MACROECONOMIC FLUCTUATIONS WITH HANK & SAM: AN ANALYTICAL APPROACH - ONLINE APPENDIX

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Abstract
This document contains 1o appendices: Characterization of the Steady-States; The Log-Linearized Model; Extension to Risk Averse Capitalists; The Euler Equation at the ZLB; Monetary Policy and the Unemployment Trap; Less Extreme Unemployment Traps; The Model with Capital Accumulation; Nominal Stickiness in Unemployment Benefits; Pricing Risky Assets; Implications for the Zero Lower Bound;.

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Appendix

A1. Steady-state Properties

A1.1. Nash Bargaining Solution. The steady-state expressions of the asset-poor households' surplus and value functions are:

$$\begin{aligned} V^n\left(1-\beta\left(1-\omega\left(1-\eta\right)\right)\right) &= \frac{w^{1-\mu}}{1-\mu}-\zeta+\beta\omega\left(1-\eta\right)V^u, \\ V^u\left(1-\beta\left(1-\eta\right)\right) &= \frac{\vartheta^{1-\mu}}{1-\mu}+\beta\eta V^n, \end{aligned}$$

where we have exploited that in equilibrium the asset-poor households are the same and consume their incomes. Now substitute out V^u in the first equation:

$$V^n\left(1-\beta\left(1-\omega\left(1-\eta\right)\right)\right)=\frac{w^{1-\mu}}{1-\mu}-\zeta+\frac{\beta\omega\left(1-\eta\right)}{1-\beta\left(1-\eta\right)}\left(\frac{\vartheta^{1-\mu}}{1-\mu}+\beta\eta V^n\right).$$

$$V^n\left(1-\beta\left(1-\omega\left(1-\eta\right)\right)-\frac{\beta\omega\left(1-\eta\right)}{1-\beta\left(1-\eta\right)}\beta\eta\right)=\frac{w^{1-\mu}}{1-\mu}-\zeta+\frac{\beta\omega\left(1-\eta\right)}{1-\beta\left(1-\eta\right)}\frac{\vartheta^{1-\mu}}{1-\mu},$$

We can now express the two values as functions of η and w:

$$\begin{aligned} V^n\left(\eta,w\right) &= \frac{\frac{w^{1-\mu}}{1-\mu}-\zeta+\frac{\beta\omega(1-\eta)}{1-\beta(1-\eta)}\frac{\vartheta^{1-\mu}}{1-\mu}}{1-\beta\left(1-\omega\left(1-\eta\right)\right)-\frac{\beta\omega(1-\eta)\beta\eta}{1-\beta(1-\eta)}} \\ V^u\left(\eta,w\right) &= \frac{\frac{\vartheta^{1-\mu}}{1-\mu}+\beta\eta V^n\left(\eta,w\right)}{1-\beta\left(1-\eta\right)} \end{aligned}$$

The first-order condition to the Nash Bargaining problem is given by

$$\left(1-\upsilon\right)S^n=\upsilon S^f\ ,$$

or,

$$\left(1-\upsilon\right)\left(V^n\left(\eta,w\right)-V^u\left(\eta,w\right)\right)=\upsilon\kappa\eta^{\alpha/(1-\alpha)}\ .$$

$$\left(V^n\left(\eta,w\right)-V^u\left(\eta,w\right)\right)=\frac{\upsilon}{1-\upsilon}\kappa\eta^{\alpha/(1-\alpha)}$$

The above is an equation in two variables, which implicitly defines the wage as a function of the job finding rate, i.e the function $w(\eta)$.

Basic properties: Consider the special case in which $\eta=0$. From the Nash bargaining solution it follows that the wage must satisfy $V^n\left(0,w(0)\right)=V^u\left(0,w(0)\right)=\frac{\frac{\vartheta^{1-\mu}}{1-\mu}}{1-\beta}$. It follows that $\frac{w(0)^{1-\mu}}{1-\mu}=\frac{\vartheta^{1-\mu}}{1-\mu}+\zeta$ and hence $w(0)>\vartheta$ whenever $\zeta>0$.

At the other extreme, under $\eta=1$ we get from the Nash Bargaining solution $V^n\left(1,w\right)=V^e\left(1,w\right)+\frac{\upsilon\kappa}{1-\upsilon}$. Also, the worker value functions imply that $V^n\left(1,w\right)-$

$V^u(1,w) = \frac{w(1)^{1-\mu}}{1-\mu} - \zeta - \frac{\vartheta^{1-\mu}}{1-\mu}$. It follows that $\frac{w(1)^{1-\mu}}{1-\mu} = \frac{\vartheta^{1-\mu}}{1-\mu} + \zeta + \frac{\upsilon}{1-\upsilon}\kappa$ and hence $w(1) > w(0)$, $V^n(1, w(1)) > V^n(0, w(0))$ and $V^u(1,w) > V^u(0,w)$.

Finally, consider a case in which the worker has no bargaining power ($\upsilon = 0$). It follows from the Nash bargaining solution that in this case $V^n(\eta, w) = V^u(\eta, w)$ which implies that $\frac{w(\eta)^{1-\mu}}{1-\mu} = \frac{\vartheta^{1-\mu}}{1-\mu} + \zeta$. As a result, the real wage does not depend of η, i.e. the real wage is sticky.

A.1.2. Positive Liquidity. We now plot the steady-state curves, as illustrated qualitatively in Figure 2, but this time for a calibrated version of the model with positive liquidity, i.e. a positive aggregate supply of bonds. Both curves are computed numerically. The purpose of the exercise is to give an example of a calibrated model for which the unemployment trap occurs, rather than a full-blown quantitative exploration.

In particular, we choose the subjective discount factor β to imply a subjective discount rate of 6 percent per annum.[1] The coefficient of risk aversion, μ, is set to 2, whereas the elasticity of substitution between goods, γ, is set to 6. To calibrate the price-stickiness parameter φ, we exploit the observational equivalence between the Calvo and Rotemberg versions of the log-linearized New Keynesian model, and target an average price duration of 12 months. The home production parameter, ϑ, is set to imply a 20 percent income drop upon unemployment.

Parameter values (monthly model)		
δ_π	1.5	Taylor rule coefficient inflation
δ_θ	0	Taylor rule coefficient tightness
μ	2	coefficient of risk aversion
γ	6	elasticity of substitution goods varieties
χ	0	real wage flexibility parameter
ω	0.02	separation rate
$\int b_{i,s} di$	0.1	aggregate bond supply
α	0.5	matching function elasticity
κ	0.08667	vacancy cost
φ	626	price adjustment cost parameter
ϑ	0.8	income drop upon unemployment

We further target a monthly job finding rate of about 0.3 in the intended steady state and set the job loss rate, ω, to 2 percent. The matching function elasticity parameter, α, is set to 0.5. Regarding the monetary policy rule, we set $\delta_\pi = 1.5$ and $\delta_\theta = 0$. For simplicity we assume sticky wages ($\chi = 0$). The vacancy cost is parameterized to target a hiring cost of about 10 percent of the monthly wage. Finally, the aggregate bond supply is set to 0.1, hence the calibration features positive liquidity.

1. In the intended steady state, the equilibrium real interest rate is about 1.4 percent per year.

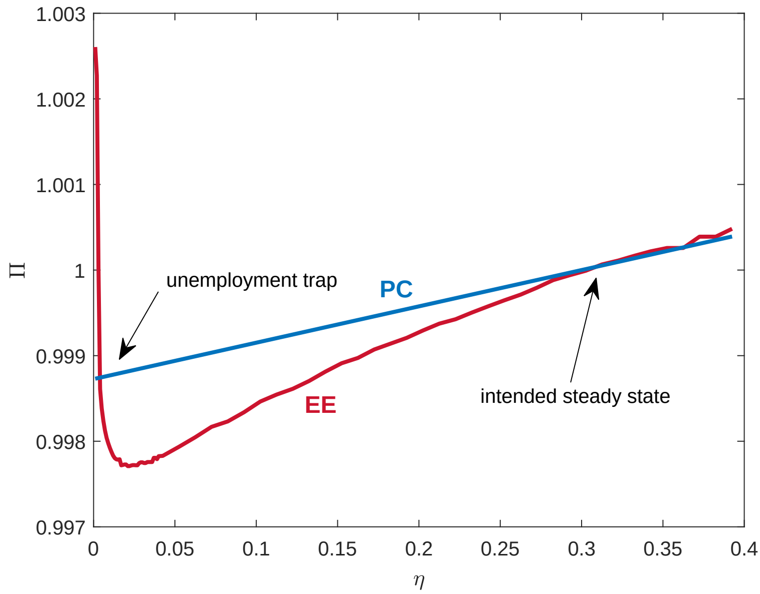


Figure 1. Steady state curves: positive liquidity (economy away from ZLB).

