

MACROECONOMIC FLUCTUATIONS WITH HANK & SAM: AN ANALYTICAL APPROACH*

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Abstract

We present a tractable New Keynesian model with incomplete asset markets and labor market frictions. The key new feature of this model is endogenous idiosyncratic earnings risk which may be countercyclical or procyclical. The latter is stabilizing while countercyclical risk induces a powerful supply-demand feedback mechanism with profound implications. First, a new high-unemployment steady state can arise. Second, monetary policy needs to be more aggressive than dictated by the Taylor principle to avoid local indeterminacy. Third, shocks are amplified and productivity increases may be inflationary. Fourth, the Zero Lower Bound may be inflationary and paradoxical outcomes may be overcome.

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1 Introduction

The New Keynesian (NK) model has gained widespread use both in academic research and in policy circles. The crux of the model is that nominal frictions induce inefficient fluctuations in the economy, which monetary and fiscal policies can be designed to address. Clarida, Galí and Gertler (1999) provide a summary of the model’s key insights, based on a highly intuitive, three-equation version. Paradoxically, however, these insights do not pertain to unemployment and distributional issues, two central aspects of many policy discussions which are widely considered to be important determinants of aggregate demand.

Recently, a new generation of NK models that addresses these deficiencies has emerged. For example, Gertler and Trigari (2009), Blanchard and Galí (2010), Ravenna and Walsh (2011), and Christiano, Eichenbaum and Trabandt (2016) introduce unemployment by incorporating Search and Matching (SAM) frictions in the labor market. Others have introduced financial market incompleteness, generating inequality in income, wealth and consumption. Kaplan, Moll and Violante (2018) have dubbed such models Heterogeneous Agents New Keynesian (HANK) models. By giving centre stage to HANK and SAM, the new models mark a clear break with the traditional “representative agent” assumption, offer a rich array of cross-sectional predictions, and allow inequality across households to matter in models of aggregate fluctuations, see e.g. McKay and Reis (2016a) and many others.¹

This paper complements the new vintage of NK models with an analytically tractable counterpart that is as simple as the model in Clarida, Galí and Gertler (1999), but nonetheless features search and matching frictions in the Diamond-Mortensen-Pissarides tradition, and incomplete markets à la Bewley, Huggett and Aiyagari. Our main purpose is to revisit core qualitative results highlighted in the New Keynesian literature and to understand how these results are affected by the interactions between HANK and SAM. A key feature of our framework is that the severity of idiosyncratic earnings risk is endogenously determined, in conjunction with aggregate demand. We demonstrate profound implications of this endogeneity for short- and long-run equilibrium determination, the response of the economy to shocks, and the implications of the Zero Lower Bound (ZLB) on the nominal interest rate.

The model’s tractability derives from a limit on participation in the equity market, com-

¹Other studies in this new vein include Auclert (2016), Bayer, Pham-Dao, Luetticke and Tjaden (2015), Beaudry, Galizia and Portier (2017), Berger, Dew-Becker, Schmidt and Takahasi (2016), Challe, Matheron, Ragot and Rubio-Ramirez (2017), den Haan, Rendahl and Riegler (2016), Heathcote and Perri (2015), Gornemann, Kuester and Nakajima (2016), Guerrieri and Lorenzoni (2017), Kekre (2016), Luetticke (2015), McKay, Nakamura and Steinsson (2016), McKay and Reis (2016b), and Ravn and Sterk (2017).

bined with a borrowing limit in the bond market. These assumptions give rise to an equilibrium with three distinct groups of households: borrowing-constrained unemployed households, unconstrained but asset-poor employed households, and asset-rich but liquidity-constrained households.² We can characterize the equilibrium outcomes analytically, which facilitates a clear understanding of the underlying economic mechanisms. Moreover, our analytical approach allows us also to address potential equilibrium multiplicity which may easily be overlooked when solving incomplete-markets models numerically. This possibility is not a mere technical artefact as fluctuations driven by “animal spirits” can arise naturally under incomplete markets and endogenous employment risk.

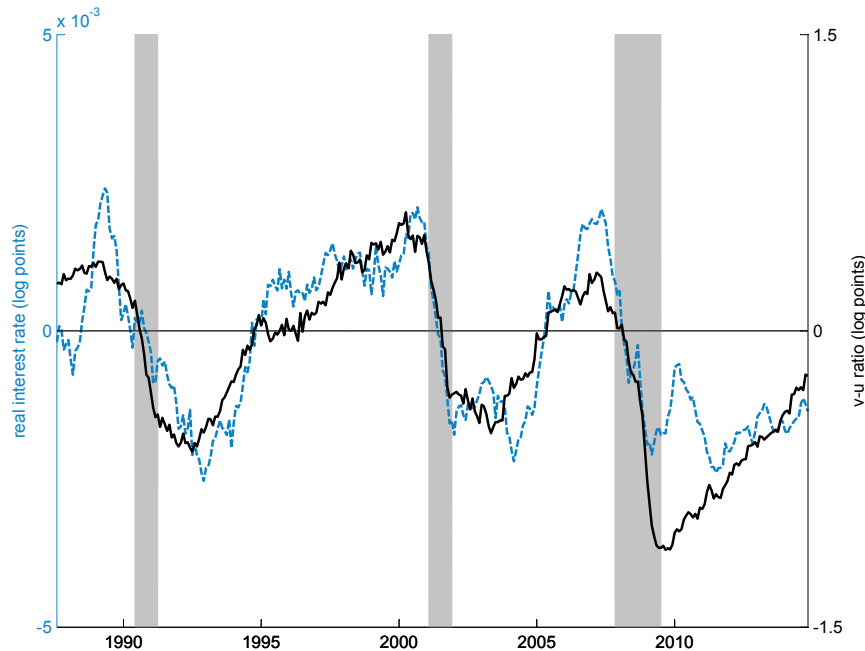
A central feature of the model is the emergence an ‘endogenous risk wedge’ in the consumption Euler equation.³ Werning (2015) similarly highlights this type of wedge in an analytical ‘aggregated’ Euler equation, but does not model explicitly how it is determined in equilibrium. We demonstrate how, in the presence of search and matching frictions, the wedge is pinned down by the tightness of the labor market, which moves over the business cycle. When the economy enters a recession, two opposing forces arise. On the one hand, the worsening labor market conditions make new jobs harder to find for job losers. This mechanism introduces *countercyclical endogenous earnings risk* because of lack of unemployment insurance. On the other hand, wages tend to fall in recessions, which makes job loss less costly and hence induces *procyclical endogenous earnings risk*. We characterize the conditions under which either of these two sources of cyclicity dominates, and relate them to primitives such as wage cyclicity, the income loss in terms of unemployment, labor market flexibility, etc.

Which of these sources is likely to dominate in practice? In models with complete markets, agents have an intertemporal savings motive which induces a negative comovement between real interest rates and labor market slackness because agents wish to save when jobs are easy to find and income is high. This tendency for negative comovement between real interest rates and job finding prospects is even stronger under incomplete markets when the procyclical earnings risk

²Our setup extends earlier work deriving tractability from assumptions on the borrowing limit, see Krusell, Mukoyama and Smith (2011), Werning (2015), McKay, Nakamura and Steinsson (2017), McKay and Reis (2016b), Bilbiie (2017) and Ravn and Sterk (2017). In these models, agents are unable to borrow. In our analysis, the employed households – who end up pricing the bonds– are in principle able to borrow but choose voluntarily not to do so.

³Like Clarida, Galí and Gertler (1999), we abstract from physical capital for reasons of tractability. Our model, however, does have a form of investment, namely investment in vacancies. In an extension of the model with capital, the precautionary savings effects would create only weak spillovers, or no spillovers at all, to capital investment, for the precise same reason that in the present model these effects do not spill over to vacancy investment: the owners of the firms are rich, which shields them from idiosyncratic risk.

Figure 1: Real interest rate (R^r) and labor market tightness (v/u) in the data.



Notes: Real interest rate and labor market tightness (vacancy-unemployment ratio) in the United States; deviations from trend. The real interest rate is expressed on a monthly basis and is computed as the Federal Funds rate minus a six-month moving average of CPI inflation. Vacancies are measured as the composite Help Wanted index from Barnichon (2010). Data series were logged and de-trended using a linear trend estimated over the period up to the end of 2007.

channel dominates. On the other hand, if earnings risk is countercyclical (when unemployment risk dominates), agents have a strong precautionary savings motive in recessions, which induces a potentially positive comovement between job finding rates and real interest rates. Figure 1 illustrates the relationship between real interest rates and labor market tightness (the ratio of job vacancies to unemployment) in the U.S. These two variables comove positively, indicating that real interest rates are low when jobs are hard to find and vice versa. This points towards dominance of the countercyclical endogenous earnings risk channels. We further carry out back-of-the-envelope calculations which support this conclusion unless wages are extremely procyclical and income losses in case of unemployment are minor.

Next, we explore the implications of endogenous earnings risk, in the presence of incomplete markets and endogenous rigidities. Countercyclical earnings risk introduces an amplification mechanism, due to a demand-supply side interaction. Intuitively, a worsening of labor market conditions increases unemployment risk, which motivates households to build more precaution-

ary savings. This reduces aggregate goods demand and increases the demand for bonds, pushing down the real interest rate. Because of nominal rigidities, firms respond to lower goods demand by cutting back on new hires. As a result, labor market conditions worsens further creating even more earnings risk, even lower goods demand, and so on. Conversely, if endogenous earnings risk is procyclical, aggregate fluctuations are stabilized. When earnings risk is acyclical, either because the two mechanisms exactly cancel out or because earnings risk is exogenous, the model’s implications for aggregate fluctuations are similar to those of standard (two-agent) New Keynesian model (see Debortoli and Galí, 2017), although the underlying mechanisms may be different (see Kaplan, Moll and Violante, 2018).

We highlight a number of key implications of the endogenous risk channel for the macro economy and for monetary policy.⁴ The first concerns the steady-state properties of the model. As in the basic NK model, there is an “intended” steady-state equilibrium as well as an unintended “liquidity trap”. In the latter steady state, the ZLB binds and output is relatively low, as in Benhabib, Schmitt-Grohé and Uribe (2001, 2002). Unlike the standard NK model, however, our model may have a third steady state, which we label the “unemployment trap.” This equilibrium can arise when the endogenous earnings risk is countercyclical and sufficiently strong. In the unemployment trap, aggregate demand is depressed to a level at which it is no longer profitable for firms to invest in vacancies, and in which inflation is moderately smaller than in the intended steady state. Therefore, hiring declines to a minimum, which perpetuates high unemployment risk and hence low demand.

We then study local determinacy properties, exploring the scope for belief-driven dynamics around steady states. We present an analytical local determinacy condition for the intended steady state. When earnings risk is procyclical, the intended steady-state is locally determinate subject to the Taylor principle. However, countercyclical earnings risk implies that local indeterminacy can arise even when the “Taylor Principle” is satisfied (see e.g. Woodford, 2003, Chapter 2). Intuitively, monetary policy must not only rule out local indeterminacy due to nominal rigidities, but also address the demand-supply interaction. Additionally, we show that the unemployment trap is determinate under a standard rule which responds more than one-for-one to inflation. Around this steady state, the monetary policy rule determines the rate of inflation, but has no grip on unemployment.

Third, we study the responses of the economy to productivity shocks and monetary policy shocks in the vicinity of the intended steady state. We solve analytically for the dynamic

⁴McKay and Reis (2016b) study optimal social insurance policy in a similar environment.

equilibrium and characterize the amplification (dampening) mechanism due to countercyclical (procyclical) earnings risk. We also show that, under countercyclical risk, positive productivity shocks may increase rather than decrease inflation, due to the ensuing boom in aggregate demand. Via the same mechanism, endogenous risk impacts on the effects of monetary policy shocks. Countercyclical risk makes monetary policy shocks more powerful. Similarly, the systematic component of monetary policy becomes more important as it can counteract the amplification mechanism, by cutting interest rates when demand is low and earnings risk is high. Importantly, the endogenous earnings channel that underlies these results is not present in the representative-agent New Keynesian models, nor in two-agent versions of that model, as in for example Galí, López-Salido and Vallés (2003) and Bilbiie (2008). In other words, market incompleteness matters for the business cycle, provided that earnings risk is endogenous.

Fourth, we revisit the role of the ZLB. In particular, we present a condition under which a *negative* productivity shock moves the nominal interest rate towards the ZLB and show that ZLB episodes are not necessarily deflationary. This happens as the real interest rate declines when unemployment increases, due to a heightened demand for precautionary savings. At the ZLB, a decline in the real interest rate implies an increase in inflation via the Fisher relation. Additionally, we revisit paradoxical properties of the representative-agent NK model which occur when the ZLB binds, see e.g. Eggertsson and Krugman (2012) and Werning (2012). Specifically, we show that our model can overturn the paradox that, at the ZLB, positive productivity shocks may be contractionary, because higher productivity dampens the precautionary savings motive. Our analysis complements McKay, Nakamura and Steinsson (2016), who study the implications of incomplete markets for the “forward guidance puzzle” at the ZLB.

The remainder of this paper is organized as follows. Section 2 presents the model. In Sections 3 and 4 we study, respectively, steady states and local fluctuations. Section 5 focuses on the role of the ZLB. Finally, in section 5 we discuss our model in light of the data, and argue that procyclical sources of earnings risk likely dominate countercyclical forces.

2 The Model

We construct a model which combines nominal rigidities in price setting, as in the NK tradition, with labor market matching frictions in the Diamond-Mortensen-Pissarides tradition, and incomplete asset markets in the Aiyagari-Bewley-Huggett tradition. The economy is made up of households who consume and work, firms which produce output, and a monetary authority in

charge of the nominal interest rate. We allow for both aggregate and idiosyncratic uncertainty and assume a lack of household insurance against idiosyncratic income risk.

2.1 Preferences and Technologies

Preferences: There is a continuum of mass 1 of infinitely lived single-member households indexed by $i \in (0, 1)$. Households consume non-durable goods, $c_{i,s}$, have disutility of work, and maximize the expected discounted present value of their utility streams:

$$\mathcal{V}_{i,t} = \mathbb{E}_t \sum_{s=t}^{\infty} \beta^{s-t} \left(\frac{c_{i,s}^{1-\mu} - 1}{1-\mu} - \zeta n_{i,s} \right), \quad (1)$$

where $\mathbb{E}_t x_s = E(x_s | I_t)$ is the date t conditional expectation of x_s , $0 < \beta < 1$ the subjective discount factor, $\mu > 0$ the measure of relative risk aversion, $n_{i,s}$ the household's employment status, and $\zeta > 0$ measures the disutility of market work. An individual household is either employed and works full-time, or does not work at all:

$$n_{i,s} = \begin{cases} 0 & \text{if not employed at date } s \\ 1 & \text{if employed at date } s \end{cases}. \quad (2)$$

The consumption level of an individual household is a CES aggregator of a basket of consumption goods, c_i^j :

$$c_{i,s} = \left(\int_j (c_{i,s}^j)^{1-1/\gamma} dj \right)^{1/(1-1/\gamma)}, \quad (3)$$

where $\gamma > 1$ is the elasticity of substitution between goods varieties. Workers who are not employed produce ϑ units of the aggregate consumption good at home.

Households decide on consumption, savings, on the financial portfolio, and on whether or not to participate in the labor force. A household not in the labor force cannot search for jobs in the market. Households who stand to lose on the net from employment declare themselves out of the labor force. We discuss the savings and portfolio problems later.

