimates, the mean duration of unemployment peaked at 40.4 weeks in July 2011, an increase of 25.8 weeks relative to the duration of unemployment in December 2007 prior to the contraction. In comparison, the counterfactual estimates indicate a peak increase in the duration of unemployment of only 11.3 weeks. Thus, the assumption of homogeneous job opportunities generates a far smaller increase in the duration of unemployment than can be observed in the data, see also Barnichon and Figura (2012) and

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10 Similar dramatic increases can be observed in the median duration of unemployment (which peaked at 24.8 weeks in late June 2010, the double of its previous high in the post-WWII sample of 12.3 weeks in May 1983) and in the share of unemployed workers who have been out of work for more 6 months or more which peaked at 45.3 percent in March 2011, up from 16-18 percent prior to the contraction.

11 Alternatively, when unemployed workers find job with the same probability, one can compute mean duration recursively from $d_t \simeq (1 - \lambda_t) (d_{t-1} + 1) (u_t/u_{t-1}) + e_t/u_t$. This approximation produces almost the same counterfactual estimate of mean duration as the inverse of the job finding rate.
Hornstein (2012). Our theoretical analysis addresses this issue by deviating from the assumption of a homogeneous labor market outlook for the stock of unemployed workers.

Another much discussed feature of the Great Recession is its impact on the Beveridge curve. Figure 4 illustrates the relationship between vacancies and unemployment using CPS estimates of unemployment and JOLTS estimates of the number of vacancies. We discriminate between the pre-Great Recession period and the period thereafter (from 2007:12). During the early parts of the recession, unemployment approximately doubled while the number of vacancies fell by around 50 percent which jointly produced a striking movement down the Beveridge curve. In the course of the initial part of the recession, the labor market conditions worsened considerably but the dynamics of unemployment and vacancies appear consistent with the pre-crisis Beveridge curve. From late 2009, however, there is instead evidence that the Beveridge curve shifted outwards, indicating a less efficient matching between workers looking for employment and firms looking for new hires, see also Barlevy (2011).

3 Model

We construct a model in which a menu of frictions comes together and investigate how labor market shocks may be amplified endogenously. The economy consists of households which may either be employed or unemployed, firms owned by entrepreneurs that hire labor, set prices and produce output, and a government which is in charge of monetary and fiscal policies.

**Households.** There is a continuum of mass 1 of households indexed by \( i \in (0, 1) \). Households are risk-averse, infinitely lived, have rational expectations and maximize the expected present value of their utility streams. A household is either working or unemployed. When employed, the household earns a real wage \( w_t \). A household that is employed at the beginning of the period (indexed by \( r_{i,t} = n \)) may lose the job at the end of the period, an event that occurs with probability \( \rho_{x,t} \in [0, 1] \). During unemployment the household searches for jobs and receives benefits \( \xi < w_t \).

A household that experiences a job loss is randomly assigned to either a ‘short-term’ (\( r_{i,t} = s \)) unemployment pool or a ‘long-term’ unemployment (\( r_{i,t} = l \)) pool. We let \( \rho_{r,t} \in [0, 1] \), \( \rho_s + \rho_l = 1 \), be the probability that a household which experiences a job loss in period \( t \) becomes state \( r_{i,t} = s, l \) unemployed. The two unemployment states differ in the probability that the household finds a new job. We assume that the job finding probabilities, \( \eta_{r,t} \), are such that \( 0 \leq \eta_{l,t} \leq \eta_{s,t} \leq 1 \) so that a short-term unemployed worker is at least as likely to receive a job offer as a long-term unemployed

The timing is as follows. At the beginning of the period, the aggregate labor market shocks are realized. After this, unemployed workers and firms match and new employment relationships are established. This is followed by production and consumption. At the end of the period, the job separations are effectuated. Thus, employed workers face idiosyncratic uncertainty within-period about the identity of job losers.\(^{12}\)

Households cannot purchase unemployment insurance contracts but can self-insure by saving in a riskless nominal bond, \(b_{i,t}^h\). Asset choices are subject to a borrowing constraint that restricts households’ real assets positions \(b_{i,t}^h \geq b_{\text{min}}^h\). They face a sequence of budget constraints:

\[
c_{i,t} + b_{i,t}^h = n_{i,t}w_t + (1 - n_{i,t})\xi + \frac{R_{t-1}}{1 + \pi_t}b_{i,t-1}^h, \ t \geq 0
\]

where \(c_{i,t}\) denotes a consumption basket, \(R_{t-1}\) is the gross nominal interest rate paid out in period \(t\) on bonds purchased in period \(t-1\), \(\pi_t\) denotes the net inflation rate in period \(t\). \(n_{i,t}\) is an indicator variable for the household’s employment state:

\[
n_{i,t} = \begin{cases} 1 & \text{if individual } i \text{ is employed in period } t \\ 0 & \text{if individual } i \text{ is unemployed in period } t \end{cases}
\]

Let \(V(b_{i}^h, r_i, S)\) be the expected present discounted utility of a household given its bond position, its labor market status, and the aggregate state vector, \(S\). The Bellman equation for an employed household is given as:

\[
V(b_{i}^h, n, S) = \max_{c_{i},b_{i}^h} \{U(c_{i}) + \beta \mathbb{E} \left[ 1 - \sum_{g=s,l} \rho_x \rho_g (1 - \eta_g') V(b_{i}^{h'}, n, S') \right]\}
\]

subject to the borrowing constraint and to the budget constraint in equation (1) setting \(n_i = 1\). \(U\) is an increasing and strictly concave utility function. \(\beta \in (0,1)\) is the subjective discount factor, and \(\mathbb{E}\) is the conditional expectations operator. \(\rho_x \rho_g\) is the probability that a worker who is employed at the beginning of the period makes a transition to unemployment state \(g\) at the beginning of the next period.

\(^{12}\)Of course, households also face idiosyncratic uncertainty about the identity of future job losers.
The Bellman equation for a type $g$ unemployed worker is:

$$
V(b^h_i, g, S) = \max_{c_i, \beta \eta'_g} \{ U(c_i) + \beta \mathbb{E} \eta'_g V(b^{h'}_i, n, S') \\
+ \beta \mathbb{E} (1 - \eta'_g) V(b^{h'}_i, g, S') \}, \ g = s, l
$$

subject to the borrowing constraint and the budget constraint in equation (1) setting $n_i = 0$. As a matter of consistency, we assume that $V(b^h, n, S) \geq V(b^h, s, S)$ for all $b^h$ and $S$ so that no unemployed household has an incentive to voluntarily leave their current job. Under the condition that $\eta'_s \geq \eta'_l$, $V(b^h, s, S) \geq V(b^h, l, S)$ for all $b^h$ and $S$.\(^{13}\)