The figure below shows plots the steady state curves for the economy away from the ZLB. The Euler Equation (EE) curve is computed by computing the steady-state real interest rate which clears the bond market, given a job finding rate η.[2] This real interest rate is then combined with the monetary policy rule in order to compute the associated rate of inflation. The Phillips Curve (PC) is computed directly from the firms' steady-state first-order condition for prices.

The figure shows two intersections between the two steady state. The right intersection is the "intended steady state" whereas the left intersection is the "unemployment trap". Interestingly, the latter steady state now occurs at a positive job finding rate, unlike in the zero liquidity model. The reason is that, with positive liquidity, the EE curve is downward sloping at very low job finding rates, i.e. high unemployment rates. When only a few households are employed, then these households own a large fraction of the total wealth in the economy and they are therefore relatively well protected against job loss. This weakens their precautionary saving motive. In this zero-liquidity version of the model, this effect does not occur, as no household owns any bonds.

2. Given η, we solve the hosueholds' problem for different real interest rates, and find the real interest rate which clears the market. This procedure is then repeated for different values of η

A2. The Log-linearized Model

Nash Bargaining Block. The first-order condition to the Nash bargaining problem, together with the asset-poor workers' value functions are given by:

$$\begin{aligned}
(1-\upsilon)\left(V_s^n - V_s^u\right) &= \upsilon\kappa\eta_s^{\alpha/(1-\alpha)}, \\
V_s^n &= \frac{w_s^{1-\mu}}{1-\mu} - \zeta + \beta\mathbb{E}_s\omega\left(1-\eta_{s+1}\right)V_{s+1}^u + \beta\mathbb{E}_s\left(1-\omega\left(1-\eta_{s+1}\right)\right)V_{s+1}^n, \\
V_s^u &= \frac{\vartheta^{1-\mu}}{1-\mu} + \beta\mathbb{E}_s\left(1-\eta_{s+1}\right)V_{s+1}^u + \beta\mathbb{E}_s\eta_{s+1}V_{s+1}^n.
\end{aligned}$$

After log-linearization, the above system can be written in the following form:

$$\boldsymbol{A}\begin{bmatrix}\widehat{V}_s^n \\ \widehat{V}_s^u \\ \widehat{w}_s\end{bmatrix} + \boldsymbol{B}\widehat{\eta}_s = \mathbb{E}_s\boldsymbol{C}\begin{bmatrix}\widehat{V}_{s+1}^n \\ \widehat{V}_{s+1}^u \\ \widehat{w}_{s+1}\end{bmatrix} + \mathbb{E}_s\boldsymbol{D}\widehat{\eta}_{s+1}$$

where $\boldsymbol{A}$ and $\boldsymbol{C}$ are 3×3 matrices and $\boldsymbol{B}$ and $\boldsymbol{D}$ are 3×1 vectors, all consisting of parameter values. Note that none of the variables $\widehat{V}_s^n$, $\widehat{V}_s^u$ and $\widehat{w}_s$ is a state variable. Provided that $\widehat{\eta}_s$ follows some linear law of motion and given the law of motion for A_s, we can apply the method of undetermined coefficients to find solutions for $\widehat{V}_s^n$, $\widehat{V}_s^u$ and $\widehat{w}_s$ as linear functions of $\widehat{\eta}_s$. We denote the solution for the wage as $\widehat{w}_s = \chi\widehat{\eta}_s$, where it follows that χ is a function of the parameters that enter $\boldsymbol{A}$, $\boldsymbol{B}$, $\boldsymbol{C}$ and $\boldsymbol{D}$.

Monetary Policy Rule, Euler Equation, Phillips Curve. The log-linerarized monetary policy rule is given by:

$$\widehat{R}_s = \delta_\pi\widehat{\Pi}_s + \delta_\theta\widehat{\theta}_s.$$

Next, consider the Euler equation of the employed households. Exploiting the fact that in Equilibrium $c_{n,s} = w_s$ and $c_{e,s} = \vartheta,$ we can express the employed workers' Euler equation, Equation (27), as:

$$w_s^{-\mu} = \beta\mathbb{E}_s\frac{R_s}{\Pi_{s+1}}\left(\omega\left(1-\eta_{s+1}\right)\vartheta^{-\mu} + \left(1-\omega\left(1-\eta_{s+1}\right)\right)w_{s+1}^{-\mu}\right),$$

and note that in the intended steady state we obtain $w^{-\mu} = \beta\overline{R}\left(\omega\left(1-\eta\right)\vartheta^{-\mu} + \left(1-\omega\left(1-\eta\right)\right)w^{-\mu}\right)$

Log-linearizing the above equation around the intended steady state gives:

$$\begin{aligned}
-\mu\widehat{w}_s &= \widehat{R}_s - \mathbb{E}_s\widehat{\Pi}_{s+1} - \beta\overline{R}\omega\eta\left(\vartheta/w\right)^{-\mu}\mathbb{E}_s\widehat{\eta}_{s+1} + \beta\overline{R}\omega\eta\mathbb{E}_s\widehat{\eta}_{s+1} - \mu\beta\overline{R}\left(1-\omega\left(1-\eta\right)\right)\mathbb{E}_s\widehat{w}_{s+1}, \\
&= -\mu\beta\overline{R}\mathbb{E}_s\widehat{w}_{s+1} + \widehat{R}_s - \mathbb{E}_s\widehat{\Pi}_{s+1} - \beta\overline{R}\omega\eta\left(\left(\vartheta/w\right)^{-\mu} - 1\right)\mathbb{E}_s\widehat{\eta}_{s+1} + \mu\beta\overline{R}\omega\left(1-\eta\right)\mathbb{E}_s\widehat{w}_{s+1}, \\
&= -\mu\beta\overline{R}\mathbb{E}_s\widehat{w}_{s+1} + \widehat{R}_s - \mathbb{E}_s\widehat{\Pi}_{s+1} - \beta\overline{R}\Theta^F\mathbb{E}_s\widehat{\eta}_{s+1},
\end{aligned}$$

where $\Theta^F = \omega\eta\left(\left(\vartheta/w\right)^{-\mu} - 1\right) - \chi\mu\omega\left(1-\eta\right)$ and where we used that $\widehat{w}_s = \chi\widehat{\eta}_s$.

Next, consider the firms' price setting condition, which can be written as:

$$\begin{aligned} & \varphi\left(\Pi_s-1\right)\Pi_s-\varphi\mathbb{E}_s\Lambda_{s,s+1}\frac{y_{s+1}}{y_s}\left(\Pi_{s+1}-1\right)\Pi_{s+1} \\ = \quad & 1-\gamma+\frac{\gamma}{\exp\left(A_s\right)}\left(w_s+\kappa\eta_s^{\alpha/(1-\alpha)}-\left(1-\omega\right)\kappa\mathbb{E}_s\Lambda_{s,s+1}\eta_{s+1}^{\alpha/(1-\alpha)}+\lambda_{v,s}\right). \end{aligned}$$

and note that at the intended steady state $\lambda_{v,s}=0$ and $\Lambda_{s,s+1}=\beta$. Log-linearizing the equation around the intended steady state with $\Pi=1$ gives:

$$\frac{\varphi}{\gamma}\widehat{\Pi}_s-\frac{\varphi}{\gamma}\beta\mathbb{E}_s\widehat{\Pi}_{s+1}=w\chi\widehat{\eta}_s+\frac{1-\gamma}{\gamma}A_s+\frac{\kappa}{q}\left(\frac{\alpha}{1-\alpha}\widehat{\eta}_s-\frac{\alpha\beta\left(1-\omega\right)}{1-\alpha}\mathbb{E}_s\widehat{\eta}_{s+1}-\beta\left(1-\omega\right)\mathbb{E}_s\widehat{\Lambda}_{s,s+1}\right),$$

where we have substituted out the wage using $\widehat{w}_s=\chi\widehat{\eta}_s$.