Production technology: Market goods are produced by a continuum of monopolistically competitive firms, indexed by $j \in (0, M)$, that each supply a differentiated good. The technology is:

$$y_{j,s} = \exp(A_s) n_{j,s}, \quad (4)$$

where $y_{j,s}$ is firms j 's output and $n_{j,s}$ its employment. A_s is an *aggregate* stochastic productivity

shock which follows a first-order autoregressive process:

$$A_s = \rho_A A_{s-1} + \sigma_A \varepsilon_s^A, \quad (5)$$

where $\rho_A \in (-1, 1)$, $\sigma_A > 0$ and $\varepsilon_s^A \sim \mathcal{N}(0, 1)$.

The law of motion of employment of firm j is:

$$n_{j,s} = (1 - \omega) n_{j,s-1} + h_{j,s}, \quad (6)$$

where ω is a constant employment separation rate and $h_{j,s}$ denotes hiring by firm j . Firms hire workers by posting vacancies, $v_{j,s}$, at cost $\kappa > 0$ per vacancy. A vacancy is filled with probability q_s . We take firms to be sufficiently large that q_s is also the fraction of vacancies that are filled.⁵ Thus, the total number of vacancies posted by firm j is given by $h_{j,s}/q_s$.

Matching technology: Agents receive information about current productivity shocks at the beginning of each period. Existing worker-firm relationships are resolved at the end of the period and new ones are formed at the beginning of the next period. Households take their consumption/saving decisions after new matches are formed. Job separations are exogenous and affect existing hires randomly, so that employees perceive ω to be the risk that they lose their current job.

New hires are produced by a matching function which relates the measure of newly formed worker-firm matches to the aggregate measures of vacancies, v_s , and job searchers, e_s , as:

$$M(e_s, v_s) = \bar{m} e_s^\alpha v_s^{1-\alpha}, \quad (7)$$

where $\bar{m} > 0$ indicates the match efficiency, $\alpha \in (0, 1)$, and $v_s = \int_j v_{j,s} dj$ is the measure of aggregate vacancies. We impose that vacancies cannot be negative:

$$v_{j,s} \geq 0, \quad (8)$$

see also Petrosky-Nadeau, Zhang and Kuehn (2017). The job finding rate, η_s , i.e. the probability that a jobless worker finds a new employer, and the vacancy filling probability, q_s , depend on

⁵This is useful because we will later assume symmetry across firms and the large firm assumption avoids having to consider that the measure of vacancies filled by individual firms is stochastic.

labor market tightness, $\theta_s \equiv \frac{v_s}{e_s}$, as:

$$\eta_s = \frac{M(e_s, v_s)}{e_s} = \bar{m}\theta_s^{1-\alpha}, \quad (9)$$

$$q_s = \frac{M(e_s, v_s)}{v_s} = \bar{m}\theta_s^{-\alpha} = \bar{m}^{\frac{1}{1-\alpha}}\eta_s^{\frac{\alpha}{\alpha-1}}. \quad (10)$$

It turns out that \bar{m} and κ enter the model equations in a way that is observationally equivalent for our purpose. Hence we impose the normalization $\bar{m} = 1$ from now on.

2.2 Price and Wage Setting

Prices: Firms set prices of their products, $P_{j,s}$, subject to a quadratic price adjustment cost as in Rotemberg (1982). The extent of nominal rigidities in price setting is parameterized by $\phi \geq 0$, which determines the size of the price adjustment costs. Let w_s denote the average real wage, y_s aggregate output, and P_s be the aggregate price level. We anticipate that in equilibrium wages are the same for all workers and hence exclude worker- and firm specific indices for the wage. Firms maximize:

$$\mathbb{E}_t \sum_{s=t}^{\infty} \Lambda_{j,t,t+s} \left[\frac{P_{j,s}}{P_s} y_{j,s} - w_s n_{j,s} - \kappa v_{j,s} - \frac{\phi}{2} \left(\frac{P_{j,s} - P_{j,s-1}}{P_{j,s-1}} \right)^2 y_s \right], \quad (11)$$

subject to (4), (6), and a demand constraint which derives from the consumers' decision problems:

$$y_{j,s} = \left(\frac{P_{j,s}}{P_s} \right)^{-\gamma} y_s, \quad (12)$$

where $y_s = \int_j y_{j,s} dj$ denotes aggregate output and $\Lambda_{j,t,t+s}$ is the discount factor of the firm's owners (discussed in Section 2.5).

Real marginal costs, $mc_{j,s}$, are the sum of the wage and hiring costs of a marginal worker (relative to productivity). To hire a marginal additional worker at date s , firms must spend κ/q_s on hiring costs but since matches persist, hiring today brings about future hiring cost savings $(1 - \omega) \kappa/q_s$ (discounted at the appropriate rate). Thus:

$$mc_{j,s} = \frac{1}{\exp(A_s)} \left(w_s + \frac{\kappa}{q_s} - \lambda_{v,j,s} - (1 - \omega) \mathbb{E}_s \Lambda_{j,s,s+1} \left\{ \frac{\kappa}{q_{s+1}} - \lambda_{v,j,s+1} \right\} \right), \quad (13)$$

where $\lambda_{v,j,s} \geq 0$ is the Kuhn-Tucker multiplier on (8), which satisfies the complementary slackness condition $\lambda_{v,j,s} v_{j,s} = 0$. Exploiting symmetry across firms, marginal costs equalize across firms

and hence we drop the firm subscript from now on. The firms' price-setting problems deliver the following first-order condition:⁶

$$1 - \gamma + \gamma mc_s = \phi (\Pi_s - 1) \Pi_s - \phi \mathbb{E}_s \Lambda_{s,s+1} \left[\frac{y_{s+1}}{y_s} (\Pi_{s+1} - 1) \Pi_{s+1} \right]. \quad (14)$$

Wages: We assume that real wages are determined by Nash bargaining between workers and firms. As discussed by Krusell, Mukoyama and Sahin (2010), financial market incompleteness and risk aversion jointly imply that the surpluses that households derive from employment generally depend on their wealth levels. Hence we label the households' value and surplus functions by i . Firms are symmetric and hence we do not include a firm index in the bargaining equations. The wage solves the following maximization problem:

$$\max (S_{i,s}^e)^v (S_s^f)^{1-v}, \quad (15)$$

where $S_{i,s}^e$ is the worker's surplus, S_s^f is the firm's surplus and $v \in (0, 1)$ is the worker's bargaining weight. We assume that were negotiations to fall through, the worker becomes unemployed while the firm can attempt to hire a new worker in the same period. The employed worker's surplus ($S_{i,s}^e$), the difference between the value of being employed ($V_{i,s}^e$) and unemployed ($V_{i,s}^u$), is then:

$$\begin{aligned} S_{i,s}^e &= V_{i,s}^e - V_{i,s}^u, \\ V_{i,s}^e &= \frac{c_{i,e,s}^{1-\mu}}{1-\mu} - \zeta + \beta \mathbb{E}_s \omega (1 - \eta_{s+1}) V_{i,s+1}^u + \beta \mathbb{E}_s (1 - \omega (1 - \eta_{s+1})) V_{i,s+1}^e, \\ V_{i,s}^u &= \frac{c_{i,u,s}^{1-\mu}}{1-\mu} + \beta \mathbb{E}_s (1 - \eta_{s+1}) V_{i,s+1}^u + \beta \mathbb{E}_s \eta_{s+1} V_{i,s+1}^e, \end{aligned}$$

where $c_{i,e,s}$ ($c_{i,u,s}$) is the consumption level optimally chosen by the household in case of employment (unemployment). Recall that separations take place at the very end of the period whereas new matches are formed at the very beginning. Accordingly, the term $1 - \omega (1 - \eta_{s+1})$ in the second equation is the probability that a worker employed in period s is still employed in period $s + 1$, either because the current match continues, or because the current breaks down but the worker immediately finds a new job in the beginning of the next period.

⁶Note that in the absence of price rigidities and search and matching frictions, the marginal cost equals $mc_s = \frac{w_s}{\exp(A_s)} = \frac{\gamma-1}{\gamma}$. To avoid trivial equilibria in which market work can generate no surplus to workers, even without labor market and price setting frictions, we assume that $\frac{\vartheta^{1-\mu}-1}{1-\mu} + \zeta < \frac{(\frac{\gamma-1}{\gamma} \exp(A_s))^{1-\mu}-1}{1-\mu}$. Strictly speaking, this requires a bound on the support of the stochastic productivity process.

Since the firm will post vacancies to hire a replacement worker should the current negotiations fail, the surplus of the match to the firm satisfies:

$$S_s^f = \frac{\kappa}{q_s}. \quad (16)$$

2.3 Monetary Policy

The monetary authority follows an interest rate rule. Specifically, the interest rate responds to inflation, given by $\Pi_s \equiv \frac{P_s}{P_{s-1}}$, and to labor market tightness. The latter variable naturally captures, inversely, the degree of labor market slack. The interest rate rule is given by:

$$R_s = \max \left\{ \bar{R} \left(\frac{\Pi_s}{\bar{\Pi}} \right)^{\delta_\pi} \left(\frac{\theta_s}{\bar{\theta}} \right)^{\delta_\theta}, 1 \right\}, \quad (17)$$

where $\bar{R}, \bar{\Pi}, \bar{\theta}, \delta_\pi, \delta_\theta \geq 0$ are policy parameters and the max operator captures the Zero Lower Bound (ZLB) on the net nominal interest rate, $R_s - 1$. We will later consider shocks to the monetary policy rule.

2.4 Financial Markets and Budget Constraints

NK models with unemployment typically assume that individual households are insured against idiosyncratic earnings shocks within large diversified families or, alternatively, that households can purchase unemployment insurance contracts at actuarially fair prices. Here we instead assume that households live in single-member families and cannot purchase unemployment insurance contracts.

Households have potentially access to two financial assets that they can invest in for self-insurance purposes. The first is a zero coupon one-period nominal bond purchased at price $1/R_s$ units of currency at date s . Let the household's purchases of bonds at date s be given by $b_{i,s}$. Households must observe a liquidity constraint:

$$b_{i,s} \geq -\psi w_{i,s}, \quad (18)$$

which allows a household to borrow up to a multiple $\psi > 0$ of its current wage income.

A second asset that is available to households is firm equity which are claims to the dividend streams of the firms. Let $x_{i,s}$ denote household i 's purchases of equity in period s . We impose

the following constraint:

$$x_{i,s} \geq 0, \tag{19}$$

which rules out the option to go short in equity.

The households choose their consumption streams and make their savings and portfolio choices subject to a sequence of budget constraints:

$$c_{i,s} + \frac{b_{i,s}}{R_s} + p_{x,s}x_{i,s} \leq w_{i,s}n_{i,s} + \vartheta(1 - n_{i,s}) + \frac{b_{i,s-1}}{\Pi_s}b_{i,s-1} + (1 - \tau_i)(p_{x,s} + d_{x,s})x_{i,s-1},$$

where $p_{x,s}$ denotes the equity price and $d_{x,s}$ is the dividend payment. This formulation allows for differences across households in equity returns through the parameter $\tau_i \in [0, 1]$.

2.5 Conditions for a Tractable Equilibrium

Without further assumptions, the model can only be solved numerically. In this paper we aim at an analytical characterization of steady-state equilibria, as well as local fluctuations around those steady states. To achieve this, we first impose that $\tau_i = \{0, 1\}$. That is, a subset of the households is unable to obtain returns from equity investment and hence is effectively blocked from participation in the equity market. We denote the fraction of households who are able to invest in equity by ξ and we assume that $\tau_i = 0$ for $i < \xi$ and $\tau_i = 1$ for $i \geq \xi$. We will assume that ξ is small, in a sense that will be detailed below. Second, we assume that the net aggregate supply of bonds is zero.

While these assumptions might seem fairly minor, they simplify the analysis considerably since they lead to equilibria in which the following two conditions hold:

$$b_{i,s} = 0 \quad \forall i, \tag{20}$$

$$1 = \beta \mathbb{E}_s \frac{R_s}{\Pi_{s+1}} \left(\omega(1 - \eta_{s+1}) \left(\frac{\vartheta}{w_s} \right)^{-\mu} + (1 - \omega(1 - \eta_{s+1})) \left(\frac{w_{s+1}}{w_s} \right)^{-\mu} \right)^{-1}. \tag{21}$$

The first of these conditions states that all individual households hold exactly zero bonds. The second equation is an asset-pricing condition, which relates the real interest rate to real wages and the job finding rate. As we will show below, this is the Euler equation of the employed households. Importantly, the equation contains only aggregate variables, which will allow us to solve for the equilibrium real interest rate without reference to the distribution of wealth.

We now verify that (20) and (21) are indeed consistent with utility maximization of all house-

holds, and with market clearing. A direct consequence of Equation (20) is that all households consume their current income. We can therefore distinguish between three distinct groups of households, with consumption levels given by:

$$c_{i<\xi,u,s} = \frac{1}{\xi} \left(y_s - \kappa v_s - w_s n_s - \frac{\phi}{2} (\Pi_s - 1)^2 y_s \right) + \vartheta, \quad (22)$$

$$c_{i\geq\xi,u,s} = \vartheta, \quad (23)$$

$$c_{i\geq\xi,e,s} = w_s. \quad (24)$$

Equation (22) is the consumption of those who can invest in equity. The first term on the right hand side is the dividend that an individual equity holder receives from the firms. Since firms make monopoly profits, the steady-state dividend is typically positive. These profits are spread out over the fraction of households who hold equity, ξ . The second term is the home production. Here we have assumed that ξ is small enough for the equity investors to have enough dividend income to drop out of the labor market. This assumption simplifies the analysis since now the equity investors are no longer exposed to idiosyncratic risk.⁷

Equation (23) is the consumption of the asset-poor employed households, whereas Equation (24) is the consumption of the asset-poor unemployed households. Since there is no heterogeneity across households conditional on their type and employment status, we drop the i -subscript and denote consumption levels as $c_{e,s} = c_{i\geq\xi,e,s}$, $c_{u,s} = c_{i\geq\xi,u,s}$, and $c_{r,s} = c_{i<\xi,u,s}$, where subscript r denotes the asset-rich households. Note also that firms now discount profits at a common rate $\Lambda_{s,s+t} = \beta (c_{r,s}/c_{r,s+t})^\mu$ since they are owned by the asset-rich households who face only aggregate risk.