The consumption index $c_i$ is a basket of consumption goods varieties:

$$
c_i = \left( \int_j (c^j_i)^{1-\gamma} \, dj \right)^{1/(1-\gamma)}
$$

where $c^j_i$ denotes household $i$’s consumption of goods of variety $j$ and $\gamma > 1$ is the elasticity of substitution between consumption goods. Variety $j$ is purchased at the nominal price $P_j$. It follows that household $i$’s demand for variety $j$ is given as:

$$
c^j_i = \left( \frac{P_j}{P} \right)^{-\gamma} c_i
$$

where $P$ is the price index associated with the consumption basket defined in (4):

$$
P = \left( \int_j P_j \, dj \right)^{1/(1-\gamma)}
$$

**Entrepreneurs.** Consumption goods are produced by a continuum of monopolistically competitive firms indexed by $j \in (0, 1)$ which are owned by risk neutral entrepreneurs. $\Psi < 1$ denotes the measure of entrepreneurs. Entrepreneurs discount utility at the rate $\beta$ and make decisions on the pricing of their goods, on vacancy postings, and on their consumption and savings policies. In return for managing (and owning) the firm, they are the sole claimants to its profits. We assume that entrepreneurs can save but face a no-borrowing constraint. This no-borrowing constraint implies that the entrepreneur finances hiring costs through retained earnings.\(^{14}\)

\(^{13}\)The formulation of the unemployed workers’ problem in equation (3) assumes that there are no flows between the two unemployment states during unemployment. However, all workers face identical job prospects upon employment. The first of these assumptions is easily relaxed and immaterial for the results as long as the flow out of unemployment is sufficiently small for type $l$ workers relative to type $s$ workers.

\(^{14}\)In the stationary equilibrium, $\beta < 1/(R/((1 + \pi)))$ so entrepreneurs will be borrowing constrained.
Output is produced according to a linear technology:

$$y_{j,t} = n_{j,t}$$  \hspace{1cm} (7)

where $n_{j,t}$ denotes entrepreneur $j$’s input of labor purchased from the households. Firms hire labor in a frictional labor market. The law of motion for employment in firm $j$ is given as:

$$n_{j,t} = (1 - \rho_{x,t-1})n_{j,t-1} + h_{j,t}$$  \hspace{1cm} (8)

where $h_{j,t}$ denotes hires made by firm $j$ in period $t$. The number of hires in turn is given as:

$$h_{j,t} = \rho_{f,t}v_{j,t}$$  \hspace{1cm} (9)

where $v_{j,t}$ is the number of vacancies posted by the firm and $\rho_{f,t}$ is the job filling probability. Firms are assumed to be sufficiently large that $\rho_{f,t}$ can be interpreted as the fraction of vacancies that leads to a hire.\(^{15}\) The cost of posting a vacancy is given by $\mu > 0$. Real marginal costs are therefore given as:

$$mc_{j,t} = w_t + \frac{\mu}{\rho_{f,t}} - \beta E_t \left[ \frac{(1 - \rho_{x,t}) \mu}{\rho_{f,t+1}} \right]$$  \hspace{1cm} (10)

which incorporates the fact that hiring in period $t$ impacts on future marginal costs through future hiring cost savings.

Following Rotemberg (1982) we assume that firms face quadratic costs of price adjustment. Given risk neutrality, entrepreneurs set prices to maximize the present discounted value of profits:

$$E_t \sum_{s=0}^{\infty} \beta^s \left( \frac{P_{j,t+s}}{P_{t+s}} - mc_{j,t+s} \right) y_{j,t+s} = \frac{\phi}{2} \left( \frac{P_{j,t+s} - P_{j,t+s-1}}{P_{j,t+s-1}} \right)^2 y_t$$  \hspace{1cm} (11)

subject to:

$$y_{jt} = \left( \frac{P_{jt}}{P_t} \right)^{-\gamma} y_t$$  \hspace{1cm} (12)

Equation (12) is the demand for goods variety $j$. $y_t$, can be interpreted as aggregate real income. In equation (11) $\phi \geq 0$ indicates the severity of nominal rigidities in price setting with $\phi = 0$ corresponding to flexible prices. The first-order condition for this problem is given as:

$$\left( 1 - \gamma + \gamma mc_{j,t} \frac{P_t}{P_{j,t}} \right) y_{j,t} = \frac{\phi}{\rho_{j,t-1}} \left( \frac{P_{j,t-1} - P_{j,t-1}}{P_{j,t-1}} \right) y_t$$

$$-\phi E_t \left[ \left( \frac{P_{j,t+1}}{P_j} \right) \left( \frac{P_{j,t+1} - P_j}{P_j} \right) P_t y_{t+1} \right]$$  \hspace{1cm} (13)

\(^{15}\)This assumption - which is equivalent to assuming that $\Psi$ is sufficiently smaller than 1, can be relaxed which would produce ex-post heterogeneity across firms. The assumption also allows us to assume that there are no indivisibility problems associated with the full-time hours assumption.
In a symmetric equilibrium, which will be the focus of our analysis, this simplifies to:
\[
(1 - \gamma) + \gamma m c_t = \phi \pi_t (1 + \pi_t) - \phi \beta \mathbb{E}_t \left[ \pi_{t+1} (1 + \pi_{t+1}) \frac{y_{t+1}}{y_t} \right]
\]

(14)

Given the entrepreneurs' decisions as a producer and price setter, she solves the following dynamic programming problem:
\[
W (b^e_j, S) = \max_{d_j, b^e_j} \{ d_j + \beta W (b^e_j', S') \}
\]

subject to the budget and borrowing constraints:
\[
d_j + b^e_j + w n_j + \frac{h_j}{\rho_f} = \frac{P_j}{\pi} n_j - T^e + \frac{R_{t-1}}{1 + \pi} b^e_j
\]

\[
b^e_j \geq 0
\]

(15)

(16)

(17)

where \(d_j\) denotes entrepreneur \(j\)'s consumption and \(b^e_j\) their bond purchases. Condition (17) imposes the no-borrowing constraint on entrepreneurs. \(T^e\) denotes a lump-sum tax imposed on employers to cover the government provided unemployment benefits.

**Labor Market.** We assume that real wages are constant, \(w_t = \bar{w}\), as long as \(\bar{w}\) is consistent with the joint match surplus being non-negative and with workers preferring to work rather than being unemployed.\(^{16}\) The constant real wage assumption serves as a useful benchmark: Our interest lies mainly in understanding recessions and we therefore interpret this assumption in terms of downward inflexible real wages. Moreover, the assumption is consistent with the behavior of “aggregate” real wages in the U.S. during the Great Recession. We later examine the extent to which this assumption matters for our results by assuming alternatively that wages are flexible and determined through Nash bargaining.

The matching technology is given as:
\[
m_t = \psi (u_{a,t})^\alpha (v_t)^{1-\alpha}
\]

(18)

where \(m_t\) denotes the measure of matches between firms with vacancies and unemployed workers at date \(t\), \(u_{a,t}\) is the measure of ‘active’ searchers, and \(v_t\) is the measure of vacancies posted by the firms. \(\psi > 0, \text{ and } \alpha \in (0,1)\) are constant parameters. The number of active searchers, in turn, is given as:
\[
u_{a,t} = \left( u_{s,t-1} + \rho_{x,t-1} u_{s,t-1} n_{t-1} \right) + q \left( u_{l,t-1} + \rho_{x,t-1} l_{t-1} n_{t-1} \right)
\]

(19)

\(^{16}\)We have checked that the match surplus is positive for all matches in all the results that we report.
where $u_{r,t}$ is the measure of type $r$ unemployed workers at date $t$. $q \in (0, 1]$ is the probability that a type $l$ unemployed worker is searching for a job at date $t$. When $q < 1$, type $l$ workers are less likely to find a job than type $s$ unemployed workers and face longer expected unemployment spells. The right hand side of this expression consists of the unemployed workers from last period plus the number of new job separations of each type corrected for the fact that type $l$ unemployed workers search only with probability $q$. $u_{a,t}$ can therefore be thought of as the number of unemployed searchers measured in efficiency units at the beginning of the period prior to the current period’s matching between firms and workers.