Reducing the Model. Under the the two assumptions ($\delta_\pi=\frac{1}{\beta}$ and risk-neutrality of the equity investors) and in the absence of productivity shocks, the log-linearized Euler equation and pricing condition become:

$$\begin{aligned} -\mu\chi\beta\widehat{\eta}_s+\mu\beta^2\overline{R}\chi\mathbb{E}_s\widehat{\eta}_{s+1} &= \widehat{\Pi}_s-\beta\mathbb{E}_s\widehat{\Pi}_{s+1}+\frac{\beta\delta_\theta}{1-\alpha}\widehat{\eta}_s-\beta^2\overline{R}\Theta^F\mathbb{E}_s\widehat{\eta}_{s+1} \\ w\chi\widehat{\eta}_s+\frac{\kappa}{q}\left(\frac{\alpha}{1-\alpha}\widehat{\eta}_s-\frac{\alpha\beta\left(1-\omega\right)}{1-\alpha}\mathbb{E}_s\widehat{\eta}_{s+1}\right) &= \frac{\varphi}{\gamma}\left(\widehat{\Pi}_s-\beta\mathbb{E}_s\widehat{\Pi}_{s+1}\right) \end{aligned}$$

where in the first equation we have substituted out the interest rate using $\widehat{R}_s=\delta_\pi\widehat{\Pi}_s+\delta_\theta\widehat{\theta}_s$, and tightness using $\widehat{\theta}_s=\frac{\widehat{\eta}_s}{1-\alpha}$. Using the first equation to substitute out $\widehat{\Pi}_s-\beta\mathbb{E}_s\widehat{\Pi}_{s+1}$ in the second equation gives:

$$\begin{aligned} & w\chi\widehat{\eta}_s+\frac{\kappa}{q}\left(\frac{\alpha}{1-\alpha}\widehat{\eta}_s-\frac{\alpha\beta\left(1-\omega\right)}{1-\alpha}\mathbb{E}_s\widehat{\eta}_{s+1}\right) \\ = \quad & \frac{\varphi}{\gamma}\left(-\mu\chi\beta\widehat{\eta}_s+\mu\beta^2\overline{R}\chi\mathbb{E}_s\widehat{\eta}_{s+1}-\frac{\beta\delta_\theta}{1-\alpha}\widehat{\eta}_s+\beta^2\overline{R}\Theta\mathbb{E}_s\widehat{\eta}_{s+1}\right). \end{aligned}$$

Collecting terms gives:

$$\mathbb{E}_s\widehat{\eta}_{s+1}=\Psi\widehat{\eta}_s,$$

where

$$\Psi=\frac{\frac{\varphi}{\gamma}\mu\chi\beta+\frac{\varphi}{\gamma}\frac{\beta\delta_\theta}{1-\alpha}+w\chi+\frac{\kappa}{q}\frac{\alpha}{1-\alpha}}{\frac{\kappa}{q}\frac{\alpha\beta(1-\omega)}{1-\alpha}+\frac{\varphi}{\gamma}\mu\beta^2\overline{R}\chi+\frac{\varphi}{\gamma}\beta^2\overline{R}\Theta^F}.$$

Productivity Shocks. With productivity shocks the model becomes:

$$\begin{aligned} & w\chi\widehat{\eta}_s+\frac{1-\gamma}{\gamma}A_s+\frac{\kappa}{q}\left(\frac{\alpha}{1-\alpha}\widehat{\eta}_s-\frac{\alpha\beta\left(1-\omega\right)}{1-\alpha}\mathbb{E}_s\widehat{\eta}_{s+1}\right) \\ = \quad & \frac{\varphi}{\gamma}\left(-\mu\chi\beta\widehat{\eta}_s+\mu\beta^2\overline{R}\chi\mathbb{E}_s\widehat{\eta}_{s+1}-\frac{\beta\delta_\theta}{1-\alpha}\widehat{\eta}_s+\beta^2\overline{R}\Theta^F\mathbb{E}_s\widehat{\eta}_{s+1}\right), \\ A_s \quad = \quad & \rho_A A_{s-1}+\sigma_A\varepsilon_s^A, \end{aligned}$$

which we can rewrite as

$$\begin{aligned} & \left(\frac{\kappa}{q}\frac{\alpha\beta\left(1-\omega\right)}{1-\alpha}+\frac{\varphi}{\gamma}\mu\beta^{2}\overline{R}\chi+\frac{\varphi}{\gamma}\beta^{2}\overline{R}\Theta^{F}\right)\mathbb{E}_{s}\widehat{\eta}_{s+1} \\ = & \left(w\chi+\frac{\kappa}{q}\frac{\alpha}{1-\alpha}+\frac{\varphi}{\gamma}\mu\chi\beta+\frac{\varphi}{\gamma}\frac{\beta\delta_{\theta}}{1-\alpha}\right)\widehat{\eta}_{s}-\frac{\gamma-1}{\gamma}A_{s}\end{aligned}$$

which gives

$$\begin{aligned} \mathbb{E}_{s}\widehat{\eta}_{s+1} &= \Psi\widehat{\eta}_{s}-\Omega A_{s}, \\ A_{s} &= \rho_{A}A_{s-1}+\sigma_{A}\varepsilon_{s}^{A},\end{aligned}$$

where

$$\Omega=\frac{\left(\gamma-1\right)/\gamma}{\frac{\kappa}{q}\frac{\alpha\beta(1-\omega)}{1-\alpha}+\frac{\varphi}{\gamma}\mu\beta^{2}\overline{R}\chi+\frac{\varphi}{\gamma}\beta^{2}\overline{R}\Theta^{F}}.$$

Monetary Policy Shocks. Now consider the model with monetary policy shocks. The log-linearized model, assuming again risk-neutral investors and $\delta_{\pi}=\frac{1}{\beta}$, becomes:

$$\begin{aligned} -\mu\chi\beta\widehat{\eta}_{s}+\mu\beta^{2}\overline{R}\chi\mathbb{E}_{s}\widehat{\eta}_{s+1} &= \widehat{\Pi}_{s}-\beta\mathbb{E}_{s}\widehat{\Pi}_{s+1}+\frac{\beta\delta_{\theta}}{1-\alpha}\widehat{\eta}_{s}-\beta^{2}\overline{R}\Theta^{F}\mathbb{E}_{s}\widehat{\eta}_{s+1}+\beta z_{s}^{R} \\ \frac{\varphi}{\gamma}\left(\widehat{\Pi}_{s}-\beta\mathbb{E}_{s}\widehat{\Pi}_{s+1}\right) &= w\chi\widehat{\eta}_{s}+\frac{\kappa}{q}\frac{\alpha}{1-\alpha}\widehat{\eta}_{s}-\beta\left(1-\omega\right)\frac{\kappa}{q}\frac{\alpha}{1-\alpha}\mathbb{E}_{s}\widehat{\eta}_{s+1} \\ z_{s}^{R} &= \rho_{R}z_{s-1}^{R}+\sigma_{R}\varepsilon_{s}^{R}\end{aligned}$$

Combining the first two equations gives:

$$\begin{aligned} & w\chi\widehat{\eta}_{s}+\frac{\kappa}{q}\left(\frac{\alpha}{1-\alpha}\widehat{\eta}_{s}-\frac{\alpha\beta\left(1-\omega\right)}{1-\alpha}\mathbb{E}_{s}\widehat{\eta}_{s+1}\right) \\ = & \frac{\varphi}{\gamma}\left(-\mu\chi\beta\widehat{\eta}_{s}+\mu\beta^{2}\overline{R}\chi\mathbb{E}_{s}\widehat{\eta}_{s+1}-\frac{\beta\delta_{\theta}}{1-\alpha}\widehat{\eta}_{s}+\beta^{2}\overline{R}\Theta^{F}\mathbb{E}_{s}\widehat{\eta}_{s+1}-\beta z_{s}^{R}\right),\end{aligned}$$

which we can re-write as

$$\begin{aligned} & \left(\frac{\kappa}{q}\frac{\alpha\beta\left(1-\omega\right)}{1-\alpha}+\frac{\varphi}{\gamma}\mu\beta^{2}\overline{R}\chi+\frac{\varphi}{\gamma}\beta^{2}\overline{R}\Theta^{F}\right)\mathbb{E}_{s}\widehat{\eta}_{s+1} \\ = & \left(w\chi+\frac{\kappa}{q}\frac{\alpha}{1-\alpha}+\frac{\varphi}{\gamma}\mu\chi\beta+\frac{\varphi}{\gamma}\frac{\beta\delta_{\theta}}{1-\alpha}\right)\widehat{\eta}_{s}+\frac{\varphi}{\gamma}\beta z_{s}^{R}.\end{aligned}$$

Which delivers which gives

$$\mathbb{E}_{s}\widehat{\eta}_{s+1}=\Psi\widehat{\eta}_{s}-\Omega^{R}z_{s}^{R},$$

where Ψ is as given in the main text and

$$\Omega^{R}=\frac{-\varphi}{\gamma\frac{\kappa}{q}\frac{\alpha(1-\omega)}{1-\alpha}+\varphi\mu\beta\overline{R}\chi+\varphi\beta\overline{R}\Theta^{F}}.$$

We again concentrate on the determinate case ($\Psi > 1$) and apply the method of undetermined coefficients and guess a solution of the form $\widehat{\eta}_s = \Gamma_\eta^R z_s^R$. Plugging this guess into the above system of equations yields the following solution:

$$\Gamma_\eta^R = \frac{\Omega^R}{\Psi - \rho_R}. \tag{1}$$

It can now be shown that, in the determinacy region of the parameter space, the job finding rate responds negatively to contractionary monetary policy shocks, i.e. $\Gamma_\eta^R < 0$. To see why, note the numerator of Equation (1) is negative and the denominator is positive under determinacy, since it then holds that $\Psi > 1 > \rho_R$.