The above outcomes are trivially consistent with clearing of the goods market and the bond market. To see why they also satisfy optimality of households decisions, consider in turn the Euler equations for bonds for the asset rich, the unemployed and the employed:

$$c_{r,s}^{-\mu} \geq \beta \mathbb{E}_s \frac{R_s}{\Pi_{s+1}} c_{r,s+1}^{-\mu}, \quad (25)$$

$$c_{u,s}^{-\mu} \geq \beta \mathbb{E}_s \frac{R_s}{\Pi_{s+1}} \left((1 - \eta_{s+1}) c_{u,s+1}^{-\mu} + \eta_{s+1} c_{e,s+1}^{-\mu} \right), \quad (26)$$

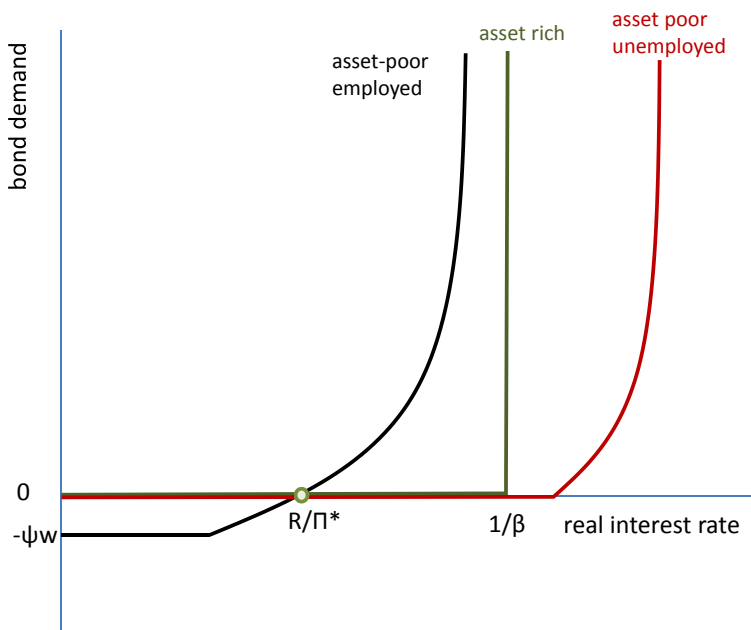
$$c_{e,s}^{-\mu} \geq \beta \mathbb{E}_s \frac{R_s}{\Pi_{s+1}} \left(\omega (1 - \eta_{s+1}) c_{u,s+1}^{-\mu} + (1 - \omega (1 - \eta_{s+1})) c_{e,s+1}^{-\mu} \right), \quad (27)$$

where each condition holds with equality if the household is not liquidity constrained and with

⁷Even if these households were to participate in the labor market, they would be relatively well insured against idiosyncratic risk due to wealth (equity holdings).

strict inequality when the liquidity constraint binds. The conjecture implies that in any steady state the real interest rate lies below the subjective discount rate, i.e. $\frac{R}{\Pi} < \frac{1}{\beta}$. Both (25) and (27) are consistent with this provided that they do not hold with equality.⁸ The Euler Equation of the employed households is also consistent with the conjecture. This can be seen by combining (27) with (23) and (24), which induces (21), the Euler equation holds with equality. The latter is necessary since otherwise the employed households would face a binding borrowing constraint. This, however, would mean that they hold positive amounts of debt, which would violate bond market clearing given that the asset-rich and the unemployed hold zero bonds.⁹

Figure 2: Illustration of steady-state bond demand schedules.



To see more intuitively why this equilibrium arises, consider Figure 2, which depicts the steady-state bond demand schedules of the three groups of households, as functions of the real interest rate. Generally these functions are upward sloping. To the far right is the demand

⁸To see this note that in a steady state (25) reduces to $\frac{R_s}{\Pi} \leq \frac{1}{\beta}$, whereas (27) reduces to $\frac{R_s}{\Pi} \leq \frac{1}{\beta}((1-\eta) + \eta(\frac{\vartheta}{w})^{-\mu})^{-1} < \frac{1}{\beta}$.

⁹One might wonder if there are related steady-state equilibria in which the asset rich and the unemployment are at the constraint, but there is heterogeneity in wealth and consumption within the group of employed households. This is not the case. To understand why, first note that on average the employed would have to hold zero bonds. Second, note that any employed household just coming out of unemployment would start with zero wealth. At the conjectured interest rate, they choose to buy exactly zero bonds. However, if the real interest rate were higher than the one conjectured, households would gradually accumulate bonds during the employment spell, monotonically converging to a certain target level of wealth. This, however, would mean that average bond holdings are positive, violating bond market clearing. Conversely, if the interest rate were lower than the one conjectured, aggregate bond holdings would be negative.

schedule of the unemployed. These households have a strong incentive to borrow, realizing that in the future they might find a job and receive more income. In the middle is the bond demand function of the asset rich. In order for these households to be willing to hold any bonds in the steady state, it must be that the real interest rate equals at least $\frac{1}{\beta}$. Below that, strictly firm equity dominates bonds in return and hence the equity holders would like to borrow in order to buy more equity. However, the liquidity constraint prevents them from doing so. The left schedule represents the employed, asset-poor households, who are most eager to save, due to their precautionary savings motive. They end up holding zero bonds, but not because they are forced by a constraint. Indeed, they are away from their constraint and could in principle save or borrow. Rather, the equilibrium real interest rate adjusts downward to a point at which they voluntarily hold zero bonds. This is required for clearing of the bond market, given that the asset rich and the unemployed hold zero bonds due to the liquidity constraint, which binds for them.

We thus arrive at a tractable steady state in which all households hold zero bonds and the asset-poor employed households determine the real interest rate, and the remaining households face a binding liquidity constraint. Importantly, individual employed households could in principle borrow or lend as they see fit, since they are away from their liquidity constraint. However, they choose not to do so since in equilibrium the real interest rate adjusts to a point at which they wish to hold zero bonds, which is required for bond market clearing. The fact that we allow those who price the bond to borrow sets us apart from earlier literature achieving tractability in incomplete-markets models, which typically assumes that no one can borrow, see Krusell, Mukoyama and Smith (2011), Werning (2015), McKay, Nakamura and Steinsson (2017), McKay and Reis (2017), Bilbiie (2017) and Ravn and Sterk (2017).¹⁰

Now consider local equilibria around the steady state, driven by aggregate shocks. Provided that these shocks are not too large, the asset rich and unemployed are liquidity constrained, whereas the employed are not. Since there is no aggregate supply of bonds in which the employed can save, the employed perpetually choose to hold zero bonds. Thus, (20) and (21) continue to hold in the neighborhoods of steady states.

Finally, consider the equilibrium labor market flows. Provided that the asset-poor are unwilling to leave the labor force, the labor market participation rate is constant and given by $1 - \xi$. In that case, the aggregate unemployment rate is given by:

¹⁰In those studies, there is typically a continuum of equilibrium real interest rates, which is not the case in our analysis.

$$u_s = 1 - n_s, \quad (28)$$

where $n_s = \frac{1}{1-\xi} \int_j n_{j,s} dj$ is the aggregate employment rate, as a fraction of the labor force. The law of motion of unemployment is given as:

$$u_s = u_{s-1} + \omega n_{s-1} - h_s, \quad (29)$$

where $h_s = \frac{1}{1-\xi} \int_j h_{j,s} dj$ is the number of new hires as a fraction of the labor force. The aggregate number of job searchers is given by $e_s = (1 - \xi) (u_{s-1} + \omega n_{s-1})$.

2.6 The Endogenous Risk Channel

Before we move on to characterize formally the equilibrium outcomes, it is useful to outline the key mechanism at play in the model, since it is central to the results that follow. There are four key relationships in the model:

$$\begin{aligned} c_{e,s}^{-\mu} &= \beta \mathbb{E}_s \frac{R_s}{\Pi_{s+1}} c_{e,s+1}^{-\mu} \left[1 + \omega (1 - \eta_{s+1}) \left((c_{u,s+1}/c_{e,s+1})^{-\mu} - 1 \right) \right], \\ 1 - \gamma + \gamma mc_s &= \phi (\Pi_s - 1) \Pi_s - \phi \mathbb{E}_s \Lambda_{s,s+1} (\Pi_{s+1} - 1) \Pi_{s+1} y_{s+1} / y_s, \\ mc_{j,s} &= \frac{1}{\exp(A_s)} (w_s + \kappa \eta_s^{\frac{-\alpha}{\alpha-1}} - (1 - \omega) \mathbb{E}_s \Lambda_{j,s,s+1} \kappa \eta_{s+1}^{\frac{-\alpha}{\alpha-1}}), \\ R_s &= \bar{R} \left(\frac{\Pi_s}{\bar{\Pi}} \right)^{\delta_\pi} \left(\frac{\theta_s}{\bar{\theta}} \right)^{\delta_\theta}. \end{aligned}$$

The first of these is the Euler equation of the employed workers, which we discuss further below. The second condition is the optimal price-setting equation, which relates inflation to real marginal costs as in the standard NK model. The third equation defines real marginal costs as a function of real wages and hiring costs, which in turn depend on the job finding rate. The last equation is the interest rate rule. We have for simplicity ignored the non-negativity constraint on vacancies and the zero lower bound on the nominal interest rate.

The first equation is key. It corresponds to (21), the Euler equation of the employed households. The term between square brackets is a wedge that arises due to market incompleteness and captures the precautionary savings motive of the employed households. The wedge collapses (to one) if there is no risk of unemployment ($\omega = 0$ or $\eta = 1$), or if markets are effectively complete ($c_{u,s+1} = c_{e,s+1}$), or when households are risk neutral ($\mu = 0$). The Euler equation then reduces to $c_{e,s}^{-\mu} = \beta \mathbb{E}_s \frac{R_s}{\Pi_{s+1}} c_{e,s+1}^{-\mu}$, coinciding precisely with the standard representative-agent

Euler equation. In that case, the model the model has no interesting implications beyond the standard NK model.

When earnings risk is endogenous, the wedge is strictly greater than one and varies with labor market conditions. In this case, the model introduces a feedback mechanism between the supply side and the demand side of the economy which induces either amplification or stabilization, depending on structural parameters. Consider a drop in the job finding rate. Lower job finding rates imply increased idiosyncratic earnings risk for employed households because they perceive lower chances of finding a job immediately upon job loss. This motivates an increase in desired precautionary saving. On the other hand, a lower job finding rate also tends to decrease the real wage, since the outside opportunities of workers have deteriorated. Lower real wages, in turn, imply a smaller drop in income upon job loss, which discourages precautionary saving.

When the first of these effects dominates, a case we refer to as countercyclical earnings risk, deteriorating labor market conditions lead employed agents to lower their goods demand at the current real interest rate, which puts downward pressure on the real interest rate. As long as the central bank operates an active rule for the nominal interest rate, the downward pressure on the real interest rate induces a drop in the inflation rate. According to the condition for optimal price setting, lower inflation requires real marginal costs to fall. Lower marginal costs, in turn, requires either real wages to drop or hiring costs to decline. In general, both real wages and hiring costs decline. The latter, in turn, requires firms to hire less, which induces a further decline in the job finding rate. This sets in motion a further reduction in demand. This feedback mechanism will operate to amplify fluctuations in the economy when the endogenous labor market risk is countercyclical. Conversely, when the wage effect dominates, the procyclical risk case, the model has a stabilizing effect because the demand for precautionary savings increases in booms and declines in recessions. Hence, the properties of the endogenous risk that arise in the model will be key for the model's implications.

Importantly, it is the endogeneity of the earning risk wedge that matters. Consider a version of the model with exogenous earnings risk. Assume, for example, that the labor market transition rates are exogenous and denote p_s^{eu} as the exogenous probability that an employed worker is unemployed next period. Moreover, assume that the drop in consumption (and income) upon job loss is exogenous and given by ϑ/w_s . We can then express the Euler equation as:

$$c_{e,s}^{-\mu} = \beta \mathbb{E}_s \frac{R_s}{\Pi_{s+1}} c_{e,s+1}^{-\mu} [1 + p_s^{eu} ((\vartheta/w_{s+1})^{-\mu} - 1)].$$

This version of the model with exogenous earnings risk still adds a precautionary savings term to the model, but it is no longer endogenous.¹¹ When p_s^{eu} rises, the risk of unemployment goes up. Similarly, when ϑ/w_{s+1} falls, the drop in consumption upon job loss increases. In both cases, goods demand from employed households is depressed, as they wish to build more precautionary savings. This demand contraction implies lower activity due to firms adjusting prices slowly. However, the supply side adjustment has no further effects on the demand side of the economy. Such precautionary savings arise, for example, in Challe and Ragot (2016) who assume exogenous job market flows or in Kaplan, Moll and Violante (2018) who incorporate exogenous idiosyncratic earnings risk.

3 Steady-state Equilibria

This section discusses the set of steady-state equilibria that can arise absent aggregate shocks and their properties.

3.1 Global Determinacy

Consider a version of the model without any aggregate shocks. An important difference vis-à-vis the extant complete-markets NK literature is that although aggregate variables are constant in the steady state, the labor market participants still face idiosyncratic risk due to lack of insurance against earnings risk.

We indicate steady-state values by removing time subscripts from variables. Define for convenience $R^* \equiv \bar{R} \bar{\Pi}^{-\delta_\pi} \bar{\theta}^{-\delta_\theta}$. The solution for the steady-state wage can be expressed as function of the job finding rate, $\mathbf{w}(\eta)$. This function is derived in Appendix A1, which also contains a characterization of some of its basic properties. Steady-state equilibria can be characterized by the solutions to:

$$\phi(1 - \beta)(\Pi - 1)\Pi = 1 - \gamma + \gamma(\mathbf{w}(\eta) + (\kappa\eta^{\alpha/(1-\alpha)} - \lambda_v)(1 - \beta(1 - \omega))), \quad (\mathbf{PC})$$

$$1 = \beta \frac{\max\{R^*\Pi^{\delta_\pi}\eta^{\delta_\theta/(1-\alpha)}, 1\}}{\Pi} \Theta^{SS}(\eta), \quad (\mathbf{EE})$$

where $\lambda_v \geq 0$ (the Kuhn-Tucker multiplier on the vacancy constraint (8)), $\lambda_v v = 0$, and where

¹¹One could modify the model explicitly to arrive at this version, by assuming that separated workers can only search for jobs with a one-period delay. In that case, p_s^{eu} equals the job separation rate, which one could make (exogenously) time-varying.

$\Theta^{SS}(\eta)$ is the steady-state endogenous risk wedge, which can be expressed as a function of the job finding rate:

$$\Theta^{SS}(\eta) \equiv 1 + \omega(1 - \eta) [(\vartheta/w(\eta))^{-\mu} - 1] \geq 1.$$

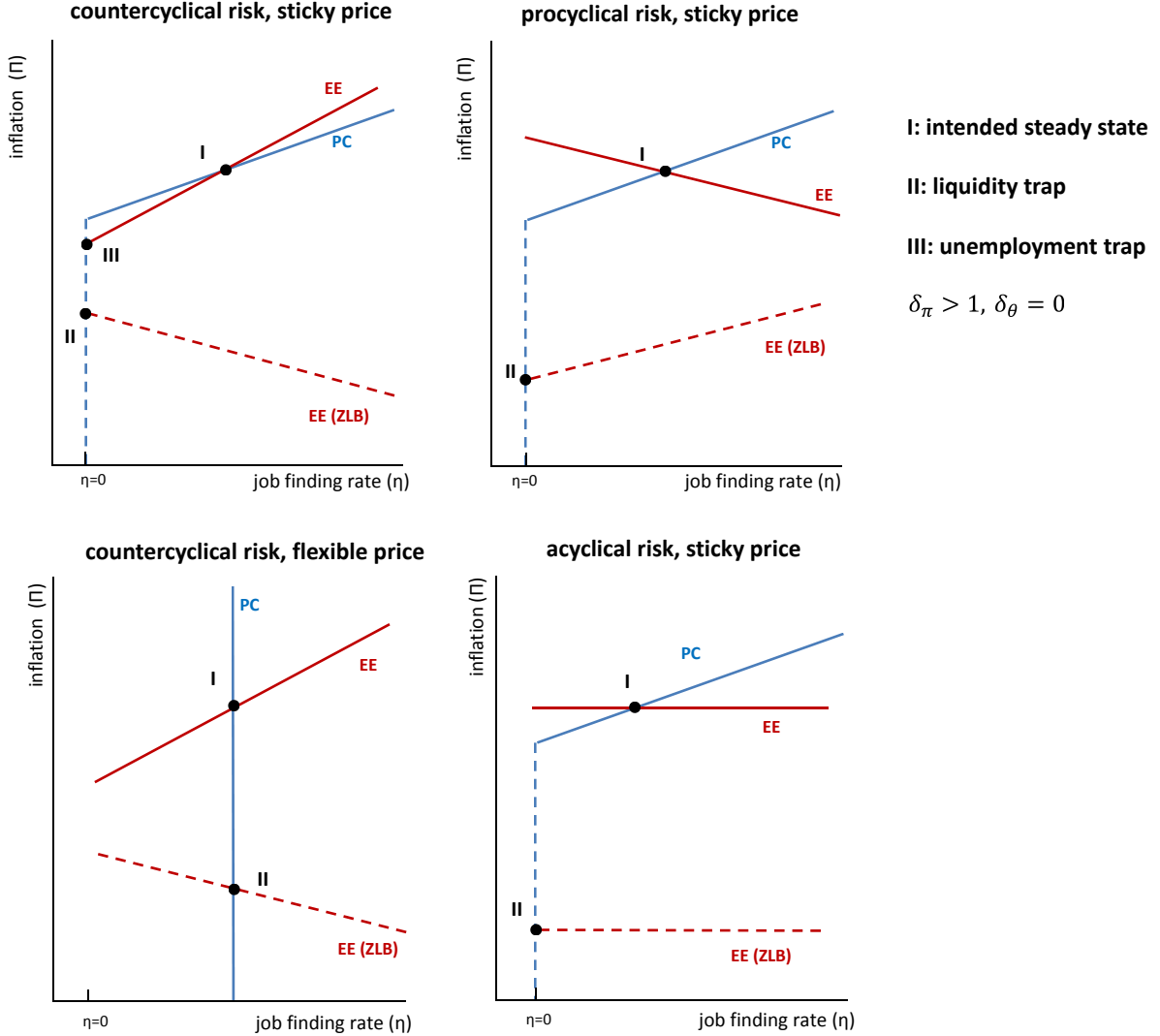
(**PC**) and (**EE**) can both be considered as defining a relationship between the job finding rate η and the inflation rate Π and the steady-state equilibria relate to the intersections of these relationships. Equation (**PC**) is the steady-state version of (14), the optimality condition for price setting (“the Phillips Curve”). It generally defines a positive relationship between inflation, Π , and the job finding rate, η . Intuitively, high job finding rates imply low vacancy filling rates and higher wages, as the competition for workers intensifies. This drives up marginal costs, and hence encourages firms to increase prices. The left-hand side of the equation is a standard steady-state sticky-price wedge, which vanishes in the absence of price adjustment costs ($\phi = 0$). In that case, the job finding rate is determined independently from the rate of inflation. A similar disconnect occurs when the non-negativity constraint on vacancies binds ($\lambda_v > 0$) and hiring freezes ($\eta = 0$).