Given the matching technology, the job filling probability and the job finding probabilities are given as:

$$
\rho_{f,t} = \psi \theta_{t}^{-\alpha} \\
\rho_{s,t} = \psi \theta_{t}^{1-\alpha} \\
\rho_{l,t} = q \rho_{s,t}
$$

where $\theta_{t} = v_{t}/u_{a,t}$ denotes labor market tightness. The laws of motion of the stocks of employed and unemployed workers are given as:

$$
n_{t} = (1 - \rho_{x,t-1}) n_{t-1} + m_{t} \\
u_{r,t} = (1 - \eta_{r,t}) \left( u_{r,t-1} + \rho_{r,t-1} \rho_{x,t-1} n_{t-1} \right), \ r = s, l
$$

**Government.** The government is in charge of monetary and fiscal policies. We assume that the government balances the budget period by period which means that:

$$
u_{t} \xi = \Psi T_{t}^{e}
$$

where $u_{t} = u_{s,t} + u_{l,t}$.

Monetary policy is specified by a rule for the short-term nominal interest rate. We assume that:

$$
R_{t} = \overline{R} \left( \frac{1 + \pi_{t}}{1 + \overline{\pi}} \right)^{\delta}
$$

where $\overline{R}$ is the long-run nominal interest rate target, $\overline{\pi}$ is the inflation target, and $\delta$ denotes the (semi-) elasticity of the nominal interest rate to deviations of inflation from its target.

**Stochastic Shocks.** We allow for shocks to the job separation rate, $\rho_{x,t}$, and to $\rho_{s,t}$, the stochastic variable which determines the share of workers affected by job terminations that become short-term
unemployed. We assume that:

\[
\begin{align*}
\rho_{x,t} &= \overline{x} + z_{x,t} \\
\rho_{s,t} &= \overline{s} + z_{s,t} \\
z_{x,t} &= \lambda_x z_{x,t-1} + \varepsilon_{x,t} \\
z_{s,t} &= \lambda_s z_{s,t-1} + \varepsilon_{s,t}
\end{align*}
\]

where \(\overline{x}, \overline{s} \in (0,1)\) are the long-run levels of job termination and the share of short-term unemployed, respectively, while \(\lambda_x, \lambda_s \in (-1,1)\) denote the persistence of shocks to the job termination rate and to the share of short-term unemployed. It is assumed that \(\varepsilon_t \sim N(0, V_\varepsilon)\) where \(\varepsilon_t = (\varepsilon_{x,t}, \varepsilon_{s,t})'\) and \(V_\varepsilon\) is assumed to be diagonal with diagonal elements \(\nu_x^2\) and \(\nu_s^2\), respectively.

**Equilibrium.** We focus upon a recursive equilibrium in which households act competitively taking all prices for given while firms act as monopolistic competitors setting the price of their own variety taking all other prices for given. In equilibrium, firms will be symmetric because of the assumptions made earlier about the absence of idiosyncratic productivity shocks, state contingent pricing and the large firm assumption. We let \(p_{j,t} = P_{j,t}/P_t\) denote the relative price of firm \(j\)’s product. Symmetry implies that this relative price equals 1 in equilibrium.

We impose that \(b^{\text{min}} = 0\). Since no household is allowed to be in debt and there is no aggregate savings vehicle, this implies that, in equilibrium, all households consume their income and that the aggregate wealth distribution is degenerate and not relevant as a state variable, see also Krusell, Mukoyama and Smith (2011). Due to this assumption, the aggregate state vector is \(S_t = (u_{t,t}, u_{s,t}, \rho_{x,t}, \rho_{s,t})\) which does not involve the wealth distribution and this simplifies the computationally aspects very considerably while it has only limited impact on the aggregate dynamics, see Ravn and Sterk (2012). We let \(c(b^h, r, S)\) and \(b^{h'}(b^h, r, S)\) denote the decision rules that solve the households’ problems (depending on their labor market status), and \(d(b^e, S)\) and \(b^{e'}(b^e, S)\) the solutions to the entrepreneurs’ consumption problem. We can now define the equilibrium formally:

**Definition 1** A recursive monopolistic competition equilibrium is defined as a state vector \(S\), pricing kernels \((w(S), \pi(S))\), decision rules \((c(b^h, r, S), b^{h'}(b^h, r, S))_{r=n,s,j}\), \((d(b^e, S), b^{e'}(b^e, S))\)

and \((p_j(b^e, S))_{j=0}^J, (n(b^e, S), h(b^e, S))\), value functions \((V_i^n(b^h_i, S), V_i^{h,s}(b^h_i, S), V_i^{h,l}(b^h_i, S))_{i=1}^1\)

and \(W(b^e, S)\), and government policies \((T^e(S), R(S))\) such that

(i) given the pricing kernel, the government policies, and the aggregate and individual states, the
household decision rules solve the households problems;

(ii) given the pricing kernel, government policies, and the aggregate state, the entrepreneur decision rules solve the entrepreneurs’ problem and \( p_j (b^c, S)^{j=0} = 1 \) for all \( j \) and all \( (b^c, S) \);

(iii) asset, goods and labor markets clear:

\[
0 = \int b_i^h \left( b_i^h, r_i, S \right) di + \Psi \int b_j^e \left( b_j^e, S \right) dj
\]

\[
\bar{y} = \int c_i^h \left( b_i^h, r_i, S \right) di + \Psi \int d_j \left( b_j^e, S \right) dj
\]

\[
y = \int n_i^h di
\]

\[
\bar{y} = y - \mu v - \frac{\phi}{2} \pi \gamma
\]

\[
n_t = (1 - \rho_{x,t-1}) n_{t-1} + \psi (u_{a,t})^\alpha (v_t)^{1-\alpha}
\]

\[
u_{a,t} = (u_{s,t-1} + \rho_{x,t-1} \rho_{s,t-1} n_{t-1}) + q (u_{i,t-1} + \rho_{x,t-1} \rho_{i,t-1} n_{t-1})
\]

\[
u_{r,t} = (1 - \eta_{r,t}) (u_{r,t-1} + \rho_{r,t-1} \rho_{x,t-1} n_{t-1}) , \ r = s, l
\]

(iv) the government budget constraint is satisfied and the nominal interest is given by the policy rule in equation (26);

4 Quantitative Results

4.1 Calibration

We solve the model numerically using a standard perturbation approach (see the Appendix for details). The calibration targets and parameter values are summarized in Tables 1 and 2. One model period corresponds to a calendar month. The household utility function is assumed to be given as:

\[
U(c_{i,t}) = \frac{c_{i,t}^{1-\sigma} - 1}{1-\sigma}, \ \sigma \geq 0
\]

\( \sigma \) determines the degree of risk aversion which matters for the household savings response to uncertainty. We set \( \sigma = 1.5 \) which is in the mid-range of empirical estimates of Attanasio and Weber (1995), Eichenbaum, Hansen, and Singleton (1988), and many others who have examined either household data or aggregate time series.