Writing out the solution for Γ_η^R explicitly gives:

$$\Gamma_\eta^R = \frac{-\varphi}{\varphi\beta\left(\frac{\delta_\theta}{1-\alpha} - \rho_R \overline{R}\Theta^F\right) + \frac{\gamma}{\beta}\frac{\kappa}{q}\frac{\alpha(1-\beta\rho_R(1-\omega))}{1-\alpha} + \left(\frac{\gamma}{\beta}w + \varphi\mu\left(1-\rho_R\beta\overline{R}\right)\right)\chi}.$$

Let us now solve for the inflation rate, guessing a solution of the form $\widehat{\Pi}_s = \Gamma_\Pi^R z_s^R$. Plugging this guess into the log-linearized Euler equation gives:

$$\Gamma_\Pi^R = \frac{\beta\left(\beta\overline{R}\Theta^F\rho_R - \frac{\delta_\theta}{1-\alpha} - \chi\left(1-\rho_R\beta\overline{R}\right)\right)\Gamma_\eta^R - \beta}{1-\beta\rho_R}$$

Belief Shocks. From Equation (51) it follows that if the equilibrium is locally determinate ($\Psi > 1$), then the only stable solution is given by $\widehat{\eta}_s = 0$ at all times. When equilibria are locally indeterminate, the solution is given by

$$\widehat{\eta}_{s+1} = \Psi\widehat{\eta}_s + \Upsilon^B \varepsilon_{s+1}^B,$$

where ε_s^B is an i.i.d. belief shock with mean zero and a standard deviation normalized to one, and Υ^B is a parameter. Thus, in a model with only belief shocks the job finding rate follows an AR(1) process. While the magnitude of the belief shocks, captured by Υ^B, is not pinned down in the model, the persistence of the effects of belief shocks on the job finding rate is captured by Ψ, and thus endogenously determined. Persistence is maximal at $\Psi = 1$, i.e. exactly at the border between the determinacy and indeterminacy region of the parameter space.

A3. Risk-Averse Capitalists

When we log-linearized the model, we have assumed for simplicity that capitalists are risk neutral. The reason is that, technically, the unemployment rate becomes a state variable for inflation and the job finding rate, once we assume risk averse capitalists. With an additional state variable, the analytical solution of the model becomes more cumbersome, detracting from the key intuitions of the model.

Below, we use numerical simulations to compare versions with risk-neutral and risk-averse capitalists, showing only very small differences. We parametrize the model

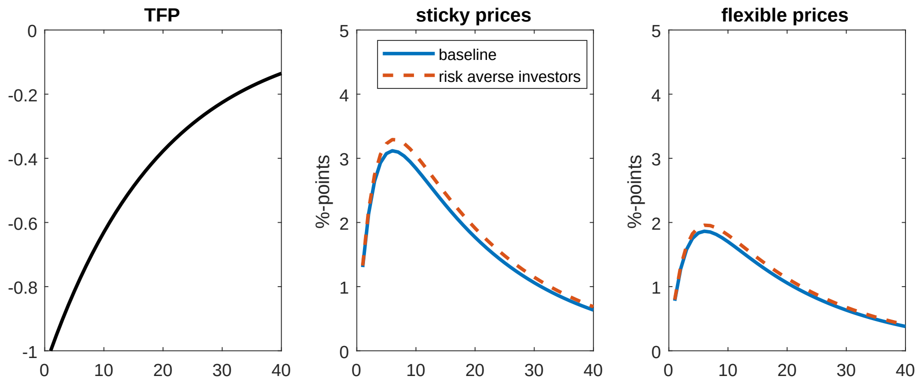


FIGURE 2. Responses to a positive technology shock. Baseline versus version with risk-averse investors (capitalists).

similar to the exercise in Appendix A.2.1, although this time we stick to the zero liquidity specification and opt for a parametrization in which the unemployment trap does not occur. In particular, this time we set the aggregate bond supply to zero, we target an average price duration of 5 months, and set calibrate the vacancy cost to 5 percent of the quarterly wage. Moreover, we now directly target a steady-state real interest rate of 3 percent per annum, using the subjective discount factor. Otherwise the calibration is the same as in Appendix A.1.2.

The left panel of the figure below plots the response of the unemployment rate to a negative technology shock under sticky prices, in the baseline model with risk neutral investors and in the version with risk averse capitalists. Quantitatively, the differences are small. Next, we consider a version of the model with flexible prices (right panel). Effectively, this removes the amplification mechanism from the model, so the increase in unemployment is considerably smaller. Again, however, the differences between the baseline and the version with risk neutral investors are minor.

A4. The Euler Equation at the ZLB

Consider the setup described in Section . For simplicity, we further assume that when the economy is in the depressed (ZLB) state, the households do not expect any further shock other than that the economy returns to the normal state with a probability p.

In the depressed state it holds, for $x = \{\eta, \Pi\}$, that $\mathbb{E}_s x_{s+1} = p\mathbb{E}_s x_{s+1}^{ZLB} + (1-p)x$, where x is the level at the indended steady state and a superscript ZLB indicates that the economy remains in the depressed state. Log-linearization of this equation around the intended steady state gives $\mathbb{E}_s \widehat{x}_{s+1} = p\mathbb{E}_s \widehat{x}_{s+1}^{ZLB}$. Note further that at the ZLB, $R_s = 1$ and hence $\widehat{R}_s = -\ln \overline{R}$.

Applying these results to the Euler equation, log-linearized around the intended steady state and as derived above, gives:

$$\left(\mu\chi\left(1-\beta\overline{R}p\right)-\beta\overline{R}\Theta^{F}p\right)\widehat{\eta}_{s}=\ln\overline{R}+p\widehat{\Pi}_{s}.$$

Here we have used that if the ZLB binds in period s then $\mathbb{E}_s\widehat{x}_{s+1}=p\mathbb{E}_s\widehat{x}_{s+1}^{ZLB}=p\widehat{x}_s$, exploiting the fact that variables remain constant as long as the depressed state persists. The Euler equation thus defines a linear relation between $\widehat{\Pi}_s$ and $\widehat{\eta}_s$, with a slope given by:

$$\frac{d\widehat{\Pi}_{s}}{d\widehat{\eta}_{s}}=\frac{\mu\chi}{p}\left(1-\beta\overline{R}p\right)-\beta\overline{R}\Theta^{F}.$$

Applying the same logic, the log-linearized Phillips Curve at the ZLB can be written as:

$$\frac{\varphi}{\gamma}\left(1-\beta p\right)\widehat{\Pi}_{s}=\left(w\chi+\frac{\kappa}{q}\frac{\alpha}{1-\alpha}\left(1-\beta\left(1-\omega\right)p\right)\right)\widehat{\eta}_{s}-wA_{s},$$

which again defines a linear relation between $\widehat{\Pi}_s$ and $\widehat{\eta}_s$, conditional on the level of productivity A_s. The slope of the Phillips Curve is given by:

$$\frac{d\widehat{\Pi}_{s}}{d\widehat{\eta}_{s}}=\frac{w\chi+\frac{\kappa}{q}\frac{\alpha}{1-\alpha}\left(1-\beta\left(1-\omega\right)p\right)}{\frac{\varphi}{\gamma}\left(1-\beta p\right)},$$

which is always positive. Note that an (unexpected) increase in productivity (A_s) shifts down the Phillips Curve, i.e. it reduces inflation, conditional on a certain level of the job finding rate.