Equation (**EE**) is the steady-state version of the employed households’ Euler equation (21). This equation also defines a relation between Π and η (consistent with employed agents maximizing utility). The slope of this schedule will have crucial implications for the properties of the steady-state equilibria and the model’s general properties. The slope depends on two factors. First, it depends on whether $\Theta^{SS}(\eta)$ is increasing or decreasing in η , which in turn depends on the cyclical nature of earnings risk. When wages are unresponsive to the job finding rate, i.e. when $w'(\eta) = 0$, earnings risk is countercyclical and $\Theta^{SS}(\eta)$ is decreasing in η .¹² Intuitively, when jobs are easier to find, employed workers have less reason to save for precautionary reasons, as unemployment is less likely. However, when wages are sufficiently elastic, i.e. when $w'(\eta)$ is positive enough, overall earnings risk becomes procyclical, so that $\Theta^{SS}(\eta)$ becomes increasing in η . More elastic wages imply that job loss results in a larger drop in income when the job finding rate is high, inducing more precautionary saving.

Second, the slope of the **EE** schedule depends on whether or not the ZLB on the nominal interest rate binds. For simplicity, we consider a case in which the interest rate rule only reacts to inflation (i.e. $\delta_\theta = 0$) and satisfies the Taylor principle ($\delta_\pi > 1$), so that away from the ZLB the real interest rate is increasing in the inflation rate. While the slope of **EE** schedule depends

¹²Formally, $\partial\Theta^{SS}(\eta)/\partial\eta = -\omega[(\vartheta/w)^{-\mu} - 1] + \mu\omega(1 - \eta)/\eta(\vartheta/w)^{-\mu}\chi$ where $\chi = (\partial w/\partial\eta)(\eta/w)$ is the elasticity of the real wage to the job finding rate.

Figure 3: Illustration of steady-state equilibria.



on $\Theta^{SS}(\eta)$, the sign of the slope reverses under a binding ZLB.¹³

Figure 3 illustrates the steady-state schedules. Under countercyclical risk, the **EE** schedule is positively sloped when the ZLB does not bind. Intuitively, when the job finding rate is high, the precautionary savings motive is weak, implying a relatively high real interest rate. Since monetary policy responds more than one-for-one to inflation, a high real interest rate implies a high rate of inflation. At the ZLB the schedule, denoted **EE(ZLB)**, slopes downward. The exact opposite is true under procyclical risk, i.e. **EE** slopes downward away from the ZLB whereas **EE(ZLB)** slopes upward. Under acyclical risk the **EE** schedule is horizontal, both at the ZLB

¹³To see this, note that, away from the ZLB, the real interest rate is given by $R^r = \Pi^{\delta_\pi - 1}$, so that $\frac{\partial R^r}{\partial \Pi} > 0$, given $\delta_\pi > 1$. At the ZLB, the real interest rate equals $R^r = 1/\Pi$, which implies $\frac{\partial R^r}{\partial \Pi} < 0$.

and away from it.

Consider now the upper left panel of Figure 3, which illustrates a case with countercyclical risk and sticky prices. Three possible steady states emerge:¹⁴

- I *Intended steady state.* This steady state occurs at the intersection of the **PC** and the **EE** schedule at $\eta > 0$. This is the “intended” steady state, at which the ZLB does not bind and the job finding rate is relatively high.
- II *Liquidity trap.* This steady state arises because of the ZLB on the nominal interest rate and occurs at the intersection of the **PC** and the **EE(ZLB)** schedule. This is the “liquidity trap” examined by Benhabib, Schmitt-Grohé and Uribe (2001, 2002) and Mertens and Ravn (2014). This steady state features a lower rate of inflation than the intended steady state, as well as a lower job finding rate. In fact, the job finding rate is zero in the illustration.
- III *Unemployment trap.* This steady state occurs at the intersection of the **PC** the **EE** schedule at $\eta = 0$. In this equilibrium, investment in vacancies comes to a complete standstill, despite the fact that the ZLB on the nominal interest rate does not bind. Note that the inflation rate in this steady state lies in between those in the intended steady state and the liquidity trap.¹⁵

The first two of these types of equilibria occur also when the endogenous risk is procyclical and in standard complete-markets representative-agent NK models. There are, however, important differences between the properties of the equilibria under complete and incomplete markets, coupled with endogenous earnings risk. Under complete markets, the steady-state real interest rate needs to equal $1/\beta$ in order to be consistent with constant consumption. Without full insurance, and regardless of the slope of $\Theta^{SS}(\eta)$, the wedge in (**EE**) exceeds unity, which reduces the equilibrium real interest rate below the inverse of the discount factor, $\frac{R}{\Pi} < \frac{1}{\beta}$. Therefore, economic policy is a co-determinant of the long-run real interest rate. As long as equilibrium wages do not depend on market incompleteness, however, the central bank can replicate the steady-state levels of unemployment and inflation that would prevail under complete

¹⁴We ignore the possibility of an additional equilibrium that occurs due to the quadratic price adjustment term, or due to non-linearities in $w(\eta)$.

¹⁵To avoid the labor market totally collapsing one can e.g. introduce the possibility that some jobs are filled without the need to post vacancies, which would lead to a strictly positive lower bound on the job finding rate.

markets. Suppose for example that the central bank targets price stability, $\bar{\Pi} = 1$, and let η^{CM} denote the steady-state job finding rate under complete markets. Then the central bank can implement the same outcome under incomplete markets by setting $\bar{\theta} = (\eta^{CM})^{\frac{1}{1-\alpha}}$ and $\bar{R} = 1/(\beta\Theta^{SS}(\eta^{CM}))$. This is not possible, however, without the use of fiscal policy if wages depend on market incompleteness, as they will in general.¹⁶

The possible emergence of a third steady state depends critically on the interaction between countercyclical risk and sticky prices. The remaining panels in Figure 3 illustrate cases with sticky prices and either procyclical or acyclical risk, or with countercyclical risk but flexible prices. With procyclical risk, the **EE** schedule becomes downward sloping, whereas under acyclical risk it is horizontal. In both case, the third steady state is ruled out. In the limit case with flexible prices, the **PC** schedule at $\eta > 0$ becomes vertical, as inflation no longer affects firms' marginal costs. As a result, the **PC** schedule at $\eta = 0$ vanishes, allowing for only two steady states. Thus, without the interaction between sticky prices and endogenous risk, the third steady state cannot exist.

The unemployment trap can arise in the presence of both sticky prices and countercyclical risk, and its likelihood is higher when monetary policy reacts little to inflation and/or labor market tightness, and hiring costs are limited.¹⁷ Intuitively, this steady state arises when endogenous risk is sufficiently countercyclical, so that expectations of poor labor market conditions trigger such an increase in desired savings that the economy spirals towards an equilibrium in which firms anticipate that posting vacancies is pointless because of lack of demand for their goods. For this to be possible, endogenous risk must be sufficiently countercyclical that the Euler equation schedule becomes steeper than the Phillips curve schedule.

The unemployment trap is an intriguing outcome. The slow recovery after the Great Recession and the very protracted nature of the surge in unemployment observed in the U.S. (and many other OECD economies) have spurred a renewed interest in “secular stagnation,” equilibrium outcomes consistent with long periods of low activity and high unemployment. Hansen (1939) argued that such outcomes (with negative natural real interest rates) were most likely produced by a combination of low rate of technological progress and population ageing implying high savings rates and low investment rates. Recently, Eggertsson and Mehrotra (2014) have

¹⁶The reason is that wages impact on the Phillips curve as well. In this case, the intended steady state can be replicated by taxing labor income and by adjusting \bar{R} .

¹⁷One intriguing issue is that the unemployment trap can be ruled out if the government allows the real interest rate to depend negatively on inflation. Such a policy, however, would make the intended steady-state locally indeterminate.

argued that deleveraging may lead to secular stagnation and exacerbate the problems that follow from an ageing population and falling investment goods prices.

The unemployment trap that can arise in our model offers an alternative perspective of secular stagnation, which ties together low real interest rates, high unemployment and low activity. Importantly, the unemployment trap can occur in our model purely because of expectations and thus does not rely on sudden changes in population growth, technological progress or financial tightening. Furthermore, while the nominal interest rate may be low in the unemployment trap, its root cause does not derive from the ZLB on nominal interest rates. Therefore, the ongoing discussions about re-design of monetary policy to prevent secular stagnation by avoiding the ZLB may be in vain.

3.2 Local Determinacy

The log-linearized model: We now log-linearize the model in order to study the local stability properties of the equilibria. Let a hat denotes a log deviation from the intended steady state, i.e. $\hat{x}_s = \ln x_s - \ln \bar{x}^I$, where \bar{x}^I denotes the value of x_s in the intended steady-state (discussed above). We assume that monetary policy parameters are such that \bar{R} , $\bar{\theta}$ and $\bar{\Pi}$ correspond to the levels of, respectively, R , θ and Π , in the intended steady state.

The log-linearized Euler equation of the employed households, (21), can be expressed as (see Appendix A2 for details):

$$\begin{aligned}
 -\mu \hat{c}_{e,s} + \mu \beta \bar{R} \mathbb{E}_s \hat{c}_{e,s+1} &= \hat{R}_s - \mathbb{E}_s \hat{\Pi}_{s+1} - \underbrace{\beta \bar{R} \Theta^F \mathbb{E}_s \hat{\eta}_{s+1}}_{\text{endogenous risk wedge}}, \\
 \Theta^F &\equiv \omega \eta \left((\vartheta/w)^{-\mu} - 1 \right) - \chi \mu \omega (1 - \eta).
 \end{aligned} \tag{30}$$

where χ is a parameter that measures the elasticity of real wages to the job finding rate, see below.

$\hat{R}_s - \mathbb{E}_s \hat{\Pi}_{s+1}$ is the real interest rate while the last term on the right-hand side is the endogenous risk wedge, which fluctuates proportionally with the expected job finding rate and captures the precautionary savings motive.¹⁸ It's strength and cyclicity is determined by Θ^F , which depends on structural parameters. The first part, $\omega \eta \left((\vartheta/w)^{-\mu} - 1 \right) > 0$, captures the impact of earnings risk due to fluctuations unemployment risk.¹⁹ According to this term, an

¹⁸The incomplete markets wedge that occurs in the log-linearized Euler equation differs from its steady state version because of a different normalization and because of the impact of wage fluctuations on savings, see below.

¹⁹The first part of Θ^F becomes zero if the steady-state job finding rate, η , equals zero. The reason for this

increase in the expected job finding rate stimulates current consumption relative to expected future consumption, at the going real interest rate. This occurs because the household perceives less risk of a lengthy unemployment spell. The second part, $-\chi\mu\omega(1-\eta) < 0$, relates to changes in earnings risk which derive from wage fluctuations. When wages are procyclical ($\chi > 0$), the income loss resulting from job loss is larger when the job finding rate is high. Via this channel, an expected increase in the job finding rate exacerbates the precautionary savings motive, tilting the households' consumption streams away from current consumption.

The sign of Θ^F depends on which of these two channels dominates. If consumption losses upon unemployment are large, e.g. in the face of little insurance, the first of these sources will tend to dominate. On the other hand, if wages are elastic and procyclical wages (high χ), the second channel dominates is more likely to dominate. We refer to $\Theta^F > 0$ as countercyclical endogenous earning risk and $\Theta^F < 0$ as procyclical earnings risk. When $\Theta^F = 0$, the endogenous risk wedge vanishes and the above equation reduces to the log-linearized Euler equation obtained in standard representative-agent models, aside from a weight $\beta\bar{R} < 1$ on next period's consumption.²⁰

Next, we log-linearize the firms' price-setting condition, Equation (14), around the intended steady state:

$$\frac{\phi}{\gamma}\widehat{\Pi}_s - \beta\frac{\phi}{\gamma}\mathbb{E}_s\widehat{\Pi}_{s+1} = w\widehat{w}_s + \frac{1-\gamma}{\gamma}A_s + \frac{\kappa}{q}\frac{\alpha}{1-\alpha}\widehat{\eta}_s - \beta(1-\omega)\frac{\kappa}{q}\left(\frac{\alpha}{1-\alpha}\mathbb{E}_s\widehat{\eta}_{s+1} + \mathbb{E}_s\widehat{\Lambda}_{v,s+1}\right), \quad (31)$$

where we have exploited that $q_s = \eta_s^{-\frac{\alpha}{1-\alpha}}$. For now, we abstract from productivity shocks, setting $A_s = A_{s+1} = 0$ at any date s . The left-hand side of the above equation is the sticky-price wedge, which vanishes in the absence of nominal rigidities ($\phi = 0$) or in the limit with perfect competition ($\gamma \rightarrow \infty$). The right-hand side is the log-linearized marginal cost, which is standard given the presence of search and matching frictions.

The log-linearized policy rule reads:

$$\widehat{R}_s = \delta_\pi\widehat{\Pi}_s + \delta_\theta\widehat{\theta}_s. \quad (32)$$

In Appendix A2, we further show that the log-linearized bargaining equations imply that:

$$\widehat{w}_s = \chi\widehat{\eta}_s, \quad (33)$$

is technical, however. Note that $\mathbb{E}_s\widehat{\eta}_{s+1}$ is the *percentage* deviation in the expected job finding rate from its steady-state value. If the steady-state value is zero, no percentage deviation represents any actual change.