We assume an annual real interest rate of 4 percent in the steady state and set the subjective discount factor equal to 0.993 for both households and entrepreneurs. This value is low relative
to standard representative agent models but because of idiosyncratic risk and incomplete markets, agents have a strong incentive to engage in precautionary savings and a low real interest rate is required to induce zero savings in equilibrium.

We target an unemployment rate of 5 percent and a 15 percent share of unemployed workers who have been out of work for 6 months or more in the stationary equilibrium, values that correspond to the mean values observed in the United States in the post-1970 period. Following Rothstein (2011), we target a monthly hazard rate from unemployment to employment for a newly unemployed worker of 43 percent and a 31 percent monthly hazard rate for workers who have been unemployed for 26 weeks or more. It follows from this that the steady-state job loss probability ($\bar{\rho}_x$) is 3.47 percent per month, that the share of workers who experience a job loss that enter the pool of high search efficiency unemployment ($\bar{s}$) equals 35 percent, and that the relative search efficiency of the long term unemployed ($q$) is 37 percent. $^{17}$

The benefit level, $\xi$, is calibrated by targeting a decline in consumption of 11 percent upon unemployment, a value which corresponds to Hurd and Rohwedder’s (2010) estimate of the household spending impact of a job loss. $^{18}$ We assume that the matching function elasticity to unemployment is equal to 50 percent ($\alpha = 0.5$), and normalize $\psi = 1$. $\mu$, the vacancy cost parameter, is calibrated by targeting an average hiring cost of 4.5 percent of the quarterly wage bill per worker. Given other parameters, this implies that $\mu = 0.13$.

We set the average mark-up equal to 20 percent which implies that $\gamma$, the elasticity of substitution between goods, is equal to 6. $\phi$, the parameter that determines the importance of price adjustment costs, is calibrated to match a price adjustment frequency of 5 months. This value is conservative but close to the value estimated by Bils and Klenow (2004). $^{19}$ This implies that $\phi = 97.42$. We assume that the government’s inflation target $\pi = 0$ so that it pursues price stability and we set $\delta = 1.5$, a conventional value in the new Keynesian literature.

Finally, we estimate the parameters of the stochastic processes that determine the persistence and volatility of the job separation rate and of the share of high search efficiency unemployed.

$^{17}$It is important to notice that due to randomness in the matching process, some of the mismatched workers find jobs quickly while some of the high search efficiency workers take a long time to find a job.


$^{19}$To be precise, we calibrate $\phi$ by exploiting the equivalence between the log-linearized Phillips curve implied by our model and the Phillips curve implied by the Calvo model.
workers from time-series data for the employment-to-unemployment flow rate and the share of workers who have been unemployed for 6 months or more, both from the period 1980-2006. For the autocorrelation coefficients, we find estimates of $\lambda_x = 0.991$ and $\lambda_s = 0.881$. This implies a half-life of job separation shocks of 77 months while shocks to the share of short term unemployed die out much faster. For the standard deviations of the innovations, we find $\nu_x = 4.7 \cdot 10^{-4}$ and $\nu_s = 0.0513$.

4.2 Results

The Impact of Labor Market Shocks. We first examine the impact of job separation and mismatch shocks. We compare the benchmark economy with two alternative economies. The first alternative economy assumes that prices are flexible ($\phi = 0$) but retains the incomplete markets assumption. Comparing this economy with the benchmark model gives insights into whether the transmission of ‘demand side’ shocks to the ‘supply side’ due to nominal rigidities matters for our results. In the second alternative economy, we retain nominal rigidities in price setting but assume that households belong to large diversified families which share their members’ idiosyncratic employment risk. In the cooperative (risk sharing) outcome, consumption is equalized across household members independently of their labor market status and the family maximizes utility subject to the single budget constraint:

$$c_t + b_t^h = n_tw_t + (1-n_t)\xi + \frac{R_{t-1}}{1+\pi_t}b_{t-1}^h, \quad t \geq 0$$

where $n_t$ is the fraction of the household members that is employed in period $t$. In this economy, insurance neutralizes the precautionary savings motive deriving from idiosyncratic employment risk. Comparing this economy with the benchmark model therefore allows us to see how labor market risks impacts on the equilibrium outcome.

Figure 5 illustrates the impact of a one standard deviation increase in job terminations. This shock sets off a persistent rise in unemployment which peaks 10-12 months after the initial increase in job separations. It is noticeable that the peak increase in unemployment is much larger than the initial impact of the shock. We also find that the fraction of longer term unemployed workers and the

---

20 We use monthly data from the BLS, detrended using the HP filter with smoothing coefficient $10^9$. The moments of the job separation rate are estimated directly from its empirical counterpart. The moments of the mismatch shock are instead estimated by matching the moments of the US time series with moments from stochastic simulations of the model.
average duration of unemployment both increase significantly. Importantly, vacancy postings drop persistently and therefore, in combination with the higher level of unemployment, imply a long-lived decrease in the job finding rate. Much of the macro-labor literature has concluded that variations in job separations are of little consequence for understanding fluctuations in unemployment since firms have a strong incentive to post more vacancies in periods of high unemployment when hiring is relatively cheap. In our economy firms hold back on vacancy postings and changes in job separations have large effects.

Figure 6 repeats the analysis for an increase in the share of job losers that flow into the unemployment state with low search efficiency (keeping the overall job termination rate constant). We find that unemployment increases stubbornly in response to this shock and so does the average duration of unemployment. Again, the persistence of the increases in the level and duration of unemployment derives from a stubborn drop in vacancy postings that produces a large decline in the job finding rate.

To understand our results, it is instructive to consider the Euler equation for employed households and the first order condition for price setting:

$$
\frac{\partial u(c^n)}{\partial c^n} = \beta \mathbb{E} \frac{R}{1 + \pi'} \left\{ \left( 1 - \sum_{r=s,l} \rho_x \rho_f (1 - \eta'_r) \right) \frac{\partial u(c^u)}{\partial c^u} + \rho_s \rho_s (1 - \eta'_s) \frac{\partial u(c^{u,s})}{\partial c^{u,s}} + \rho_x \rho_l (1 - \eta'_l) \frac{\partial u(c^{u,l})}{\partial c^{u,l}} \right\}
$$

where

$$
1 - \gamma + \gamma \left( w + \frac{\mu}{\rho_f} - \beta \mathbb{E} (1 - \rho_x) \frac{\mu}{\rho'_f} \right) = \phi (1 + \pi) \pi - \beta \phi \mathbb{E} (1 + \pi') \pi' \frac{y'}{y}
$$

21 Initially the increase in job separations produces a drop in average unemployment duration and in the fraction of long-term unemployed workers because of the inflow of newly unemployed workers. This is, however, quickly reversed.
the amplification mechanism occurs because lower vacancy postings implies even worse job finding prospects therefore introducing an even stronger precautionary savings motive which in turn lowers aggregate demand.