A5. Monetary Policy and the Unemployment Trap

Here we consider the impact of systematic monetary policy on the steady-state properties of the model. Notice first that the responses of monetary policy to inflation and labor market tightness have no direct influence on the model once the nominal interest rate is constrained by the zero lower bound. Thus, we concentrate on the case in which the net nominal interest rate is positive.

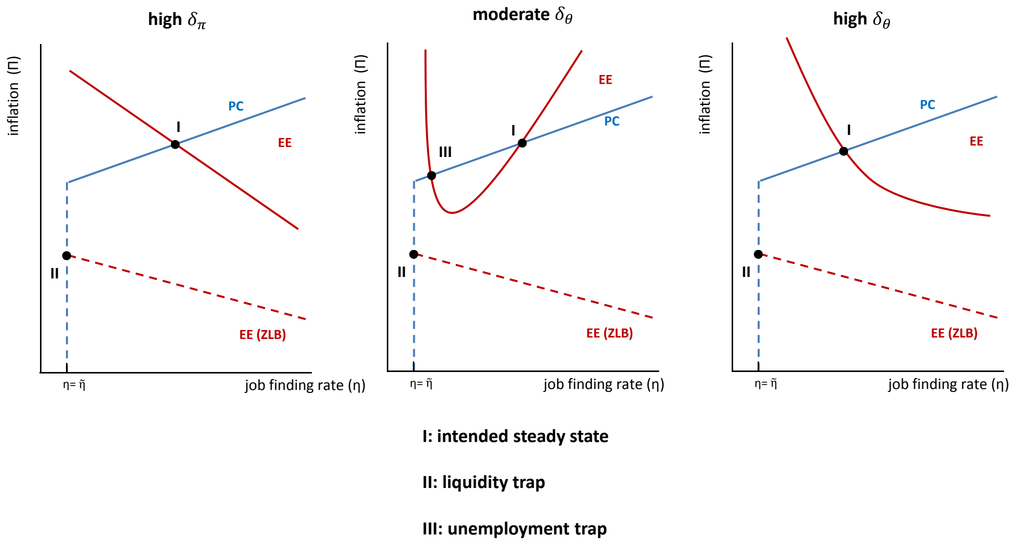
For positive net nominal interest rates, the relationship between inflation and the job finding rate along the **EE** curve can be expressed as

$$\Pi=\left[\beta\overline{R}\left(\frac{\eta}{\overline{\eta}}\right)^{\delta_{\theta}/(1-\alpha)}\overline{\Pi}^{-\delta_{\pi}}\mathbf{\Theta}^{SS}\left(\eta\right)\right]^{-1/(\delta_{\pi}-1)}$$

where $\overline{\eta}=\overline{\theta}^{1-\alpha}$. Notice that $-1/\left(\delta_{\pi}-1\right)<0$ since we imposed $\delta_{\pi}>1$.

Assume first that $\delta_{\theta}=0$ and consider the impact of variations in δ_{π}. Assume for simplicity that $\overline{\Pi}=1$. In this case, an increase in δ_{π} flattens the **EE** relationship tilting it around the intended steady state of the economy. Recall that existence of the unemployment trap requires the endogenous earning risk wedge to be sufficiently countercyclical, $\partial\mathbf{\Theta}^{SS}\left(\eta\right)/\partial\eta<0$ so that the **EE** curve is steeper than the **PC**. Minor

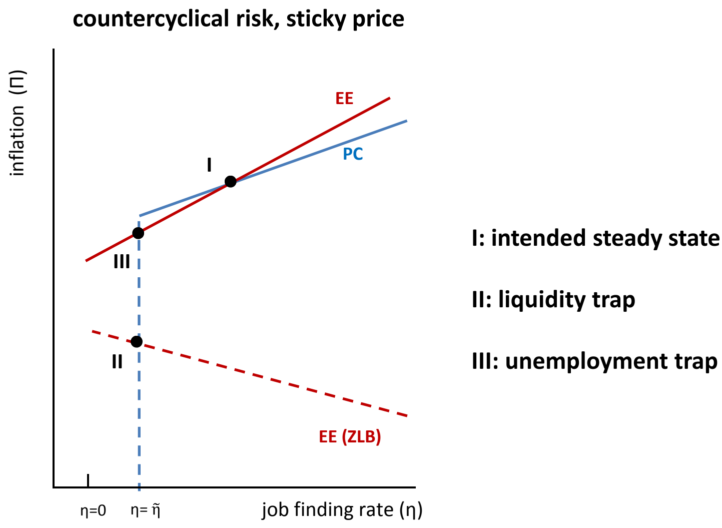
FIGURE 3. Illustration of steady-state equilibria: alternative monetary policy rules.



variations in δ_π will have no impact on the existence of the unemployment trap but a sufficiently large value of δ_π will mean that this bad long run equilibrium will cease to exist. Intuitively, aggressive policy manipulates agents' expectations so that they realize that any decline in inflation will be accompanied by a sufficiently large decline in real interest rates that savings will actually decline thereby preventing the spiral towards the unemployment trap. Figure 3 below, left panel, shows one such situation.

Consider now the impact of δ_θ. Notice that once $\delta_\theta > 0$, the **EE** will depend on labor market tightness and therefore on the job finding rate on top of the incomplete markets wedge, $\mathbf{\Theta}^{SS}(\eta)$. Notice also that the relationship between inflation and job finding rates along the **EE** curve implies that the **EE** curve becomes vertical as $\eta \to 0$. This implies that, if the unemployment trap exists, it is close to, but not exactly at $\eta = 0$. Marginal increases in δ_θ will then fail to rule out the existence of the unemployment trap but make this equilibrium slightly less bad. A sufficiently large value of δ_θ will, however, similarly to the impact of δ_π, rule out the existence of the unemployment trap. Intuitively, when δ_θ is large enough, the central bank can rule out the unemployment trap by signaling that any deterioration in the labor market will be accompanied by large cuts in the nominal interest rate which stimulate the economy ruling out a spiral towards the bad equilibrium. The middle and right panels of Figure 3 show these policy configurations graphically.

FIGURE 4. Illustration of steady-state equilibria: model with some costless vacancies.



A6. Less Extreme Unemployment Traps

The unemployment trap discussed above is an extreme outcome in which firms do not hire at all. However, less extreme unemployment traps are also possible, under minor modifications of the model setup. One example, shown above, is the case in which monetary policy responds moderately to labor market tightness, as shown in the middle panel of Figure 3. In that case, the unemployment trap occurs at a low but positive job finding rate, i.e. unemployment is below 100 percent. Another example, shown in Appendix A.1.2. is a model with positive liquidity.

An alternative setting with a similar outcome is one in which there is some frictionless hiring. Suppose each firms receives a limited number of costless vacancies, capturing the reality that some hiring takes place via informal channels which do not require explicit recruitment costs. In the intended steady state, firms then exhaust all their costless vacancies and top them up with costly vacancies. In the unemployment trap, however, firms only use their costless vacancies. Figure 4 illustrates this case: there is still hiring in the unemployment trap, as the job finding rate drops to $\widetilde{\eta} > 0$. The associated unemployment rate is high, relative to the intended steady state, but does not reach 100 percent.[3]

3. There might be additional equilibria in which firms choose not to even make use of the costless vacancies.

A7. The Model With Capital Accumulation

We now consider a version of the model in which firms invest not only in vacancies, but also in physical capital. We assume a Cobb-Douglas production function: $y_{j,s} = \exp(A_s)k_{j,s}^{\gamma_k}n_{j,s}^{1-\gamma_k}$, where $\gamma_k \in [0,1]$ is the production elasticity with respect to capital. The marginal cost of production now becomes:

$$mc_{j,s} = \frac{1}{mpl_{j,s}}\left(w_s + \frac{\kappa}{q_s} - \lambda_{v,j,s} - (1-\omega)\,\mathbb{E}_s\Lambda_{j,s,s+1}\left\{\frac{\kappa}{q_{s+1}} - \lambda_{v,j,s+1}\right\}\right),$$

where $mpl_{j,s}$ is the marginal product of labor, which is given by $mpl_{j,s} = (1-\gamma_k)\,y_{j,s}/n_{j,s}$. The stock of capital evolves as:

$$k_{j,s+1} = (1-\delta_k)k_{j,s} + i_{j,s},$$

where $\delta_k \in (0,1]$ is the depreciation rate of capital and $i_{j,s}$ denotes capital investment. The Euler equation for capital investment is given by:

$$1 = \mathbb{E}_s\Lambda_{j,s,s+1}(1-\delta_k + mpk_{j,s+1}),$$

where $mpk_{j,s+1} = \gamma_k y_{j,s}/k_{j,s}$ is the marginal product of capital.