²⁰This weight is not unimportant, however. McKay, Nakamura and Steinsson (2017) show that it is instrumental in alleviating the "forward guiding puzzle".

where χ is a convolution of the model's deep parameters, which captures the sensitivity of the wage to fluctuations the job finding rate and depends critically on the bargaining parameter v . Finally, note that in equilibrium the employed households consume their wage, i.e. $\widehat{c}_{e,s} = \widehat{w}_s$.

Reducing the model to a single equation: For maximal tractability, we introduce two further assumptions which allow us to reduce the model to a single equation. First, we set the monetary policy coefficient equal to $\delta_\pi = \frac{1}{\beta} > 1$. This is inconsequential, since the coefficient on tightness, δ_θ , is left unrestricted.²¹ Second, we assume that the households who can invest in equity (i.e. those with index $i < \xi$) are risk neutral. In this case, the log-linearized model has no endogenous state variables. In Appendix A3, we relax this assumption. The results suggest that allowing for risk-averse equity investors has only very limited implications.

The log-linearized model can now be reduced just one dynamic equation for the job finding rate (see Appendix A2 for a derivation):

$$\begin{aligned} \mathbb{E}_s \widehat{\eta}_{s+1} &= \Psi \widehat{\eta}_s, \\ \Psi &\equiv \frac{\phi\gamma^{-1}\mu\chi\beta + \phi\gamma^{-1}\frac{\beta\delta_\theta}{1-\alpha} + w\chi + \frac{\kappa}{q}\frac{\alpha}{1-\alpha}}{\frac{\kappa}{q}\frac{\alpha\beta(1-\omega)}{1-\alpha} + \phi\gamma^{-1}\mu\beta^2\overline{R}\chi + \phi\gamma^{-1}\beta^2\overline{R}\Theta^F} = \frac{\Psi^N}{\Psi^D}. \end{aligned} \tag{34}$$

Both the numerator, Ψ^N , and the denominator, Ψ^D , are positive, and while the expression for Ψ seems complicated at a first glance, it turns out to deliver very intuitive results, which we present below.²²

Determinacy around the intended steady state under inelastic real wages: How does the presence of incomplete markets impact on the possibility of local self-fulfilling equilibria? Recall the interaction between demand and supply by which higher job uncertainty lowers aggregate demand, which in turn reduces the incentives to post vacancies. The reduction in vacancies in turn reduces the job finding rate, further increasing unemployment risk. Monetary policy must intervene in this feedback mechanism to rule out the possibility that exogenous changes in beliefs, or “sunspot fluctuations,” can generate self-fulfilling equilibria.

The model formalizes the condition under which such fluctuations can occur. For simplicity, we start with a version with inelastic real wages ($\chi = 0$). In this case, $\Theta^F > 0$ and the endogenous earning risk is countercyclical. Since the job finding rate is not a state variable, the equilibrium

²¹The log-linearized model contains no endogenous state variables and hence for any desire pair of values δ_π and δ_θ one can find a value δ_θ^* such that the same solution is obtained under the restriction that $\delta_\pi = \frac{1}{\beta}$.

²² Ψ^N is easily seen to be positive since it involves only positive or non-negative terms. The denominator is also confirmed to be positive by insertion of the expression for Θ^F .

is locally determinate if and only if $\Psi > 1$, i.e. if and only if:

$$\frac{\phi}{\gamma} \left(\beta^2 \bar{R} \Theta^F - \frac{\beta \delta_\theta}{1 - \alpha} \right) < \frac{\kappa}{q} \frac{\alpha}{1 - \alpha} (1 - \beta(1 - \omega)).$$

This condition clarifies the importance of the various market frictions and their interaction. The occurrence of local indeterminacy depends on four types of market frictions present in the model, as well as on monetary policy:

- (i) *Price rigidity.* If prices are fully flexible ($\phi = 0$) the equilibrium is always determinate since the left-hand side collapses to zero and the right-hand side is strictly positive. The stickier are prices ($\phi > 0$), the more likely is the possibility that the equilibrium becomes locally indeterminate.
- (ii) *Imperfect competition.* Under perfect competition ($\gamma \rightarrow \infty$) the equilibrium is always determinate, since prices will be flexible. Less substitution across goods (lower γ) impacts on the determinacy condition symmetrically to larger nominal rigidities.
- (iii) *Endogenous earnings risk.* Under inelastic real wages, the endogenous risk parameter collapses to $\Theta^F = \omega \eta ((\vartheta/w)^{-\mu} - 1) \geq 0$. Thus, when real wages are inelastic, a larger endogenous risk wedge unambiguously demands more aggressive monetary policy to ensure local determinacy of the intended equilibrium. When $\Theta^F = 0$, as would occur under risk neutrality ($\mu = 0$), full insurance ($\vartheta = w$), or vanishing idiosyncratic risk ($\omega = 0$ or $\eta = 1$), the equilibrium is always locally determinate.
- (iv) *Labor adjustment cost.* The term $\frac{\kappa}{q} \frac{\alpha}{1 - \alpha} (1 - \beta(1 - \omega))$ denotes the steady-state marginal cost of hiring a worker today rather than tomorrow, so we can think of it as a labor adjustment cost, i.e. a real labor rigidity. Note that this cost is proportional to the steady-state hiring cost $\frac{\kappa}{q}$.
- (v) *Monetary policy.* The more aggressively monetary policy responds to tightness, i.e. the higher δ_θ , the less likely indeterminacy is to occur.

There are two main differences between the incomplete markets model with endogenous earnings risk and the standard model with insurance against idiosyncratic risk. The first is simply that the conditions for determinacy are more stringent under incomplete markets. With complete markets, a sufficient conditions for local determinacy is that $\delta_\pi > 1$ as we have assumed. Intuitively, in the standard NK model, when $\delta_\pi > 1$ self-fulfilling inflationary expectations are

ruled out in the vicinity of the intended equilibrium because higher expected inflation leads to lower actual inflation, due to the hike in the nominal interest rate. Under incomplete markets and countercyclical endogenous earnings risk this is no longer a sufficient condition, because higher expected inflation also stimulates goods demand when $\Theta^F > 0$ and higher goods demand is, in turn, inflationary. Thus, monetary policy needs to be even more aggressive to rule out expectational equilibria.

Secondly, the wedges interact in important ways. As long as monetary policy dominates the endogenous risk wedge, $\Theta^F < \beta\delta_\theta$, the sticky-price wedge and the labor market wedge are irrelevant and the intended equilibrium is locally determinate. However, once the endogenous risk wedge dominates the monetary policy effect, $\Theta^F > \beta\delta_\theta$, the three wedges all matter and there is complementarity between the sticky-price wedge and the endogenous risk wedge. In particular, market incompleteness, nominal rigidities and risk aversion become complements, making local indeterminacy increasingly likely in combination.

An intriguing insight regards the impact of labor market frictions since the higher is the labor adjustment cost, the *less* likely it is for indeterminacy to happen. Thus, less flexible labor markets imply less amplification. The reason for this is that when it is costly for firms to adjust on the labor margin, they are more likely to adjust prices which neutralizes the feedback mechanism.

The above analysis complements a literature which has studied local determinacy in New Keynesian Models with both forward-looking and “rule-of-thumb households”, see for example Galí, López-Salido and Vallés (2003) and Bilbiie (2008). A crucial difference with our environment, however, is that in these models there is no idiosyncratic risk and hence no precautionary savings motive. In our model, the precautionary motive, coupled with endogenous risk, is the key source behind the breakdown of the Taylor principle, as demonstrated above.

Determinacy around the intended steady state under flexible real wages: The determinacy condition becomes somewhat more involved when we introduce elastic wages ($\chi > 0$):

$$\frac{\phi}{\gamma} \left(\beta^2 \bar{R} \Theta^F - \frac{\beta \delta_\theta}{1 - \alpha} \right) - w\chi - \frac{\phi}{\gamma} \mu \beta (1 - \beta \bar{R}) \chi < \frac{\kappa}{q} \frac{\alpha}{1 - \alpha} (1 - \beta(1 - \omega)).$$

Wage responses affect determinacy via three channels. First, it does so via an *endogenous risk channel*. Recall that $\Theta^F = \omega\eta((\vartheta/w)^{-\mu} - 1) - \chi\mu\omega(1 - \eta)$. Hence, wage flexibility reduces the endogenous risk wedge and makes it more likely that the intended equilibrium is locally determinate. Furthermore, if wage flexibility is sufficiently high that $\Theta^F < 0$, local determinacy

is guaranteed as long as $\delta_\theta \geq 0$.

Second, wage flexibility creates a *marginal cost channel*, as it pushes down wage costs during times of low market tightness, pushing up vacancy posting. This channel comes in via the term $-w\chi$. Finally, wage flexibility generates a *discounting channel*, which enters via the term $-\phi\gamma^{-1}\mu\beta(1-\beta\bar{R})\chi$. This term arises only under incomplete markets, but does not require job risk to be endogenous. It emerges due to the Euler equation “discount” on future income (consumption), $\beta\bar{R} < 1$. See McKay, Nakamura and Steinsson (2017) for a discussion of this discount in relation to the “forward guidance puzzle”.

Finally, note that through all three channels wage flexibility pushes the model towards the determinacy region of the parameter space. In conclusion, real wage flexibility is stabilizing in the vicinity of the intended steady state.

Determinacy around the unemployment trap: We now consider local determinacy around the unemployment trap. To this end, we exploit that the non-negativity constraint on vacancies binds. Hence, we can drop Equation (31) and set η_s equal to 0 (or equal to some lower bound if some frictionless hiring is introduced)²³. Thus, the job finding rate is trivially determined. The Euler equation, log-linearized around the unemployment trap, is given by:

$$0 = \delta_\pi \hat{\Pi}_s - \mathbb{E}_s \hat{\Pi}_{s+1}.$$

It follows immediately that the equilibrium is unique if and only if $\delta_\pi > 1$, i.e. the interest rate elasticity with respect to inflation exceeds unity. Thus, the unemployment trap is determinate under a standard Taylor rule which responds more than one-for-one to inflation.

4 Fluctuations

We now solve for the local dynamics in the vicinity of the intended steady state in response to aggregate shocks. We focus on the impact of productivity shocks and monetary policy shocks, but it is not difficult to derive the implications for other shocks, such as mark-up shocks or non-fundamental “belief shocks.”²⁴

²³One might for, for example, assume that firms can fill some jobs without posting vacancies which would create a lower bound on η .

²⁴We outline the implications for belief shocks in Appendix A2. For an analysis of technology shocks in the standard New Keynesian model, see Galí (1999).

4.1 Productivity shocks

The model with productivity shocks can be written as:

$$\begin{aligned}\mathbb{E}_s \widehat{\eta}_{s+1} &= \Psi \widehat{\eta}_s - \Omega A_s, \\ A_s &= \rho_A A_{s-1} + \sigma_A \varepsilon_s^A, \\ \Omega &= \frac{(\gamma - 1)/\gamma}{\Psi^D},\end{aligned}$$

where $\Omega > 0$ since we established above that $\Psi^D > 0$ and $\gamma > 1$. See Appendix A2 for a derivation.

We concentrate on the determinate case ($\Psi > 1$). Apply the method of undetermined coefficients and guess a solution of the form $\widehat{\eta}_s = \Gamma_\eta^A A_s$. Plugging this guess into the above system of equations yields the following solution:

$$\Gamma_\eta^A = \frac{\Omega}{\Psi - \rho_A}. \quad (35)$$

It can now be shown that, in the determinacy region of the parameter space, the job finding rate responds positively to productivity shocks, i.e. $\Gamma_\eta^A > 0$. To see why, recall that the numerator of Equation (36) is positive and note that for the denominator to be positive as well, it is required that $\Psi > \rho_A$, which is satisfied automatically in the determinacy given that $\Psi > 1$ and $\rho_A < 1$.

Writing out the solution for Γ_η^A explicitly gives:

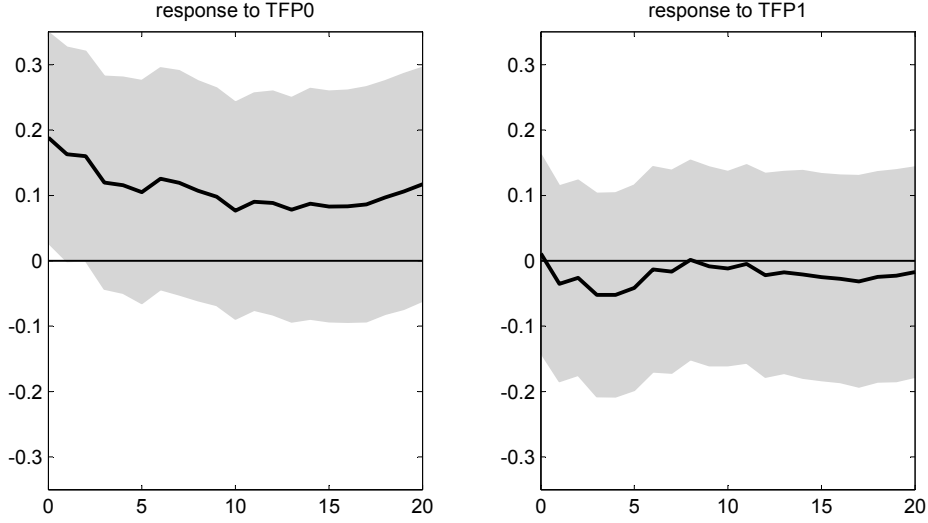
$$\Gamma_\eta^A = \frac{\gamma - 1}{\phi\beta \left(\frac{\delta_\theta}{1-\alpha} - \rho_A \beta \bar{R} \Theta^F \right) + \gamma \frac{\kappa}{q} \frac{\alpha(1-\rho\beta(1-\omega))}{1-\alpha} + (\gamma w + \phi\mu\beta(1 - \rho_A \beta \bar{R})) \chi}.$$

Given $\Gamma_\eta^A > 0$, it holds that $\frac{\partial \Gamma_\eta^A}{\partial \Theta^F} \geq 0$, i.e. a higher value of the endogenous risk parameter Θ^F amplifies the impact of productivity on the job finding rate. Thus, incomplete markets and endogenous risk are important for the response output and employment to productivity shock, via the intuitive amplification/dampening mechanism described earlier.

When the endogenous earnings risk is countercyclical ($\Theta^F > 0$), there is complementarity between the endogenous risk wedge and sticky prices, giving rise to amplification. In this case, more aggressive monetary policy dampens the response, since $\frac{\partial \Gamma_\eta^A}{\partial \delta_\theta} \leq 0$, but only when prices are sticky. Elastic real wages dampen the response of the job finding rate to productivity shocks, i.e. $\frac{\partial \Gamma_\eta^A}{\partial \chi} < 0$, since $\beta \bar{R} \leq 1$, $\rho_A \in (-1, 1)$ and $\frac{\partial \Theta^F}{\partial \chi} < 0$.

We can now solve for the inflation rate, guessing a solution of the form $\widehat{\Pi}_s = \Gamma_\Pi^A A_s$. Plugging

Figure 4: Response of CPI inflation to a positive TFP shock.