The results can be further understood by considering the two alternative economies described above. When prices are flexible (but agents are exposed to idiosyncratic risk), job separations shocks impact much less on unemployment and lead to a much smaller increase in the mean duration of unemployment than in the benchmark economy. In this economy, the increase in job separations has little impact on vacancy postings beyond the first period because firms take advantage of low hiring costs in response to the initial increase in unemployment. The lack of amplification is even starker when considering the mismatch shock which is largely neutralized when prices are flexible. In the flexible price economy there is very little transmission from the demand side to equilibrium quantities because firm cut prices in response to declining demand. Thus, while households still want increase precautionary savings in response to either of the labor market shocks, there is little need to cut back on vacancy postings in equilibrium.

In the economy with insurance within the family, the main impact of increases in either job separations or in the incidence of mismatch is to decrease expected future family income which produces an intertemporal savings motive while the uncertainty effect no longer is relevant. The intertemporal savings motive is very small in the case of mismatch shocks which accordingly have very minor effects in this alternative economy. Job separations have a large effect in the short run but thereafter leads to a muted response of the economy that is very similar to what we found for the flexible price economy.

We conclude from this analysis that the combination of idiosyncratic employment risk, incomplete asset markets and nominal rigidities provide a powerful mechanism through which labor market shocks are amplified.

**The Great Recession.** We now consider the extent to which the mechanisms of the model may be useful for thinking about the features of the Great Recession discussed in Section 2. We derive estimates of the sequences of innovations to job termination and to the fraction of workers that flow into mismatch unemployment, \((\varepsilon_{x,t}, \varepsilon_{s,t})_{t=2007:1}^{2013:1}\) by matching the observed U.S. time-series on the employment-to-unemployment transition rate and the number of unemployed workers who have been out of work for 6 months or more relative to the labor force. In order to avoid having too erratic shocks, we smooth both data series with a 6 months moving average filter. We then feed
the resulting shock processes into the model economy and simulate the economy in response to this particular sequence of shocks.

Figure 7a illustrates the shocks that we estimate for the Great Recession and their impact on the level and average duration of unemployment, the number unemployed workers out of work for 6 months or more (relative to the labor force), and on vacancies. As discussed in Section 2, the Great Recession witnessed a spur of job separations which peaked in early 2009. Conditional upon this shock, we find that the fraction of workers that become mismatched remained flat until 2009 and then increased 25 percentage points from 65 percent in the steady-state to 90 percent in mid-2010. Thus, according to our estimates, the mismatch shock reinforced the negative labor market conditions as the job separation shock started to level out.

In response to the labor market shocks, the benchmark model implies a large and persistent rise in unemployment that is very similar to what was observed in the U.S. economy in the aftermath of the financial crisis. By comparing the results with the effects that we obtain when assuming no mismatch shocks, we see that while the initial increase in unemployment derives from the increase in job separations in the early part of the crisis, the mismatch shock accounts for both the high level of unemployment and its persistence from mid-2009 onwards. We also find that the model can account for a large fraction of the fall in vacancies that occurred during the Great Recession even if it misses the full extent of the vacancy decline in mid-2009. The mismatch shock is even more important for understanding the drop in vacancies. Eliminating this shock, we find only a modest initial decline in vacancies.

The uncertainty channel is key for understanding why the depressed labor market conditions triggered the large increase in unemployment and the decline in vacancies. When we eliminate the idiosyncratic risk mechanism, we find a very muted increase in unemployment because firms respond to higher unemployment levels by increasing vacancy postings. As a result, this economy also predicts a very modest increase in the share of long term unemployed. Nominal rigidities also have an important role to play. Assuming perfectly flexible prices, the model predicts that vacancies remain approximately constant over the duration of the Great Recession and therefore implies a much smaller increase in unemployment than what was observed in the U.S.

Figure 7b illustrates the Beveridge curve observed in the U.S. data and its counterfactual equivalents. One might suspect that the mismatch shock introduces a too large outward shift of the Beveridge curve relative to what is observed in U.S. data but we find that the benchmark model provides a surprisingly good fit to actual data. Similarly to what can be observed in the U.S. data,
the model economy experiences a large slide down the Beveridge curve during the early part of the recession and an outward shift of this curve in the latter part of the recession. Interestingly, when we eliminate the mismatch shock, the model implies a very modest slide down the Beveridge curve but a sizeable outward shift of this relationship. Thus, the decline in search efficiency that we estimate is mainly responsible for generating the large increase in unemployment and a decline in vacancies that can be observed in the U.S. during the Great Recession. When we assume flexible prices or insurance within the family, the model fails to generate a Beveridge curve and implies either a flat relationship between unemployment (when prices are flexible) and vacancies or a positive sloped Beveridge curve (when households can insure).

**The Early 1990’s Recession.** An important check on our analysis is its applicability to other recessions. We now study the early 1990’s recession which was much milder than the Great Recession. As above we feed in the observed U.S. time-series for job separations and estimate the mismatch shock to match the U.S. time-series for the fraction of workers who have been out of work for 6 months more. The resulting series for the job separation rate and the mismatch shock are shown in the top part of Figure 8. We note that a main difference between the early 1990’s recession and the Great Recession is that the shocks that hit the economy were much smaller in the former of these recessions. On the other hand, similarly to the Great Recession, we find that the mismatch shock sets in late after the trough of the contraction thus providing an important source of persistence of the labor market impact of the downturn.

The model can account for much of the rise in unemployment during the early 1990’s recession as well and for the decline in vacancies in the last part of the recession. As in the Great Recession considered earlier we find that the benchmark model generates a much better account of the dynamics of unemployment and vacancies than the flexible price model and the model with insurance against idiosyncratic shocks. Moreover, we also find that the mismatch shock has important quantitative implications. Hence, the mechanisms we have stressed above appear to be relevant also in the early 1990’s recession even if the uncertainty effect appears much more prominent in the Great Recession due to the scale of the shocks that affected the U.S. economy.

5 **Extensions and Robustness Analysis**

This section extends our analysis to investigate three further issues. First we examine the importance of the downward inflexible wage assumption. Next, we investigate how the monetary policy
response matters for the amplification mechanism. We then go one step further and consider the impact of imposing the ZLB on nominal interest rates. We also examine how different monetary policy rules perform in a ZLB environment.

**The Role of Wage Flexibility.** Our benchmark model assumes that real wages are downward inflexible. This assumption appears consistent with BLS evidence on real wages during the Great Recession. Figure 9 illustrates BLS estimates of the average real compensation per hour worked in the Business Sector. According to these data, real wages grew by approximately 25 percent from the mid-1990’s to the beginning of 2007 but has since remained flat. Yet, it is still interesting to investigate the importance of our assumption regarding downward inflexible real wages and therefore now assume alternatively that wages are determined by Nash bargaining.

We assume that once workers and firms have been matched, a worker flows to the two unemployment pools with the same probability as newly unemployed workers so that the past unemployment state is irrelevant for the outside option. This assumption combined with the borrowing constraint, implies that the equilibrium wealth distribution is degenerate making the outcome under Nash bargaining particularly simple.