We compare amplification in the model with and without capital. To this end we consider two additional versions, for both the model with and without capital. First, we consider a version with flexible prices. Second, we consider a version with sticky prices, but with exogenous unemployment risk. The latter version is obtained by assuming that those who become unemployed can only become employed again with a one-month lag. In that case, unemployment risk is purely determined by the separation rate, which is an exogenous parameter. For comparability with the baseline model, we then re-calibrate the separation rate such that the steady-state unemployment inflow probability is the same as in the baseline.

We solve the models numerically, as the model with capital can no longer be solved analytically. The calibration follows Appendix A3. In the model we capital, we further set $\gamma_k = 0.3$ and $\delta_k = 0.01$. Figure 5 displays response of the unemployment rate to a negative shock to Total Factor Productivity. The left panel shows that in the model without capital, there is substantial amplification in the baseline, relative to the versions with sticky prices and exogenous risk. The right panel shows that this amplification is still present once we introduce capital. In fact, the unemployment responses are substantially stronger than in the version without capital.

A8. Nominal Stickiness in Unemployment Benefits

In this section, we study a version of the model with nominal stickiness in the unemployment benefit. This may have stabilizing effects. Intuitively, if prices decline during a recession but benefits are nominally sticky, the real value of the benefit increases. This effects makes income risk less countercyclical, provided that prices fall during recessions. To study this case, we modify the model to include the following

Figure 5. Responses of the unemployment rate to a TFP shock.

law of motion for the real value unemployment benefits:

$$\vartheta_t = \iota\overline{\vartheta} + (1-\iota)\frac{\vartheta_{t-1}}{1+\pi_t},$$

where $\iota \in [0,1]$ is a parameter which controls the degree of nominal stickiness and $\overline{\vartheta}$ is the steady-state level of the benefit. If we set $\iota = 1$, we obtain the baseline model with a constant real benefit. If $\iota > 0$, there is nominal stickiness in the benefit, and a decline in inflation increases the benefit. If $\iota = 0$, benefits are fully nominally rigid.

Note that in this version of the model, we obtain an extra state variable, so we solve the model numerically. We calibrate the model as in Appendix A3 (assuming a fully rigid real wage), and set $\iota = 0.2$, implying substantial nominal stickiness.

Figure 6 plots the responses to a negative productivity shock in the baseline model, a version with a sticky benefit as described above (as well as sticky prices), and a version with flexible prices (but a sticky benefit). As expected, the version with a nominally sticky benefit features less amplification than the baseline (both relative to the flexible price case). The reduced amplification follows from the fact that prices to decline following the productivity shock, due to the ensuing decline in aggregate demand. With a nominally sticky benefit, the decline in prices increase the real value of benefits, which dampens the increase in income risk. Therefore, unemployment increases by less than in the baseline. Nonetheless, substantial amplification is left, despite the considerable nominal stickiness in the benefit.

A9. Pricing Risky Assets

This section explores asset pricing implications of the model. We show that the model generates a positive risk premium, but only if markets are incomplete. Intuitively,

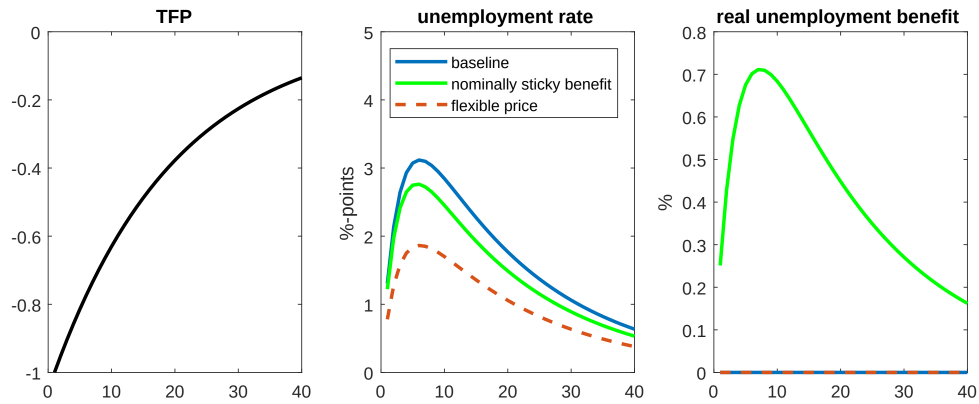


FIGURE 6. Responses to a positive technology shock.

agents dislike asset with returns that co-move negatively with the probability of becoming unemployed, and hence require a discount relative to asset with acyclical returns.

For simplicity, consider the model with sticky wages ($\chi = 0$) and no sunspots. We focus on equilibria around the intended steady state. The stochastic discount factor of an employed household is given by $\Lambda_{n,s,s+1} = \beta\omega\left(1-\eta_{s+1}\right)\left(\vartheta/w\right)^{-\mu} + \beta\left(1-\omega\left(1-\eta_{s+1}\right)\right)$. Note that the period-$s$ conditional correlation between $\Lambda_{n,s,s+1}$ and η_{s+1} (and hence between $\Lambda_{s,s+1}$ and A_{s+1}) is perfectly negative, due to the fact that $\vartheta < w$. The conditional variance of the stochastic discount factor is given by:

$$Var_s\left\{\Lambda_{n,s,s+1}\right\} = \beta^2\left(\Theta^F\right)^2\Gamma_\eta^2\sigma_A^2.$$

Note that under complete markets ($\Theta^F = 0$), we obtain $Var_s\left\{\Lambda_{n,s,s+1}\right\} = 0,$ i.e. the stochastic discount factor is constant. Intuitively, when agents' income is fully insured against unemployment risk and wages are sticky, their income, and hence their desire to save, is completely constant. When markets are incomplete, the precautionary savings motive emerges and fluctuates with the cycle since the amount of unemployment and wage risk varies over the business cycle.

Exogenous payoffs: We now use the model to price risky assets with simple payoff structures. First, consider a risky asset that pays off $1 + A_{s+1} - \rho A_s$ in period $s+1$. We choose this payoff structure as it has the simplifying property that the expected payoff is one, while at the same time payoffs increase after an expansionary shock to productivity.

To obtain analytical tractability, we again assume that the asset is in zero net supply and that households cannot go short in the asset. As a result, the employed asset-poor households are the ones pricing the asset at the margin, whereas the other two types of households are in equilibrium at the no-short sale constraint.

Krusell, Mukoyama and Smith (2011) exploit a similar setup to price risky asset under incomplete markets, but in an economy with exogenous endowments. Here, we analyze the importance of the endogenous feedback mechanism created by HANK and SAM, and study the effects of monetary policy on asset prices.

Below we show that the employed households' stochastic discount factor and the solution of the log-linearized model imply that the price of the risky asset, denoted z_s, is given by:

$$z_s = \mathbb{E}_s \Lambda_{n,s,s+1} - \beta \Theta^F \Gamma_\eta \sigma_A^2.$$

In the above equation, the term $\beta \Theta^F \Gamma_\eta \sigma_A^2$ is the discount relative to a riskless asset. To see this, consider a riskless asset that pays out one unit of goods in the next period regardless of the state of the world (i.e. a real bond). Again imposing the no-shortsale constraint, it follows immediately from the households' discount factor that the price of the riskless asset is given by $\mathbb{E}_s \Lambda_{n,s,s+1}$.

The above equation thus makes clear that if the endogenous earnings risk is countercyclical, i.e. $\Theta^F > 0$, there is a risk premium, which emerges despite the fact that the above equation is based on the solution of the log-linearized model.[4] Further, recall that Γ_η is the response of the job finding rate to a productivity shock. The magnitude of Γ_η depends on the strength of the endogenous interaction between HANK and SAM, as well as on the monetary policy rule. By responding more aggressively to economic shocks, the central bank stabilizes the economy, reducing the strength of the precautionary savings mechanism and thereby the risk premium. Finally, note that without shocks, i.e. $\sigma_A = 0$, there is no risk premium.