Notes: IRF of $400 \cdot \log(\text{cpit}/\text{cpit}-1)$ to change in \log TFP as estimated by Fernald <http://www.frbsf.org/economic-research/publications/working-papers/2016/wp2016-07.pdf> using local projection. The sample starts in 1980 and we included 4 lags. TFP0 (TFP1) refers to Fernald estimate for Total Factor Productivity without (with) control for factor utilization. Shaded areas denote error bands of two standard deviations.

this guess into the log-linearized Euler equation gives:

$$\Gamma_{\Pi}^A = \frac{\beta^2 \bar{R} \Theta^F \rho_A - \frac{\beta \delta_{\theta}}{1-\alpha} - \mu \chi \beta (1 - \rho_A \beta \bar{R})}{1 - \beta \rho_A} \Gamma_{\eta}^A.$$

It follows that inflation increases following a positive technology shock (i.e. $\Gamma_{\Pi} > 0$) if and only if $\beta^2 \bar{R} \Theta^F \rho_A > \frac{\beta \delta_{\theta}}{1-\alpha} + \mu \chi \beta (1 - \rho_A \beta \bar{R})$. Thus, unlike the response of the job finding rate, the sign of the inflation response is ambiguous. Evidently, when the endogenous earnings risk is procyclical or acyclical, inflation declines following positive technology shocks, as long as either $\delta_{\theta} > 0$ or $\chi > 0$. In these cases, higher productivity lower real marginal costs, which has a negative impact on inflation and may even stimulate precautionary savings if wages are very elastic.

However, higher productivity sets off higher inflation when the above inequality is reversed, i.e. when earnings risk is sufficiently countercyclical. Intuitively, this possibility comes from a demand channel: the increase in vacancy posting pushes up job finding rates, reducing the precautionary savings motive. This creates a boom in demand which pushes up prices, which may more than offset the marginal cost effect of the technology shock.

The possibility that higher productivity produces higher inflation is not a mere theoretical curiosity. In Figure 4 we show the impulse response of CPI inflation to TFP shocks where the latter correspond to those estimated by Fernald and Wang (2016). Using local projection, we regressed (400 times) quarterly (log) changes in the CPI on TFP (log) growth (times 100) for a sample that starts in 1980. Depending on whether one controls for movements in factor utilization or not, higher TFP either leaves inflation unchanged or gives rise to higher inflation. While the empirical results come with a fair amount of uncertainty, they do suggest that a positive inflation response is not simply an odd feature of our model.²⁵

4.2 Monetary policy shocks

Above we have shown that the systematic components of monetary policy are crucial for the impact of technology shocks. We now consider the effects of monetary policy shocks. In particular, we introduce an exogenous shock e_t^R to the interest rate rule:

$$R_s = \max \left\{ \bar{R} \left(\frac{\Pi_s}{\bar{\Pi}} \right)^{\delta_\pi} \left(\frac{\theta_s}{\bar{\theta}} \right)^{\delta_\theta} \exp(e_t^R), 1 \right\}.$$

We assume that e_t^R follows an AR(1) process with persistence parameter ρ_R . The solutions for the job finding rate and the inflation rate when log-linearizing the model around the intended steady state are:

$$\begin{aligned} \hat{\eta}_s &= \Gamma_\eta^R e_s^R, \\ \hat{\Pi}_s &= \Gamma_\Pi^R e_s^R, \\ \Gamma_\eta^R &= \frac{-\phi}{\phi\beta \left(\frac{\delta_\theta}{1-\alpha} - \rho_R \bar{R} \Theta^F \right) + \frac{\gamma \kappa \alpha (1-\beta \rho_R (1-\omega))}{\beta q} + \left(\frac{\gamma w}{\beta} + \phi \mu (1 - \rho_R \beta \bar{R}) \right) \chi}, \\ \Gamma_\Pi^R &= \frac{\beta (\beta \bar{R} \Theta^F \rho_R - \frac{\delta_\theta}{1-\alpha} - \chi (1 - \rho_R \beta \bar{R})) \Gamma_\eta^R - \beta}{1 - \beta \rho_R}. \end{aligned}$$

Following the same logic as above, we can verify that $\Gamma_\eta^R < 0$. That is, a contractionary monetary policy shock triggers a decline in the job finding rate and hence in output, as in the standard NK model. This decline is amplified by the presence of countercyclical risk, since $\frac{\partial \Gamma_\eta^R}{\partial \Theta^F} > 0$ provided that there is some persistence in the monetary policy shocks ($\rho_R > 0$). Intuitively,

²⁵The result holds also for the core PCE and here the positive response holds regardless of the TFP measure. The results also hold true for a sample period that starts in 1984, the sample split that Fernald and Wang (2016) focus upon.

the boom in demand created by a monetary expansion reduces idiosyncratic risk, creating a further boom in demand. In effect, monetary policy shocks thus become more powerful.

Now consider the effect on inflation. When $\Theta^F > 0$, the presence of the endogenous risk wedge strengthens the inflation response. This strengthening comes on top of the amplification generated by the endogenous risk wedge via Γ_η^R . If instead $\Theta^F < 0$ so that the endogenous earnings risk is procyclical, Γ_Π^R is further weakened. Under these circumstances, an expansionary monetary policy shock stimulates the labor market and pushes up wages. This, however, implies a larger drop in income in case of job loss. Hence the increase in wages strengthens the precautionary savings motive and tilts the desired consumption stream away from the present, which dampens the inflation response.

5 Implications for the Zero Lower Bound

Our analysis thus far has focused on the implications of the endogenous risk channel when the economy is away from the ZLB on the nominal interest rate. In this section, we analyze how the channel impacts on paths into the ZLB, and economic outcomes once the ZLB is reached.

5.1 Contractionary Shocks and the ZLB

A recent literature has emerged on the effects of the Zero Lower Bound (ZLB) in the New Keynesian model, see e.g. Christiano, Eichenbaum and Rebelo (2011), Krugman and Eggertsson (2012) and Farhi and Werning (2013). Often, such analyses start off from a premise that some exogenous and transitory shock brings the economy temporarily to the ZLB. The specific shock introduced for this purpose is typically an exogenous shock to the discount factor, making agents temporarily more patient. The increase in patience drives down the aggregate demand, putting downward pressure on inflation and the real interest rate. Via the interest rate rule, this results in a decline in the nominal interest rate, which may hit the ZLB if the shock is large enough (and at that point induces a potentially significant recession in the economy).

To appreciate the purpose of this specific shock, it helps to note that more conventional recessionary shocks, such as negative productivity shocks, typically will not lead to a decline in the nominal interest rate. There are two reasons for this. First, recessionary shocks reduce aggregate income and in a representative-agent model, lower current income (relative to expected future income) reduces households' desire to save inducing upward pressure on real and nominal interest rates, see e.g. Galí (2015, Chapter 3). A negative technology shock additionally increases

real marginal costs, which puts further upward pressure on inflation and, via the Taylor rule, also the nominal interest rate. Thus, in a standard NK model without other sources of shocks, *expansionary* rather than *recessionary* technology shocks are required to produce a decline in the nominal interest rate. For that reason, much research in the NK literature has introduced discount factor shocks when studying ZLB dynamics.

The precautionary savings mechanism that arises under endogenous risk can radically alter the cyclicity of the real interest rate, avoiding the need for discount factor shocks. Mechanically, the endogenous risk wedge acts as a shock to the discount factor in the Euler equation, but is determined endogenously rather than exogenously. Assume that $\Theta^F > 0$. As economic conditions worsen, the risk of becoming unemployed increases, driving down aggregate demand and increasing agents' desire to save. If the precautionary savings mechanism is strong enough, the nominal interest rate may decline, as argued by for example Werning (2015).²⁶

Here, we can exploit the solution to the full model to obtain an explicit condition for the nominal interest rate to decline in response to a negative productivity shock. For simplicity, let us assume that monetary policy only responds to inflation ($\delta_\theta = 0$) and abstract from monetary policy shocks.²⁷ The log-linearized interest rate rule is given as

$$\widehat{R}_s = \delta_\pi \widehat{\Pi}_s = \delta_\pi \Gamma_{\Pi}^A A_s,$$

where $\delta_\pi > 1$. In the previous subsection, we have shown that Γ_{Π}^A is negative when $\Theta^F = 0$. That is, under complete markets (or exogenous earning risk) the inflation rate, and hence the nominal interest rate, responds positively to a negative technology shock. However, when $\Theta^F > \frac{\mu\chi}{\beta R \rho} (1 - \rho\beta\bar{R})$, i.e. when markets are sufficiently incomplete and the endogenous earnings risk is countercyclical, Γ_{Π}^A is positive. Under this condition, a negative technology shock drives down inflation and the nominal interest rate. If the shock is large enough, the ZLB may become binding.

²⁶The working paper version of Ravn and Sterk (2017) make a similar point based on numerical simulations and, but do not consider productivity shocks. Werning (2015) presents analytical arguments, but not a fully fledged model.

²⁷We further assume that $\Gamma_\eta > 0$, i.e. the job finding rate responds positively to a positive productivity shock. As shown above, this is always the case in the determinacy region of the parameter space, and may be the case in the indeterminacy region.

5.2 Understanding Missing Deflation

Although inflation has been moderate in the aftermath of the financial crisis, no country has experienced persistent deflation. This is not easily reconciled with the standard NK model: Under the assumption of complete markets ($\Theta^{SS}(\eta) = 0$), the deterministic steady-state real interest rate is given by $R/\Pi = 1/\beta$ and it follows that, when the ZLB binds in a steady state, the gross inflation rate must equal $\beta < 1$. Temporary episodes at the ZLB will be even more deflationary than this since the stochastic Euler equation in that case will only be satisfied as long as $\Pi < \beta$ during the ZLB regime.²⁸ It is important to notice that these implications are independent of the arguments that enter the interest rate rule.

The incomplete markets NK model has different implications. As explained earlier, the steady-state real interest rate under incomplete markets is:

$$\frac{R}{\Pi} = \frac{1}{\beta \Theta^{SS}(\eta)} < \frac{1}{\beta},$$

which implies that the steady-state real interest rate depends on labor market conditions. When the ZLB binds, the steady-state Euler equation and the policy rule for the interest rate imply that the following two conditions must be satisfied in a liquidity trap (LT):

$$\begin{aligned} \Pi^{LT} &= \beta \Theta^{SS}(\eta^{LT}) > \beta, \\ \Pi^{LT} &< \overline{\Pi} \overline{\theta}^{\delta_\theta/\delta_\pi} \overline{R}^{-1/\delta_\pi} (\eta^{LT})^{-(\delta_\theta/\delta_\pi)/(1-\alpha)}. \end{aligned}$$

Notice that if $\delta_\theta = 0$, the policy rule implies that $\Pi^{LT} < \overline{\Pi} \overline{R}^{-1/\delta_\pi} < 1$, so that the liquidity trap is deflationary, given that in the intended steady state $\Pi = \overline{\Pi} = 1$ and $R = \overline{R} > 1$. When $\delta_\theta > 0$, however, inflation may be positive or negative in the liquidity trap. In particular, steady-state inflation is likely to be positive if $(\vartheta/w(\eta^{LT}))^{-\mu} \gg 1$ and wages are not too responsive to the job finding rate, i.e. when the endogenous risk wedge is sufficiently countercyclical. Intuitively, under these circumstances, deteriorating labor market conditions (worsening tightness) induces both lower nominal interest rates and lower goods demand which in turn implies a further decline in tightness and in nominal rates the end-product of which may be that the ZLB may be reached at a positive inflation rate.

²⁸Suppose that the ZLB regime persists with probability p while the intended steady-state is absorbing. In that case, the inflation rate during the ZLB episode is determined as $\Pi^{LT} = \beta \left(p + (1-p)(c^I/c^{LT})^{-\mu} \right)$ where Π^{LT} is the inflation rate during the liquidity trap, c^I is consumption in the intended steady-state and c^{LT} is consumption in the liquidity trap. This condition implies $\Pi^{LT} < \beta$ as long as $c^I > c^{LT}$.

5.3 Paradoxes at the Zero Lower Bound

It is well known that at the ZLB, the representative-agent NK model has some paradoxical properties, see e.g. Eggertsson (2010), Eggertsson and Krugman (2012) and Werning (2012). One prominent example is the “supply shock paradox”: at the ZLB, positive shocks to the supply side of the economy can trigger a contraction in real activity.²⁹

The paradox arises from the fact that a positive supply shock pushes down production costs and hence inflation. The increase in inflation, in turn, creates paradoxical effects which can be understood from the consumption Euler equation. Consider, for simplicity, the complete-markets Euler equation under perfect foresight at the ZLB:

$$\left(\frac{c_{s+1}}{c_s}\right)^\mu = \beta \frac{1}{\Pi_{s+1}}.$$

The effect of a decline in expected inflation, at the ZLB, is that the real interest rate, $\frac{1}{\Pi_{s+1}}$, increases. The above Euler equation makes clear that this implies an increase in expected consumption growth, c_{s+1}/c_s . Given that the decline in inflation is transitory however, an increase in expected consumption growth implies a *decline* in the current level of consumption, i.e. an economic contraction.³⁰

The joint presence of incomplete markets and countercyclical earnings risk, however, can overturn these results. Mechanically, the endogenous risk wedge in the Euler equation can absorb the effect of a decline in the real interest rate. Intuitively, an increase in output implies an increase in hiring, which reduces the precautionary savings motive. This makes an expansion in output compatible with an increase in the real interest rate.

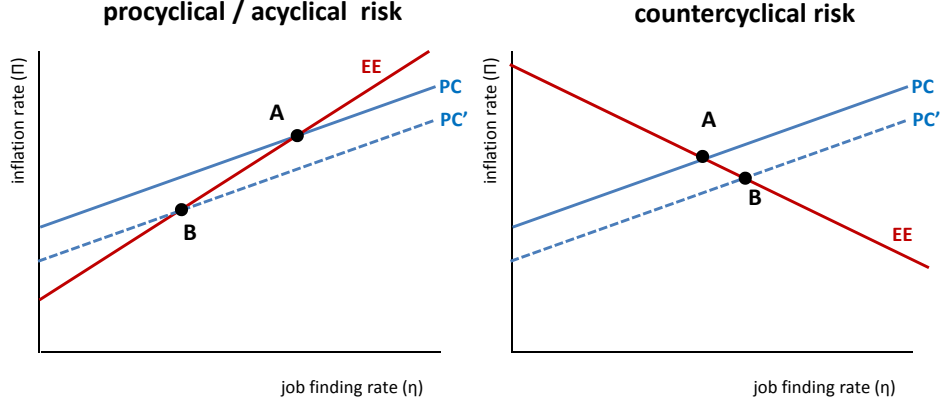
We now formalize these arguments. Suppose that the economy fluctuates discretely between a “depressed state” at which the ZLB binds, and a “normal state” which coincides with the intended steady state. Let $p \in (0, 1)$ be the probability that the ZLB regime persists and let the normal state be absorbing. In Appendix A4 we derive the relation between inflation and the job finding rate implied by the Euler equation, illustrated by lines labeled “**EE**” in Figure 4. The slope is given by:

$$\frac{d\widehat{\Pi}_s}{d\widehat{\eta}_s} = \frac{\mu\chi}{p} (1 - \beta\bar{R}p) - \beta\bar{R}\Theta^F.$$

²⁹Another important and closely related example is the “paradox of flexibility” which states that, at the ZLB, a higher degree of price flexibility creates a larger drop in output.

³⁰Throughout this subsection, we consider equilibria which ultimately lead to the intended steady state. Properties of equilibria leading to the liquidity trap steady state can be very different, see e.g. Mertens and Ravn (2014).

Figure 5: Supply Shock Paradox at the ZLB: illustration.



Under acyclical risk ($\Theta^F = 0$), or under procyclical endogenous earnings risk ($\Theta^F < 0$) the elasticity is positive since $\mu\chi > 0$ and $\beta\bar{R}p < 1$. Thus, any additional shock which reduces inflation must create a labor market contraction. As explained above, this is the source of the paradox. However, when $\Theta^F > \frac{\mu\chi}{p}(\beta^{-1}\bar{R}^{-1} - 1)$, i.e. when the endogenous earnings risk is highly countercyclical, the slope is negative. In that case, a reduction in inflation coincides with a labor market expansion.