It is well known that the properties of matching models depend crucially on the distribution of the match surplus (i.e. the bargaining power), see e.g. Hagedorn and Manovskii (2008). For that reason we report results for a wide range of values of the workers’ bargaining power which includes both Hagedorn and Manovskii’s (2008) calibration of workers receiving 5 percent of the match surplus to ‘traditional’ values of this parameter of 50 percent.

Figure 10 illustrates the impact of a job separation shock on unemployment and on real wages. We report the maximum increase in unemployment and the maximum decline in the real wage as a percentage of the steady-state real wage. We find that higher bargaining power on the

22 Alternatively, one can assume that workers retain their unemployment status but in this case the outcome would be that there arises one-period wage dispersion which seems unreasonable.

23 The generalized Nash bargaining solution satisfies:

\[ \phi \frac{\mu}{\rho f_t} = (1 - \phi) \left[ V^n (b^h_i, S) - \rho_i V^{n,l} (b^h_i, S) - (1 - \rho_i) V^{n-s} (b^h_i, S) \right] c^n (b^h_i, S) \]

where \( \phi \) is the bargaining power of the worker. Here, \( \frac{\mu}{\rho f_t} \) is the surplus of a match to an entrepreneur, which is equal to the (expected) cost of hiring a new worker. The term between square brackets on the right-hand side is the surplus of a match to a household.

24 We assume that workers enjoy leisure when unemployed and calibrate the utility value of leisure so that the
part of workers implies higher wage flexibility in equilibrium and a significantly smaller maximum response of unemployment. For example, when workers and firms have the same bargaining power, the maximum response of unemployment is less than 40 percent of the corresponding response in the benchmark economy. Assuming instead that workers’ bargaining power is 5 percent, implies a maximum increase in unemployment that is only 30 percent lower than the increase we found when assuming inflexible real wages. To understand these results, recall that workers’ outside options worsen when labor market conditions deteriorate. For that reason, real wages will tend to fall after an increase in the job separation rate and the associated decline in marginal costs helps alleviating the need for the job filling rate to increase to restore equilibrium. The higher is workers’ bargaining power, the larger is the fall in the real wage and therefore the amount of stabilization provided by real wage flexibility.

As we have noted above, real wages did not decline much, if at all, during the Great Recession. The results presented here imply that this may either be consistent with workers have little bargaining power or with real wages are downward inflexible. These two alternative scenarios have very similar implications for equilibrium quantities and would therefore be difficult to disentangle empirically.

The Role of Monetary Policy. It is standard intuition in the monetary economics literature that aggressive nominal interest rate responses to inflation can neutralize the inefficiencies that derive from nominal rigidities while too weak responses to inflation produce locally indeterminate equilibria. In our setting, these issues are more complicated because of the combination of incomplete markets and nominal rigidities, but the policy relevance is obviously just as pertinent.

Figure 11 reports how job separation rate shocks impact on unemployment in the benchmark economy as a function of two key parameters, $\delta$ and $\sigma$. $\delta$ determines the response of the nominal interest rate to deviations of inflation from its target while $\sigma$ determines the extent to which households respond to employment risk. We indicate by different colors the amplification of the labor market shocks in the benchmark economy by normalizing the maximum impact on unemployment of the job separation shock with the equivalent response in a flexible price economy. A dark blue color means no amplification relative to the flexible price economy with lighter shades of blue and yellow and orange colors indicating ever increasing degrees of amplification. The white area corre-

---

22 steady-state equilibrium real wage implies a 5 percent unemployment rate.

25 The results for mismatch shocks are nearly identical so we do not report them here.
sponds to combinations of $\delta$ and $\sigma$ that are inconsistent with local determinacy of the equilibrium where inflation is on target.

We find that sufficiently aggressive monetary policy rules succeed in neutralizing the amplification mechanism while interest rate rules similar to those typically assumed in the New Keynesian literature instead produce a large amount of amplification. In our calibration ($\delta = \sigma = 1.5$), both shocks are significantly amplified but increasing $\delta$ to around 3 neutralizes much of the amplification relative to the flexible price allocation. More aggressive of monetary policy responses provide stabilization by moderating the agents’ expectations regarding the impact of the shocks on equilibrium inflation and vacancy postings and thus impact directly on the mechanism through which labor market shocks are amplified.

Our results also show that higher degrees of risk aversion demand more aggressive policy rules in order to provide stabilization and that, the higher is the degree of risk aversion, the more aggressive has the interest rate rule to be in order to ensure local indeterminacy of the intended equilibrium. These features derive from the impact of risk aversion on precautionary savings. When agents are more risk averse, there is a stronger impact of labor market uncertainty on precautionary savings which motivates the need for a more aggressive monetary policy stance in order to stabilize the economy.\footnote{\$\sigma$ also impacts on the determinacy region in standard New Keynesian models, see e.g. Gali (2008), but only because it impacts on the response of consumption to real interest rates. In our model, a more important source of impact is through precautionary savings.} In the indeterminacy region, equilibria can exist in which agents’ expectations of worsening labor market outcomes and low inflation drives down aggregate demand thereby motivating firms to hire less labor and leading the economy to a high-unemployment-cum-low-inflation self-fulfilling equilibria. Thus, the design of the monetary reaction function is critical in the incomplete markets set-up analyzed in this paper.

**The Zero Lower Bound.** During the early stages of the Great Recession, the Federal Reserve engaged in successive cuts of nominal interest rates and the ZLB on nominal interest rates has now been binding since late 2008. It is still unclear which underlying shocks and mechanisms have led the economy into this liquidity trap so and it is therefore interesting to confront our model with the ZLB to examine whether labor market uncertainties may have been a contributory factor due to their impact on savings.
We modify the monetary policy rule:

\[
R_t = \max \left[ R \left( \frac{1 + \pi_t}{1 + \pi} \right)^{\delta}, 1 \right]
\]

which assumes that the central bank sets interest rates according to the interest rate rule specified earlier as long as the nominal interest rate is positive. Similar specifications have been assumed in much of the literature that has examined the ZLB under interest rate rules.

We make one further adjustment to the model. At the ZLB, the nominal interest rate can no longer fall and an equilibrium with positive vacancy posting may fail to exist. In this case, the real wage no longer belongs to the bargaining set.\(^{27}\) For that reason we allow the real wage to fall temporarily as long as the ZLB binds. We assume the following wage rule:

\[
w = \min (\bar{w}, \bar{w} + \xi (\eta_{s,t} - \bar{\eta}))
\]

where \(\xi, \bar{\eta} \geq 0\) are constant parameters. This specification implies that wages drop once the job finding rate amongst searchers falls below a certain threshold, \(\bar{\eta}\). We assume that \(\bar{\eta} = 0.6\) and that \(\xi = 0.03\). We follow the same steady-state calibration strategy as for our benchmark model and choose the same values for the same persistence parameters of the shock processes. Given the larger degree of amplification in the model with ZLB, we reduce the variances of the innovations to the job termination rate and the fraction into mismatch to \(\nu_x = 1 \cdot 10^{-4}\) and \(\nu_s = 0.001\), respectively.\(^{28}\) We also lower the persistence of shocks to the job termination rate to \(\lambda_x = 0.9\). We solve the non-linear model using a time iteration projection method (see the Appendix for details).