Endogenous payoffs: Consider now another risky asset with an payoff equal to $1 + \widehat{\eta}_{s+1} - \rho \widehat{\eta}_s$. Note that, again, the expected payoff is one and that the payoff is increasing in next period's job finding rate. Again, we impose the no-shortsale constraint. Below we show that the price of the asset is given by:

$$z_s = \mathbb{E}_s \Lambda_{n,s,s+1} - \beta \Theta^F \Gamma_\eta^2 \sigma_A^2.$$

Note that in the return of the risky asset we now observe Γ_η^2 rather than Γ_η. This reflects the fact that the payoff of the asset is now endogenous. As a result, market frictions and monetary policy affect the risk premium via two channels: through the households' stochastic discount factor (via their unemployment risk) and through the asset payoff (via the equilibrium effects of household demand).

Derivations: Consider the stochastic discount factor of the employed, asset-poor households:

$$\Lambda_{n,s,s+1} = \beta \omega \left(1 - \eta_{s+1}\right) \left(\vartheta / w\right)^{-\mu} + \beta \left(1 - \omega \left(1 - \eta_{s+1}\right)\right).$$

4. In representative agent models risk premia typically vanish after log-linearization since in the steady state there is no risk. Recall that in our model, by contrast, there is still idiosyncratic risk in the steady state.

Given the solution, the job finding rate is –up to a first-order approximation– given by $\eta_s = \eta + \eta\Gamma_\eta A_s$. We exploit this to write the period$-s$ conditional expectation and variance of $\Lambda_{e,s,s+1}$, respectively, as:

$$\begin{aligned}
\mathbb{E}_s\Lambda_{n,s,s+1} &= \beta\omega\left(1-\mathbb{E}_s\eta_{s+1}\right)(\vartheta/w)^{-\mu}+\beta\left(1-\omega\left(1-\mathbb{E}_s\eta_{s+1}\right)\right), \\
&= \beta\omega\left(1-\eta-\rho_A\eta\Gamma_\eta A_s\right)(\vartheta/w)^{-\mu}+\beta\left(1-\omega\left(1-\eta-\rho_A\eta\Gamma_\eta A_s\right)\right),
\end{aligned}$$

and

$$\begin{aligned}
Var_s\left\{\Lambda_{n,s,s+1}\right\} &= \beta^2\omega^2\left(1-(\vartheta/w)^{-\mu}\right)^2 Var_s\left\{\eta_{s+1}\right\}, \\
&= \beta^2\omega^2\left(1-(\vartheta/w)^{-\mu}\right)^2 \eta^2\Gamma_\eta^2 Var_s\left\{\rho_A A_s+\sigma_A\varepsilon^A_{s+1}\right\}, \\
&= \beta^2\omega^2\left(1-(\vartheta/w)^{-\mu}\right)^2 \eta^2\Gamma_\eta^2\sigma_A^2, \\
&= \beta^2\left(\Theta^F\right)^2\Gamma_\eta^2\sigma_A^2.
\end{aligned}$$

Exogenous payoffs: The pricing equation for the asset that pays off $1+A_{s+1}-\rho A_s$ in period $s+1$ reads:

$$\begin{aligned}
z_s &= \mathbb{E}_s\left\{\Lambda_{n,s,s+1}\left(1+A_{s+1}-\rho_A A_s\right)\right\} \\
&= \mathbb{E}_s\Lambda_{n,s,s+1}\mathbb{E}_s\left(1+A_{s+1}-\rho_A A_s\right)+Cov_t(\Lambda_{n,s,s+1},1+A_{s+1}-\rho_A A_s) \\
&= \mathbb{E}_s\Lambda_{n,s,s+1}-\sqrt{Var_s\left\{\Lambda_{e,s,s+1}\right\}Var_s\left\{1+A_{s+1}-\rho_A A_s\right\}} \\
&= \mathbb{E}_s\Lambda_{n,s,s+1}-\beta\Theta^F\Gamma_\eta\sigma_A^2
\end{aligned}$$

where we exploited the fact that the $Cor_s\left\{\Lambda_{n,s,s+1},A_{s+1}\right\}=-1$, that $1+\mathbb{E}_sA_{s+1}-\rho_A A_s=1$, and that $Var_s\left\{1+A_{s+1}-\rho_A A_s\right\}=\sigma_A^2$.

Endogenous payoffs: Consider now another risky asset with an payoff equal to $1+\widehat{\eta}_{s+1}-\rho\widehat{\eta}_s$. The pricing equation for this asset reads:

$$\begin{aligned}
z_s &= \mathbb{E}_s\left\{\Lambda_{n,s,s+1}\left(1+\widehat{\eta}_{s+1}-\rho_A\widehat{\eta}_s\right)\right\}, \\
&= \mathbb{E}_s\left\{\Lambda_{n,s,s+1}\left(1+\Gamma_\eta A_{s+1}-\rho_A\Gamma_\eta A_s\right)\right\}, \\
&= \mathbb{E}_s\Lambda_{n,s,s+1}-\sqrt{Var_s\left\{\Lambda_{e,s,s+1}\right\}Var_s\left\{1+\Gamma_\eta A_{s+1}-\rho_A\Gamma_\eta A_s\right\}}, \\
&= \mathbb{E}_s\Lambda_{n,s,s+1}-\beta\Theta^F\Gamma_\eta^2\sigma_A^2.
\end{aligned}$$

A10. Implications for the Zero Lower Bound

Our analysis thus far has focused on the implications of the endogenous risk channel when the economy is away from the ZLB on the nominal interest rate. In this section, we analyze how the channel impacts on paths into the ZLB, and economic outcomes once the ZLB is reached.

Contractionary Shocks and the ZLB. A recent literature has emerged on the effects of the Zero Lower Bound (ZLB) in the New Keynesian model, see e.g. Christiano, Eichenbaum and Rebelo (2011), Krugman and Eggertsson (2012) and Farhi and Werning (2013). Often, such analyses start off from a premise that some exogenous and transitory shock brings the economy temporarily to the ZLB. The specific shock introduced for this purpose is typically an exogenous shock to the discount factor, making agents temporarily more patient. The increase in patience drives down aggregate demand, putting downward pressure on inflation and the real interest rate. Via the interest rate rule, this results in a decline in the nominal interest rate, which may hit the ZLB if the shock is large enough (and at that point induces a potentially significant recession in the economy).

To appreciate the purpose of this specific shock, it helps to note that more conventional recessionary shocks, such as negative productivity shocks, typically will not lead to a decline in the nominal interest rate. There are two reasons for this. First, recessionary shocks reduce aggregate income and in a representative-agent model, lower current income (relative to expected future income) reduces households' desire to save inducing upward pressure on real and nominal interest rates, see e.g. Galí (2015, Chapter 3). A negative technology shock additionally increases real marginal costs, which puts further upward pressure on inflation and, via the Taylor rule, also the nominal interest rate. Thus, in a standard NK model without other sources of shocks, *expansionary* rather than *recessionary* technology shocks tend to be required to produce a decline in the nominal interest rate. For that reason, much research in the NK literature has introduced discount factor shocks when studying ZLB dynamics.

The precautionary savings mechanism that arises under endogenous risk can radically alter the cyclicality of the real interest rate, avoiding the need for discount factor shocks. Mechanically, the endogenous risk wedge acts as a shock to the discount factor in the Euler equation, but is determined endogenously rather than exogenously. Assume that $\Theta^F > 0$. As economic conditions worsen, the risk of becoming unemployed increases, driving down aggregate demand and increasing agents' desire to save. If the precautionary savings mechanism is strong enough, the nominal interest rate may decline, as argued by for example Werning (2015).[5]

Here, we can exploit the solution to the full model to obtain an explicit condition for the nominal interest rate to decline in response to a negative productivity shock. For simplicity, let us assume that monetary policy only responds to inflation ($\delta_\theta = 0$) and abstract from monetary policy shocks.[6] The log-linearized interest rate rule is given as

$$\widehat{R}_s = \delta_\pi \widehat{\Pi}_s = \delta_\pi \Gamma_\Pi^A A_s,$$

5. The working paper version of Ravn and Sterk (2017) make a similar point based on numerical simulations and, but do not consider productivity shocks. Werning (2015) presents analytical arguments, but not a fully fledged model.

6. We further assume that $\Gamma_\eta > 0$, i.e. the job finding rate responds positively to a positive productivity shock. As shown above, this is always the case in the determinacy region of the parameter space, and may be the case in the indeterminacy region.

where $\delta_\pi > 1$. In the previous subsection, we have shown that Γ^A_Π is negative when $\Theta^F = 0$. That is, under complete markets (or exogenous earning risk) the inflation rate, and hence the nominal interest rate, responds positively to a negative technology shock. However, when $\Theta^F > \frac{\mu\chi}{\beta\overline{R}\rho}\left(1-\rho\beta\overline{R}\right)$, i.e. when markets are sufficiently incomplete and the endogenous earnings risk is countercyclical, Γ^A_Π is positive. Under this condition, a negative technology shock drives down inflation and the nominal interest rate. If the shock is large enough, the ZLB may become binding.