In order to study explicitly the effect of a change in productivity, consider now the supply side of the economy. The Phillips Curve implies a positive relation between inflation and the job finding rate, see Appendix A4 for details. The lines in Figure 4 labeled “**PC**” illustrate this relation. An increase in productivity shifts down the **PC** curve and moves the equilibrium from point A to point B.

The left panel of Figure 4 depicts an economy with acyclical/procyclical risk and illustrates the paradox that arises also under complete markets: the increase in productivity reduces the job finding rate, and hence employment. The right panel illustrates a case with a downward-sloping **EE** curve, due to countercyclical risk. In this case, the job finding rate increases in response to the productivity increase. Thus, the presence of incomplete markets and countercyclical risk can overturn the supply shock paradox.

6 An Empirical Perspective

A central implication of the model is that the presence of endogenous earnings risk can create either dampening or amplification of fluctuations. Dampening arises under procyclical risk

($\Theta^F < 0$), whereas amplification occurs under countercyclical risk ($\Theta^F > 0$). We now shed more light on the question which of these two cases is empirically most likely.

To do so we exploit the analytical formula for the endogenous risk parameter Θ^F in Equation (30). We consider a time period of one month and evaluate the steady-state job finding rate η and the job loss rate ω , as the average of their counterparts in the Current Population Survey (CPS) over the period January 1990 until September 2017.³¹ Recall further that $1 - \vartheta/w = 1 - c_u/c_e$ is the decline in consumption upon job loss. Following Karabarbounis and Chodorow-Reich (2017), we assume that consumption drops 20 percent upon job loss. However, we also consider a much smaller drop of only 5 percent. For the risk aversion parameter μ we consider both $\mu = 0.5$ and $\mu = 2$, as estimates of this parameters vary across studies in the literature.

The final parameter that matters is the wage flexibility parameter, $\chi = \frac{\partial \ln w_s}{\partial u_s} \frac{\partial u_s}{\partial \ln \eta_s}$. We can obtain the second term by differentiating the transition equation for unemployment with respect to the (log of the) job finding rate. Evaluated at the steady state this gives $\frac{\partial u_s}{\partial \ln \eta_s} = -\eta u - \omega \eta (1 - u)$, which we evaluate using CPS data. The semi-elasticity of the wage with respect to unemployment, $\frac{\partial \ln w_s}{\partial u_s}$, has been estimated in several studies. Gertler, Huckfeldt and Trigari (2016) estimate this elasticity to be $\frac{\partial \ln w_s}{\partial u_s} = -0.16$ for job stayers (see their Table 2, fourth column). We take this number as our baseline, since Θ^F captures the expected wage of those currently employed, in the event they remain employed. However, we also consider a much larger elasticity of -1.5 , which is in the ballpark of the estimates which Gertler et al. estimate specifically for new hires from unemployment (-0.164) and job switchers (-2.085).³²

Table 1 shows the results. In most cases, the countercyclical effect of unemployment risk dominates the procyclical effect of wage risk (i.e. $\Theta^F > 0$). Only when we assume *both* a small consumption drop (5 percent) and a very elastic wage ($\frac{\partial \ln w_s}{\partial u_s} = -1.5$), do we find that the effect of wage risk slightly dominates. Given that these values are relatively unlikely in the light of most studies in the literature, we conclude that procyclical earnings risk is the more relevant case.

To get a sense of the magnitudes, note that an increase in the wedge in Equation (30) has the same partial-equilibrium effect on consumption growth of the employed as a change in the real interest rate. Consider also the fact that during the Great Recession, the job finding rate

³¹In particular, we measure η_t as the unemployment-to-employment transition rate, $\omega_t = \frac{u_t - u_{t-1}(1 - \eta_t)}{(1 - u_{t-1})(1 - \eta_t)}$, given a series for the unemployment rate u_t , consistent with the timing assumptions in our model.

³²We also estimated χ directly by running a regression of w_s on the job finding rate η_s , and a time trend. Here, we measured w_s as average hourly earnings of production and nonsupervisory employees, deflated by the CPI. While results varied across specifications, the largest wage elasticity we found was $\hat{\chi} = 0.03$ which corresponds to about $\frac{\partial \ln w_s}{\partial u_s} = -1.5$.

Table 1: Cyclicity of earnings risk in the log-linearized model.

I. baseline wage elasticity ($\frac{\partial \ln w_t}{\partial u_t} = -0.16$)				
consumption loss upon job loss	baseline (20%)		low (5%)	
coefficient of risk aversion	$\mu = 0.5$	$\mu = 2$	$\mu = 0.5$	$\mu = 2$
1) Θ^F : unemployment	0.0436	0.2079	0.0096	0.0399
2) Θ^F : wage	-0.0017	-0.0067	-0.0017	-0.0067
3) Θ^F : total	0.0420	0.2013	0.0079	0.0333
II. high wage elasticity ($\frac{\partial \ln w_t}{\partial u_t} = -1.5$)				
consumption loss upon job loss	baseline (20%)		low (5%)	
coefficient of risk aversion	$\mu = 5$	$\mu = 2$	$\mu = 0.5$	$\mu = 2$
1) Θ^F : unemployment	0.0436	0.2079	0.0096	0.0399
2) Θ^F : wage	-0.0167	-0.0666	-0.0167	-0.0666
3) Θ^F : total	0.0270	0.1413	-0.0071	-0.0267

Notes: $\Theta^F > 0$ implies countercyclical earnings risk. 1): $\omega\eta((\vartheta/w)^{-\mu} - 1)$, 2): $-\chi\mu\omega(1 - \eta)$, 3): $\omega\eta((\vartheta/w)^{-\mu} - 1) - \chi\mu\omega(1 - \eta)$. All results have been multiplied by 100.

fell by about fifty percent. Given this decline, the baseline parametrization under $\mu = 2$ implies a change in the endogenous risk wedge which is equivalent to a fall in the annual real interest rate of about 120 basis points.

An alternative way of taking the model to the data is to consider the implied relation between the real interest rate and market tightness, as plotted in Figure 1. Consider the model with only productivity shocks. Using the log-linearized Euler equation, the real interest rate can be expressed as $\widehat{R}_s^r = (1 - \alpha) \left(- (1 - \beta\bar{R}\rho_A) \mu\chi + \Theta^F \beta\bar{R}\rho_A \right) \widehat{\theta}_s$, where $\widehat{R}_s^r \equiv \widehat{R}_s - \mathbb{E}_s \widehat{\Pi}_{s+1}$. This equation provides a direct relation between market tightness and the real interest rate, which we can confront with the data. Recall that the relation between \widehat{R}_s^r and $\widehat{\theta}_s$ is overwhelmingly positive. The above expression implies that in the model this can only be the case if Θ^F positive and sufficiently large. Again, the data suggest that procyclical earnings risk is the more relevant case. Intuitively, increased unemployment risk encourages households to save more during recessions, pushing down the equilibrium real interest rate.

7 Conclusion

We have proposed a simple and intuitive heterogeneous-agents New Keynesian (NK) model with endogenous unemployment, and highlighted that the interaction between market frictions can give rise to belief-driven fluctuations. Moreover, the interaction between these frictions produces

potentially a significant amount of amplification of shocks to the economy. This amplification occurs when the endogenous earnings risk is countercyclical and we have argued that this is the empirically plausible case. The essence of the interaction is that incomplete markets produces movements in aggregate demand in response to fluctuations in the job finding rate which impact on the supply side when there are nominal rigidities and creates a feedback mechanism. In particular, weak labor demand produces low goods demand which in itself produces low labor demand. The combination of HANK and SAM therefore has fundamental consequences and puts labor markets in the centre of the amplification and transmission mechanism.

We have also shown that the new NK model with countercyclical earnings risk can resolve a number of puzzles that have arisen in the macroeconomic literature. These involve the existence of persistent low growth equilibria with low but positive inflation, the impact of supply shocks on inflation dynamics, and various paradoxes at the ZLB. Intriguingly, the model can also provide a coherent framework for understanding the positive relationship between real interest rates and labor market tightness which can be observed in the US.

In Appendix A5, we demonstrate that under incomplete markets the NK model becomes useful to analyze the link between monetary policy and financial asset prices. While we limit the analysis to simple analytical exercises, it would be interesting to evaluate the extent to which a full-scale heterogeneous-agents NK can explain observed asset prices. Vice versa, financial markets data may be useful to impose empirical discipline on the new generation of NK models.

Throughout the analysis, we have assumed that government policies are summarized by a simple interest rate rule, subject to the zero lower bound. It would be interesting to use the framework to obtain insights into the stabilization effects of other government policies, such as fiscal policy or labor market policies. Also, the framework could be used to consider optimal policies. We leave these issues for future research.

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Appendix

A1. Steady-state Nash bargaining solution

The steady-state expressions of the asset-poor households' surplus and value functions are:

$$\begin{aligned} V^e (1 - \beta (1 - \omega (1 - \eta))) &= \frac{w^{1-\mu}}{1-\mu} - \zeta + \beta\omega (1 - \eta) V^u, \\ V^u (1 - \beta (1 - \eta)) &= \frac{\vartheta^{1-\mu}}{1-\mu} + \beta\eta V^e, \end{aligned}$$

where we have exploited that in equilibrium the asset-poor households are the same and consume their incomes. Now substitute out V^u in the first equation:

$$\begin{aligned} V^e (1 - \beta (1 - \omega (1 - \eta))) &= \frac{w^{1-\mu}}{1-\mu} - \zeta + \frac{\beta\omega (1 - \eta)}{1 - \beta (1 - \eta)} \left(\frac{\vartheta^{1-\mu}}{1-\mu} + \beta\eta V^e \right). \\ V^e \left(1 - \beta (1 - \omega (1 - \eta)) - \frac{\beta\omega (1 - \eta)}{1 - \beta (1 - \eta)} \beta\eta \right) &= \frac{w^{1-\mu}}{1-\mu} - \zeta + \frac{\beta\omega (1 - \eta)}{1 - \beta (1 - \eta)} \frac{\vartheta^{1-\mu}}{1-\mu}, \end{aligned}$$

We can now express the two values as functions of η and w :

$$\begin{aligned} V^e (\eta, w) &= \frac{\frac{w^{1-\mu}}{1-\mu} - \zeta + \frac{\beta\omega(1-\eta)}{1-\beta(1-\eta)} \frac{\vartheta^{1-\mu}}{1-\mu}}{1 - \beta (1 - \omega (1 - \eta)) - \frac{\beta\omega(1-\eta)\beta\eta}{1-\beta(1-\eta)}} \\ V^u (\eta, w) &= \frac{\frac{\vartheta^{1-\mu}}{1-\mu} + \beta\eta V^e (\eta, w)}{1 - \beta (1 - \eta)} \end{aligned}$$

The first-order condition to the Nash Bargaining problem is given by

$$(1 - v) S^e = v S^f,$$

or,

$$(1 - v) (V^e (\eta, w) - V^u (\eta, w)) = v \kappa \eta^{\alpha/(1-\alpha)}.$$

$$(V^e (\eta, w) - V^u (\eta, w)) = \frac{v}{1 - v} \kappa$$

The above is an equation in two variables, which implicitly defines the wage as a function of the job finding rate, i.e the function $w(\eta)$.

Basic properties: Consider the special case in which $\eta = 0$. From the Nash bargaining solution it follows that the wage must satisfy $V^e (0, w(0)) = V^u (0, w(0)) = \frac{\vartheta^{1-\mu}}{1-\beta}$. It follows that

$\frac{w(0)^{1-\mu}}{1-\mu} = \frac{\vartheta^{1-\mu}}{1-\mu} + \zeta$ and hence $w(0) > \vartheta$ whenever $\zeta > 0$.

At the other extreme, under $\eta = 1$ we get from the Nash Bargaining solution $V^e(1, w) = V^u(1, w) + \frac{v\kappa}{1-v}$. Also, the worker value functions imply that $V^e(1, w) - V^u(1, w) = \frac{w(1)^{1-\mu}}{1-\mu} - \zeta - \frac{\vartheta^{1-\mu}}{1-\mu}$. It follows that $\frac{w(1)^{1-\mu}}{1-\mu} = \frac{\vartheta^{1-\mu}}{1-\mu} + \zeta + \frac{v}{1-v}\kappa$ and hence $w(1) > w(0)$, $V^e(1, w(1)) > V^e(0, w(0))$ and $V^u(1, w) > V^u(0, w)$.

Finally, consider a case in which the worker has no bargaining power ($v = 0$). It follows from the Nash bargaining solution that in this case $V^e(\eta, w) = V^u(\eta, w)$ which implies that $\frac{w(\eta)^{1-\mu}}{1-\mu} = \frac{\vartheta^{1-\mu}}{1-\mu} + \zeta$. As a result, the real wage does not depend of η , i.e. the real wage is sticky.

A2. Log-linearized model

Nash Bargaining block

The first-order condition to the Nash bargaining problem, together with the asset-poor workers' value functions are given by:

$$\begin{aligned} (1-v)(V_s^e - V_s^u) &= v\kappa\eta_s^{\alpha/(1-\alpha)}, \\ V_s^e &= \frac{w_s^{1-\mu}}{1-\mu} - \zeta + \beta\mathbb{E}_s\omega(1-\eta_{s+1})V_{s+1}^u + \beta\mathbb{E}_s(1-\omega(1-\eta_{s+1}))V_{s+1}^e, \\ V_s^u &= \frac{\vartheta^{1-\mu}}{1-\mu} + \beta\mathbb{E}_s(1-\eta_{s+1})V_{s+1}^u + \beta\mathbb{E}_s\eta_{s+1}V_{s+1}^e. \end{aligned}$$

After log-linearization, the above system can be written in the following form:

$$\mathbf{A} \begin{bmatrix} \widehat{V}_s^e \\ \widehat{V}_s^u \\ \widehat{w}_s \end{bmatrix} + \mathbf{B}\widehat{\eta}_s = \mathbb{E}_s \mathbf{C} \begin{bmatrix} \widehat{V}_{s+1}^e \\ \widehat{V}_{s+1}^u \\ \widehat{w}_{s+1} \end{bmatrix} + \mathbb{E}_s \mathbf{D}\widehat{\eta}_{s+1}$$

where \mathbf{A} and \mathbf{C} are 3×3 matrices and \mathbf{B} and \mathbf{D} are 3×1 vectors, all consisting of parameter values. Note that none of the variables \widehat{V}_s^e , \widehat{V}_s^u and \widehat{w}_s is a state variable. Provided that $\widehat{\eta}_s$ follows some linear law of motion and given the law of motion for A_s , we can apply the method of undetermined coefficients to find solutions for \widehat{V}_s^e , \widehat{V}_s^u and \widehat{w}_s as linear functions of $\widehat{\eta}_s$. We denote the solution for the wage as $\widehat{w}_s = \chi\widehat{\eta}_s$, where it follows that χ is a function of the parameters that enter \mathbf{A} , \mathbf{B} , \mathbf{C} and \mathbf{D} .