Figure 12 repeats the Great Recession exercise now imposing the ZLB. As above, we feed in the observed job termination process and we calibrate the mismatch shock so that we reproduce the observed U.S. time-series for long-term unemployment as a fraction of unemployment. The

\(^{27}\)To appreciate this point, recall that the equilibrium must satisfy the subsystem comprised by Equations (29) and (30). No endogenous state variable enters this system, so its solution is a function of the exogenous state only. When \(R = 1\) and real wages are fixed, there is no current-period variable that can adjust to ensure that Equation (29) holds in any state of the world. It is straightforward to show that in a perfect-foresight version of the model without persistence in the exogenous variables, the Euler equation generally has no solution at the ZLB if real wages are fixed. Expectedly, we were also unable to find a numerical solution more general versions of the model with the ZLB and a fixed real wage.

\(^{28}\)The solution to the benchmark model does not depend on the variances of the exogenous shocks, due to the certainty equivalence property implied by the first-order perturbation technique. The non-linear solution to the model with the zero lower bound does depend on the variances of the shocks.
resulting process for the mismatch shock which is very similar to what we showed in figure 7. With these shocks, the model reproduces quite precisely the observed path of the nominal interest rate with the ZLB binding from late 2008 until the end of the sample. Thus, according to these results, the labor market shocks were sufficiently large that the increase in precautionary savings triggered a drop in nominal interest rates that took the economy into a liquidity trap. We further find that the real wage must fall but only very marginally in response to these shocks (the largest decline in the real wage is less than 1 percent in our simulation) while unemployment rises by 11 percentage points at the peak in early 2010. The further amplification of the labor market shocks produced by the ZLB derives from two mechanisms. First, real wages fall because $\eta_{s,t} < \bar{\eta}$ which ensures that the solution is consistent with the real wage belonging to the bargaining set. This in itself implies a drop in demand. Moreover, at the ZLB, equilibrium output has to fall even further in order to clear the asset market and this produces even more job uncertainty – and therefore precautionary savings – as firms cut back on hiring. The implied increase in unemployment is larger than what has been observed in the U.S. economy during the Great Recession. One interpretation of this result is that there were other shocks affecting the economy or that other policy interventions such as unconventional monetary policies or fiscal policies may have had a stabilizing effect.

We showed above that a more aggressive interest rate rule stabilizes the economy in the benchmark economy and one may wonder whether this is also the case when we impose the ZLB. A priori the answer to this question does not seem obvious. On the one hand, a more aggressive of monetary policy rule moderates the agents’ expectations regarding the impact of the shocks on equilibrium inflation and therefore provides stabilization. On the other hand, a more aggressive rule may push the economy faster towards the ZLB by implementing a larger interest rate cut in response to a fall in inflation than a less aggressive rule. Thus, to examine the impact of following a more aggressive policy stance, Figure 13 repeats the Great Recession experiment when setting $\delta = 3$ feeding in the same shocks as in Figure 12. We find that the stabilizing effect dominates and that the economy avoids the ZLB when the interest rate rule is more aggressive. As a consequence, we find a dramatically smaller increase in unemployment and in the share of long-term unemployed workers which now (at their peaks) increase by 2 and 5 percentage points, respectively. The fact that the ZLB is avoided does, of course, depend on the shocks that we feed in to compute the counterfactual. However, the result that a more aggressive rule provides stabilization and may avoid the ZLB where other more accommodating monetary policy rules may allow for a liquidity

\footnote{See also Christiano, Eichenbaum and Rebelo (2011).}
6 Conclusions and Summary

We have shown how frictions in labor markets that interact with goods and financial markets frictions can lead to a significant amplification of labor market shocks in a general equilibrium framework. At the heart of our theory is the idea that labor market shocks lead to job uncertainty which triggers households to engage in precautionary savings thereby impacting on the level of aggregate demand. A calibrated version of the model can account not only for the increase in unemployment observed in the U.S. during the Great Recession but also for much of the movements in the Beveridge curve. It is the transmission of weak aggregate demand due to job uncertainty to aggregate supply that produces these results because of an endogenous amplification mechanism. We have shown that this amplification mechanism is strong when downward wage flexibility is limited and when monetary policy is insufficiently aggressive. We also demonstrated that job uncertainty may be a candidate for the shock that has taken the U.S. economy to a liquidity trap from which it is yet to escape.

Our emphasis on job uncertainty deriving from idiosyncratic employment risk and uncertain outcomes of labor market search offers an additional route through which macroeconomic uncertainty can impact on the economy. Much recent literature has focused upon uncertainty shocks deriving from changes in the volatility of aggregate variables (such as TFP or policy related variables) and shown that such uncertainty can be important for understanding aggregate fluctuations, see e.g. Bloom (2009). As Carroll and Dunn (1997) we have instead focused on the idiosyncratic uncertainty implications of first moment shocks to job loss and job finding probabilities. We plan in future work to further examine the importance of this channel using micro level data on consumption and labor supply.

Our theory has abstracted from aggregate savings and we imposed that households cannot go into debt. These assumptions are appealing from a computational perspective but it would be interesting to relax them both so that one can also evaluate the impact on aggregate savings and investment. It would also be interesting to investigate the impact of unemployment insurance policies. Unemployment insurance duration is usually extended during U.S. recessions and the Great Recession is no exception to this. Our exercise does take this into account because we focus entirely upon recessions and the calibration of the level of benefits targets Hurd and Rohwedder’s
estimates of the consumption loss due to unemployment shocks during the Great Recession. Yet, it would be of interest to investigate further how such cyclical variations in unemployment benefits impact on the aggregate outcomes. We plan to pursue each of these issues in future work.

7 References


Kroft, Kory, Fabian Lange, Matthew J. Notowidigo and Lawrence F. Katz. 2013. “Long-
Term Unemployment and the Great Recession: The Role of Composition, Duration Dependence, and Non-Participation.” Manuscript, University of Toronto.


7.1 Appendix 1: Solving the benchmark model

Given the borrowing constraint that we impose, it follows that in equilibrium, the individual workers’ consumption levels are given as:

\[ c^u = c^s = \xi \]
\[ c^n = w \]

Given the equilibrium definition, we can reduce the model down to the following two equations:

\[
\frac{\partial u(c^n)}{\partial c^n} = \beta E \frac{R}{1 + \pi} \left\{ 1 - \sum_{r=s,l} \rho_x \rho_r (1 - \eta'_r) \right\} \frac{\partial u(c^n)}{\partial c^n} \\
+ \rho_x \rho_s (1 - \eta'_s) \frac{\partial u(c^{u,s})}{\partial c^{u,s}} + \rho_x \rho_l (1 - \eta'_l) \frac{\partial u(c^{u,l})}{\partial c^{u,l}} \]

\[ (1 - \gamma) + \gamma \left( w + \frac{\mu}{\rho_f} - \beta E (1 - \rho_x) \frac{\mu}{\rho'_f} \right) = \phi (1 + \pi) \pi - \beta E (1 + \pi') \pi' \]

plus the laws of motion for the stocks of unemployment and the monetary policy rule. This system can be further simplified using the expression for consumption above and realizing that \( \eta_s = q \mu_l = \psi^{1/\alpha} \rho_f^{(\alpha-1)/\alpha} \). We solve this system of equations using a standard perturbation approach (note that expected output growth drops out from equation (36) when it is linearized). All other endogenous variables can be derived from the remaining equilibrium conditions of the model.