Understanding Missing Deflation. Although inflation has been moderate in the aftermath of the financial crisis, no country has experienced persistent deflation. This is not easily reconciled with the standard NK model: Under the assumption of complete markets ($\boldsymbol{\Theta}^{SS}(\eta) = 0$), the deterministic steady-state real interest rate is given by $R/\Pi = 1/\beta$ and it follows that, when the ZLB binds in a steady state, the gross inflation rate must equal $\beta < 1$. Temporary episodes at the ZLB will be even more deflationary than this since the stochastic Euler equation in that case will only be satisfied as long as $\Pi < \beta$ during the ZLB regime.[7] It is important to notice that these implications are independent of the arguments that enter the interest rate rule.

The incomplete markets NK model has different implications. As explained earlier, the steady-state real interest rate under incomplete markets is:

$$\frac{R}{\Pi} = \frac{1}{\beta\boldsymbol{\Theta}^{SS}(\eta)} < \frac{1}{\beta},$$

which implies that the steady-state real interest rate depends on labor market conditions. When the ZLB binds, the steady-state Euler equation and the policy rule for the interest rate imply that the following two conditions must be satisfied in a liquidity trap (LT):

$$\begin{aligned}\Pi^{LT} &= \beta\boldsymbol{\Theta}^{SS}\left(\eta^{LT}\right) > \beta, \\ \Pi^{LT} &< \overline{\Pi}\overline{\theta}^{\delta_\theta/\delta_\pi}\overline{R}^{-1/\delta_\pi}\left(\eta^{LT}\right)^{-(\delta_\theta/\delta_\pi)/(1-\alpha)}.\end{aligned}$$

Notice that if $\delta_\theta = 0$, the policy rule implies that $\Pi^{LT} < \overline{\Pi}\overline{R}^{-1/\delta_\pi} < 1$, so that the liquidity trap is deflationary, given that in the intended steady state $\Pi = \overline{\Pi} = 1$ and $R = \overline{R} > 1$. When $\delta_\theta > 0$, however, inflation may be positive or negative in the liquidity trap. In particular, steady-state inflation is likely to be positive if $\left(\vartheta/w\left(\eta^{LT}\right)\right)^{-\mu} \gg 1$ and wages are not too responsive to the job finding rate, i.e. when the endogenous risk wedge is sufficiently countercyclical. Intuitively, under these circumstances, deteriorating labor market conditions (worsening tightness) induces

7. Suppose that the ZLB regime persists with probability p while the intended steady-state is absorbing. In that case, the inflation rate during the ZLB episode is determined as $\Pi^{LT} = \beta\left(p+(1-p)\left(c^I/c^{LT}\right)^{-\mu}\right)$ where Π^{LT} is the inflation rate during the liquidity trap, c^I is consumption in the intended steady-state and c^{LT} is consumption in the liquidity trap. This condition implies $\Pi^{LT} < \beta$ as long as $c^I > c^{LT}$.

both lower nominal interest rates and lower goods demand which in turn implies a further decline in tightness and in nominal rates the end-product of which may be that the ZLB may be reached at a positive inflation rate.

Paradoxes at the Zero Lower Bound. It is well known that at the ZLB, the representative-agent NK model has some paradoxical properties, see e.g. Eggertsson (2010), Eggertsson and Krugman (2012) and Werning (2012). One prominent example is the "supply shock paradox": at the ZLB, positive shocks to the supply side of the economy can trigger a contraction in real activity.[8]

The paradox arises from the fact that a positive supply shock pushes down production costs and hence inflation. The increase in inflation, in turn, creates paradoxical effects which can be understood from the consumption Euler equation. Consider, for simplicity, the complete-markets Euler equation under perfect foresight at the ZLB:

$$\left(\frac{c_{s+1}}{c_s}\right)^{\mu} = \beta\frac{1}{\Pi_{s+1}}.$$

The effect of a decline in expected inflation, at the ZLB, is that the real interest rate, $\frac{1}{\Pi_{s+1}}$, increases. The above Euler equation makes clear that this implies an increase in expected consumption growth, c_{s+1}/c_s. Given that the decline in inflation is transitory however, an increase in expected consumption growth implies a *decline* in the current level of consumption, i.e. an economic contraction.[9]

The joint presence of incomplete markets and countercyclical earnings risk, however, can overturn these results. Mechanically, the endogenous risk wedge in the Euler equation can absorb the effect of a decline in the real interest rate. Intuitively, an increase in output implies an increase in hiring, which reduces the precautionary savings motive. This makes an expansion in output compatible with an increase in the real interest rate.

We now formalize these arguments. Suppose that the economy fluctuates discretely between a "depressed state" at which the ZLB binds, and a "normal state" which coincides with the intended steady state. Let $p \in (0,1)$ be the probability that the ZLB regime persists and let the normal state be absorbing. In Appendix A4 we derive the relation between inflation and the job finding rate implied by the Euler equation, illustrated by lines labeled "**EE**" in Figure 4 . The slope is given by:

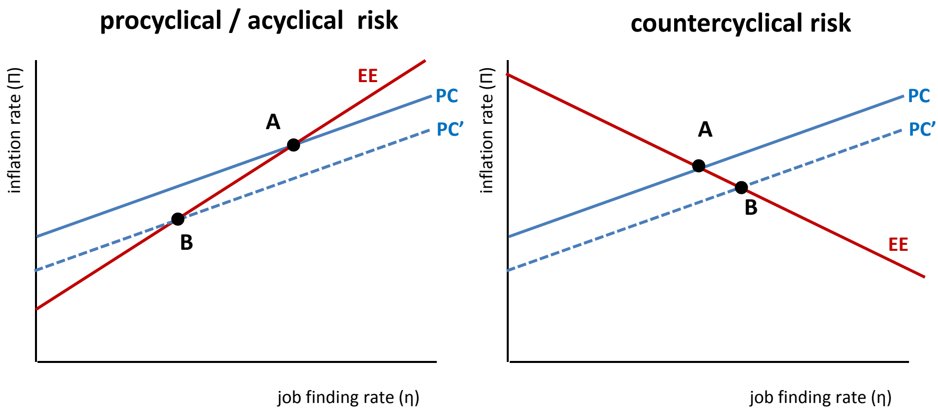
$$\frac{d\widehat{\Pi}_s}{d\widehat{\eta}_s} = \frac{\mu\chi}{p}\left(1-\beta\overline{R}p\right) - \beta\overline{R}\Theta^F.$$

Under acyclical risk ($\Theta^F = 0$), or under procyclical endogenous earnings risk $\left(\Theta^F < 0\right)$ the elasticity is positive since $\mu\chi > 0$ and $\beta\overline{R}p < 1$. Thus, any additional shock which

8. Another important and closely related example is the "paradox of flexibility" which states that, at the ZLB, a higher degree of price flexibility creates a larger drop in output.

9. Throughout this subsection, we consider equilibria which ultimately lead to the intended steady state. Properties of equilibria leading to the liquidity trap steady state can be very different, see e.g. Mertens and Ravn (2014).

Figure 7. Supply Shock Paradox at the ZLB: illustration.



reduces inflation must create a labor market contraction. As explained above, this is the source of the paradox. However, when $\Theta^F > \frac{\mu\chi}{p}(\beta^{-1}\overline{R}^{-1} - 1)$, i.e. when the endogenous earnings risk is highly countercyclical, the slope is negative. In that case, a reduction in inflation coincides with a labor market expansion.

In order to study explicitly the effect of a change in productivity, consider now the supply side of the economy. The Phillips Curve implies a positive relation between inflation and the job finding rate, see Appendix A4 for details. The lines in Figure 4 labeled "**PC**" illustrate this relation. An increase in productivity shifts down the **PC** curve and moves the equilibrium from point A to point B.

The left panel of Figure 4 depicts an economy with acyclical/procyclical risk and illustrates the paradox that arises also under complete markets: the increase in productivity reduces the job finding rate, and hence employment. The right panel illustrates a case with a downward-sloping **EE** curve, due to countercyclical risk. In this case, the job finding rate increases in response to the productivity increase. Thus, the presence of incomplete markets and countercyclical risk can overturn the supply shock paradox.