7.2 Appendix 2: Solving the ZLB model

Our solution procedure is based on (35) – (36), in addition to the real wage rule and the monetary policy rule. We approximate (36) as:

\[ 1 - \gamma + \gamma \left( w + \mu \eta_s - \beta E (1 - \rho_x) \mu \eta'_s \right) = \phi (1 + \pi) \pi - \beta E (1 + \pi') \pi' \]

which differs from (36) only by not including the output growth term. We verify the accuracy of this approximation.

We solve the model using a time iteration algorithm with the following steps:

1. Construct a grid for the exogenous state variables. We use 100 grid points.

2. Guess initial policy functions for the job finding and for the inflation rate as functions of the two exogenous shock variables. We use third-order polynomials to approximate these policy functions.
3. At each grid point:

- Evaluate the conditional expectations, using the guesses for the policy rules. We use a Gauss-Hermite quadrature approximation with 16 nodes (4 in each dimension).\(^{30}\)

- Solve for \( R \) from the household’s Euler equation.
  
  - If \( R > 1 \), solve for \( \pi \) from the interest rate rule. Also, set \( w = \bar{w} \) and solve for \( \eta_s \) from the firm’s pricing equation (see additional notes below).
  
  - If \( R = 1 \), solve for \( w \) from the household’s Euler equation. Use the wage rule to solve for \( \eta_s \) and solve for \( \pi \) from the firm’s pricing equation.

4. Update the guesses for the policy rule using a regression.

5. Go back to step 3 until the policy rules no longer change with new iterations

6. Check that in equilibrium it is the case that \( R = 1 \) iff \( \eta_s \leq \bar{\eta} \).

We verify the accuracy of our approximation as follows. Along the simulated path, we compute two alternative time series for inflation, using the firm’s pricing equation. The first series, labeled \( A \), is computed by evaluating the right hand side of the first equation below, using the policy rules and Gauss-Hermite quadrature. Inflation is then solved for from the left hand side. The second series, labeled \( B \), we compute in the same way, but now we replace the term \( \beta \phi \mathbb{E} (1 + \pi') \pi' \frac{\bar{y}}{y} \) by its approximation, \( \beta \phi \mathbb{E} (1 + \pi') \pi' \):

\[
\phi \left(1 + \pi^A\right) \pi^A = 1 - \gamma + \gamma \left(w + \mu \eta_s - \beta \mathbb{E} (1 - \rho_x) \mu \eta'_s\right) + \beta \phi \mathbb{E} (1 + \pi') \pi' \frac{\bar{y}}{y} \\
\phi \left(1 + \pi^B\right) \pi^B = 1 - \gamma + \gamma \left(w + \mu \eta_s - \beta \mathbb{E} (1 - \rho_x) \mu \eta'_s\right) + \beta \phi \mathbb{E} (1 + \pi') \pi'
\]

Finally, we also show the path for inflation that we get directly from the policy function. The plot below shows the results.

\(^{30}\)The shock innovations are assumed to be normally distributed.
First, note that the series for $\pi^A$ and $\pi^B$ are nearly on top of each other. Hence, the approximation loss due to the first-order Taylor approximation of the term on the right-hand side of the firm’s pricing condition appears minimal. Also, the two series are extremely similar to the series obtained directly from the third-order polynomial approximation of the policy rule.
### 7.3 Tables and Figures

#### Table 1: Targets and Parameter Values

<table>
<thead>
<tr>
<th>Targets</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.05  unemployment rate</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.4   average job finding rate among newly unemployed</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.3   average job finding rate among unemployed &gt; 6m</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.045 hiring cost as a fraction of the quarterly wage</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.15  fraction of long-term unemployed</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.11  consumption loss upon unemployment $(\xi, \xi')$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0     net inflation rate</td>
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<td></td>
</tr>
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</table>

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{p}_x$ steady state job termination rate</td>
<td>0.035</td>
<td></td>
</tr>
<tr>
<td>$\bar{p}_l$ steady state fraction into mismatch</td>
<td>0.65</td>
<td></td>
</tr>
<tr>
<td>$\lambda_x$ Persistence of shocks to job termination rate</td>
<td>0.991</td>
<td></td>
</tr>
<tr>
<td>$\nu_x$ Standard deviation of shocks to job termination rate</td>
<td>4.7\times 10^{-4}</td>
<td></td>
</tr>
<tr>
<td>$\lambda_s$ Persistence of shocks to fraction into mismatch</td>
<td>0.881</td>
<td></td>
</tr>
<tr>
<td>$\nu_s$ Standard deviation of shocks to fraction into mismatch</td>
<td>0.0513</td>
<td></td>
</tr>
<tr>
<td>$\psi$ ratio of vacancy cost over matching efficiency parameter</td>
<td>0.13</td>
<td></td>
</tr>
<tr>
<td>$q$ probability of search for mismatched unemployed</td>
<td>0.37</td>
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</tr>
<tr>
<td>$\bar{w}$ real wage</td>
<td>0.83</td>
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</tr>
<tr>
<td>$\xi$ unemployment benefit</td>
<td>0.74</td>
<td></td>
</tr>
<tr>
<td>$\bar{r}$ steady state nominal interest rate</td>
<td>0.0033</td>
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</table>

#### Table 2: Stationary State Values

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Meaning</th>
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</thead>
<tbody>
<tr>
<td>$c_e$</td>
<td>0.829</td>
<td>consumption employed</td>
</tr>
<tr>
<td>$c_{u,s}$</td>
<td>0.737</td>
<td>consumption regular unemployed</td>
</tr>
<tr>
<td>$c_{u,l}$</td>
<td>0.737</td>
<td>consumption mismatch unemployed</td>
</tr>
<tr>
<td>$\eta_s$</td>
<td>0.848</td>
<td>job finding rate searchers</td>
</tr>
<tr>
<td>$\eta_l$</td>
<td>0.310</td>
<td>job finding rate mismatched</td>
</tr>
<tr>
<td>$u_s$</td>
<td>0.0020523</td>
<td>mass of regular unemployed</td>
</tr>
<tr>
<td>$u_l$</td>
<td>0.0226742</td>
<td>mass of mismatch unemployed</td>
</tr>
</tbody>
</table>
Figure 1: The Civilian Unemployment Rate

Figure 2: Estimates of Job Separation and Job Finding Rates
Figure 3: The Inverse Instantaneous Job Finding Rate and Mean Unemployment Duration

Figure 4: The Beveridge Curve
Figure 5: The Impact of Job Separation Shocks
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Figure 7a: The Great Recession (all variables in deviations from their steady-state)
Figure 7b: Actual and Counterfactual Beveridge Curves
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Figure 11: Amplification and the Monetary Policy Rule
Figure 12: The Great Recession and the ZLB (all variables apart from interest rates and wages in deviations from steady-state)
Figure 13: The Great Recession and the ZLB with Aggressive Monetary Policy (all variables apart from interest rates and wages in deviations from steady-state)