Monetary Policy rule, Euler equation, Phillips Curve

The log-linearized monetary policy rule is given by:

$$\widehat{R}_s = \delta_\pi \widehat{\Pi}_s + \delta_\theta \widehat{\theta}_s.$$

Next, consider the Euler equation of the employed households. Exploiting the fact that in Equilibrium $c_{e,s} = w_s$ and $c_{u,s} = \vartheta$, we can express the employed workers' Euler equation, Equation (21), as:

$$w_s^{-\mu} = \beta \mathbb{E}_s \frac{R_s}{\Pi_{s+1}} \left(\omega (1 - \eta_{s+1}) \vartheta^{-\mu} + (1 - \omega (1 - \eta_{s+1})) w_{s+1}^{-\mu} \right),$$

and note that in the intended steady state we obtain $w^{-\mu} = \beta \bar{R} (\omega (1 - \eta) \vartheta^{-\mu} + (1 - \omega (1 - \eta)) w^{-\mu})$. Log-linearizing the above equation around the intended steady state gives:

$$\begin{aligned} -\mu \widehat{w}_s &= \widehat{R}_s - \mathbb{E}_s \widehat{\Pi}_{s+1} - \beta \bar{R} \omega \eta (\vartheta/w)^{-\mu} \mathbb{E}_s \widehat{\eta}_{s+1} + \beta \bar{R} \omega \eta \mathbb{E}_s \widehat{\eta}_{s+1} - \mu \beta \bar{R} (1 - \omega (1 - \eta)) \mathbb{E}_s \widehat{w}_{s+1}, \\ &= -\mu \beta \bar{R} \mathbb{E}_s \widehat{w}_{s+1} + \widehat{R}_s - \mathbb{E}_s \widehat{\Pi}_{s+1} - \beta \bar{R} \omega \eta ((\vartheta/w)^{-\mu} - 1) \mathbb{E}_s \widehat{\eta}_{s+1} + \mu \beta \bar{R} \omega (1 - \eta) \mathbb{E}_s \widehat{w}_{s+1}, \\ &= -\mu \beta \bar{R} \mathbb{E}_s \widehat{w}_{s+1} + \widehat{R}_s - \mathbb{E}_s \widehat{\Pi}_{s+1} - \beta \bar{R} \Theta^F \mathbb{E}_s \widehat{\eta}_{s+1}, \end{aligned}$$

where $\Theta^F = \omega \eta ((\vartheta/w)^{-\mu} - 1) - \chi \mu \omega (1 - \eta)$ and where we used that $\widehat{w}_s = \chi \widehat{\eta}_s$.

Next, consider the firms' price setting condition, which can be written as:

$$\begin{aligned} &\phi (\Pi_s - 1) \Pi_s - \phi \mathbb{E}_s \Lambda_{s,s+1} \frac{y_{s+1}}{y_s} (\Pi_{s+1} - 1) \Pi_{s+1} \\ &= 1 - \gamma + \frac{\gamma}{\exp(A_s)} \left(w_s + \kappa \eta_s^{\alpha/(1-\alpha)} - (1 - \omega) \kappa \mathbb{E}_s \Lambda_{s,s+1} \eta_{s+1}^{\alpha/(1-\alpha)} + \lambda_{v,s} \right). \end{aligned}$$

and note that at the intended steady state $\lambda_{v,s} = 0$ and $\Lambda_{s,s+1} = \beta$. Log-linearizing the equation around the intended steady state with $\Pi = 1$ gives:

$$\frac{\phi}{\gamma} \widehat{\Pi}_s - \frac{\phi}{\gamma} \beta \mathbb{E}_s \widehat{\Pi}_{s+1} = w \chi \widehat{\eta}_s + \frac{1 - \gamma}{\gamma} A_s + \frac{\kappa}{q} \left(\frac{\alpha}{1 - \alpha} \widehat{\eta}_s - \frac{\alpha \beta (1 - \omega)}{1 - \alpha} \mathbb{E}_s \widehat{\eta}_{s+1} - \beta (1 - \omega) \mathbb{E}_s \widehat{\Lambda}_{s,s+1} \right),$$

where we have substituted out the wage using $\widehat{w}_s = \chi \widehat{\eta}_s$.

Reducing the model

Under the the two assumptions ($\delta_\pi = \frac{1}{\beta}$ and risk-neutrality of the equity investors) and in the absence of productivity shocks, the log-linearized Euler equation and pricing condition become:

$$\begin{aligned} -\mu\chi\beta\hat{\eta}_s + \mu\beta^2\bar{R}\chi\mathbb{E}_s\hat{\eta}_{s+1} &= \hat{\Pi}_s - \beta\mathbb{E}_s\hat{\Pi}_{s+1} + \frac{\beta\delta_\theta}{1-\alpha}\hat{\eta}_s - \beta^2\bar{R}\Theta^F\mathbb{E}_s\hat{\eta}_{s+1} \\ w\chi\hat{\eta}_s + \frac{\kappa}{q}\left(\frac{\alpha}{1-\alpha}\hat{\eta}_s - \frac{\alpha\beta(1-\omega)}{1-\alpha}\mathbb{E}_s\hat{\eta}_{s+1}\right) &= \frac{\phi}{\gamma}\left(\hat{\Pi}_s - \beta\mathbb{E}_s\hat{\Pi}_{s+1}\right) \end{aligned}$$

where in the first equation we have substituted out the interest rate using $\hat{R}_s = \delta_\pi\hat{\Pi}_s + \delta_\theta\hat{\theta}_s$, and tightness using $\hat{\theta}_s = \frac{\hat{\eta}_s}{1-\alpha}$. Using the first equation to substitute out $\hat{\Pi}_s - \beta\mathbb{E}_s\hat{\Pi}_{s+1}$ in the second equation gives:

$$w\chi\hat{\eta}_s + \frac{\kappa}{q}\left(\frac{\alpha}{1-\alpha}\hat{\eta}_s - \frac{\alpha\beta(1-\omega)}{1-\alpha}\mathbb{E}_s\hat{\eta}_{s+1}\right) = \frac{\phi}{\gamma}\left(-\mu\chi\beta\hat{\eta}_s + \mu\beta^2\bar{R}\chi\mathbb{E}_s\hat{\eta}_{s+1} - \frac{\beta\delta_\theta}{1-\alpha}\hat{\eta}_s + \beta^2\bar{R}\Theta^F\mathbb{E}_s\hat{\eta}_{s+1}\right).$$

Collecting terms gives:

$$\mathbb{E}_s\hat{\eta}_{s+1} = \Psi\hat{\eta}_s,$$

where

$$\Psi = \frac{\frac{\phi}{\gamma}\mu\chi\beta + \frac{\phi}{\gamma}\frac{\beta\delta_\theta}{1-\alpha} + w\chi + \frac{\kappa}{q}\frac{\alpha}{1-\alpha}}{\frac{\kappa}{q}\frac{\alpha\beta(1-\omega)}{1-\alpha} + \frac{\phi}{\gamma}\mu\beta^2\bar{R}\chi + \frac{\phi}{\gamma}\beta^2\bar{R}\Theta^F}.$$

Productivity shocks

With productivity shocks the model becomes:

$$\begin{aligned} w\chi\hat{\eta}_s + \frac{1-\gamma}{\gamma}A_s + \frac{\kappa}{q}\left(\frac{\alpha}{1-\alpha}\hat{\eta}_s - \frac{\alpha\beta(1-\omega)}{1-\alpha}\mathbb{E}_s\hat{\eta}_{s+1}\right) \\ = \frac{\phi}{\gamma}\left(-\mu\chi\beta\hat{\eta}_s + \mu\beta^2\bar{R}\chi\mathbb{E}_s\hat{\eta}_{s+1} - \frac{\beta\delta_\theta}{1-\alpha}\hat{\eta}_s + \beta^2\bar{R}\Theta^F\mathbb{E}_s\hat{\eta}_{s+1}\right), \\ A_s = \rho_A A_{s-1} + \sigma_A \varepsilon_s^A, \end{aligned}$$

which we can rewrite as

$$\left(\frac{\kappa}{q}\frac{\alpha\beta(1-\omega)}{1-\alpha} + \frac{\phi}{\gamma}\mu\beta^2\bar{R}\chi + \frac{\phi}{\gamma}\beta^2\bar{R}\Theta^F\right)\mathbb{E}_s\hat{\eta}_{s+1} = \left(w\chi + \frac{\kappa}{q}\frac{\alpha}{1-\alpha} + \frac{\phi}{\gamma}\mu\chi\beta + \frac{\phi}{\gamma}\frac{\beta\delta_\theta}{1-\alpha}\right)\hat{\eta}_s - \frac{\gamma-1}{\gamma}A_s$$

which gives

$$\begin{aligned}\mathbb{E}_s \widehat{\eta}_{s+1} &= \Psi \widehat{\eta}_s - \Omega A_s, \\ A_s &= \rho_A A_{s-1} + \sigma_A \varepsilon_s^A,\end{aligned}$$

where

$$\Omega = \frac{(\gamma - 1)/\gamma}{\frac{\kappa \alpha \beta (1 - \omega)}{q} + \frac{\phi}{\gamma} \mu \beta^2 \bar{R} \chi + \frac{\phi}{\gamma} \beta^2 \bar{R} \Theta^F}.$$

Monetary policy shocks

Now consider the model with monetary policy shocks. The log-linearized model, assuming again risk-neutral investors and $\delta_\pi = \frac{1}{\beta}$, becomes:

$$\begin{aligned}-\mu \chi \beta \widehat{\eta}_s + \mu \beta^2 \bar{R} \chi \mathbb{E}_s \widehat{\eta}_{s+1} &= \widehat{\pi}_s - \beta \mathbb{E}_s \widehat{\pi}_{s+1} + \frac{\beta \delta_\theta}{1 - \alpha} \widehat{\eta}_s - \beta^2 \bar{R} \Theta^F \mathbb{E}_s \widehat{\eta}_{s+1} + \beta e_s^R \\ \frac{\phi}{\gamma} \left(\widehat{\pi}_s - \beta \mathbb{E}_s \widehat{\pi}_{s+1} \right) &= w \chi \widehat{\eta}_s + \frac{\kappa}{q} \frac{\alpha}{1 - \alpha} \widehat{\eta}_s - \beta (1 - \omega) \frac{\kappa}{q} \frac{\alpha}{1 - \alpha} \mathbb{E}_s \widehat{\eta}_{s+1} \\ e_s^R &= \rho_R e_{s-1}^R + \sigma_R \varepsilon_s^R\end{aligned}$$

Combining the first two equations gives:

$$\begin{aligned}w \chi \widehat{\eta}_s + \frac{\kappa}{q} \left(\frac{\alpha}{1 - \alpha} \widehat{\eta}_s - \frac{\alpha \beta (1 - \omega)}{1 - \alpha} \mathbb{E}_s \widehat{\eta}_{s+1} \right) \\ = \frac{\phi}{\gamma} \left(-\mu \chi \beta \widehat{\eta}_s + \mu \beta^2 \bar{R} \chi \mathbb{E}_s \widehat{\eta}_{s+1} - \frac{\beta \delta_\theta}{1 - \alpha} \widehat{\eta}_s + \beta^2 \bar{R} \Theta^F \mathbb{E}_s \widehat{\eta}_{s+1} - \beta e_s^R \right),\end{aligned}$$

which we can re-write as

$$\left(\frac{\kappa \alpha \beta (1 - \omega)}{q} + \frac{\phi}{\gamma} \mu \beta^2 \bar{R} \chi + \frac{\phi}{\gamma} \beta^2 \bar{R} \Theta^F \right) \mathbb{E}_s \widehat{\eta}_{s+1} = \left(w \chi + \frac{\kappa}{q} \frac{\alpha}{1 - \alpha} + \frac{\phi}{\gamma} \mu \chi \beta + \frac{\phi}{\gamma} \frac{\beta \delta_\theta}{1 - \alpha} \right) \widehat{\eta}_s + \frac{\phi}{\gamma} \beta e_s^R.$$

Which delivers which gives

$$\mathbb{E}_s \widehat{\eta}_{s+1} = \Psi \widehat{\eta}_s - \Omega^R e_s^R,$$

where Ψ is as given in the main text and

$$\Omega^R = \frac{-\phi}{\gamma \frac{\kappa \alpha (1 - \omega)}{q} + \phi \mu \beta \bar{R} \chi + \phi \beta \bar{R} \Theta^F}.$$

We again concentrate on the determinate case ($\Psi > 1$) and apply the method of undetermined

coefficients and guess a solution of the form $\widehat{\eta}_s = \Gamma_\eta^R e_s^R$. Plugging this guess into the above system of equations yields the following solution:

$$\Gamma_\eta^R = \frac{\Omega^R}{\Psi - \rho_R}. \quad (36)$$

It can now be shown that, in the determinacy region of the parameter space, the job finding rate responds negatively to contractaionary monetary policy shocks, i.e. $\Gamma_\eta^R < 0$. To see why, note the numerator of Equation (36) is negative and the denominator is positive under determinacy, since it then holds that $\Psi > 1 > \rho_R$.

Writing out the solution for Γ_η^R explicitly gives:

$$\Gamma_\eta^R = \frac{-\phi}{\phi\beta \left(\frac{\delta_\theta}{1-\alpha} - \rho_R \bar{R} \Theta^F \right) + \frac{\gamma \kappa}{\beta q} \frac{\alpha(1-\beta\rho_R(1-\omega))}{1-\alpha} + \left(\frac{\gamma}{\beta} w + \phi\mu (1 - \rho_R \beta \bar{R}) \right) \chi}.$$

Let us now solve for the inflation rate, guessing a solution of the form $\widehat{\Pi}_s = \Gamma_\Pi^R e_s^R$. Plugging this guess into the log-linearized Euler equation gives:

$$\Gamma_\Pi^R = \frac{\beta \left(\beta \bar{R} \Theta^F \rho_R - \frac{\delta_\theta}{1-\alpha} - \chi (1 - \rho_R \beta \bar{R}) \right) \Gamma_\eta^R - \beta}{1 - \beta \rho_R}$$

Belief shocks

From Equation (34) it follows that if the equilibrium is locally determinate ($\Psi > 1$), then the only stable solution is given by $\widehat{\eta}_s = 0$ at all times. When equilibria are locally indeterminate, the solution is given by

$$\widehat{\eta}_{s+1} = \Psi \widehat{\eta}_s + \Upsilon^B \varepsilon_{s+1}^B,$$

where ε_s^B is an i.i.d. belief shock with mean zero and a standard deviation normalized to one, and Υ^B is a parameter. Thus, in a model with only belief shocks the job finding rate follows an AR(1) process. While the magnitude of the belief shocks, captured by Υ^B , is not pinned down in the model, the persistence of the effects of belief shocks on the job finding rate is captured by Ψ , and thus endogenously determined. Persistence is maximal at $\Psi = 1$, i.e. exactly at the border between the determinacy and indeterminacy region of the parameter space.

A3. Risk-averse investors

When we log-linearized the model, we have assumed for simplicity that the asset-rich firm owners are risk neutral. The reason is that, technically, the unemployment rate becomes a state variable for inflation and the job finding rate, once we assume risk averse investors. With an additional state variable, the analytical solution of the model becomes more cumbersome, detracting from the key intuitions of the model.

Below, we use numerical simulations to compare versions with risk-neutral and risk-averse investors, showing only very small differences. We parametrize the model as follows. We choose the subjective discount factor β target a steady-state interest rate of 3 percent per annum. The coefficient of risk aversion, μ , is set to 2, whereas the elasticity of substitution between goods, γ , is set to 6. To calibrate the price-stickiness parameter ϕ , we exploit the observational equivalence between the Calvo and Rotemberg versions of the log-linearized New Keynesian model, and target an average price duration of 5 months. The home production parameter, ϑ , is set to imply a 20 percent consumption drop upon unemployment.

The vacancy cost is parametrized to target a steady-state hiring cost of about 5 percent of the quarterly wage. We further target a monthly job finding rate of 0.3 and set the job loss rate, ω , to 2 percent. The matching function elasticity parameter, α , is set to 0.5. Regarding the monetary policy rule, we set $\delta_\pi = 1.5$ and $\delta_\theta = 0$. The persistence parameter of the technology shock is set to $\rho_A = 0.95$. For simplicity we assume sticky wages ($\chi = 0$).

The left panel of the figure below plots the response of the job finding rate to a positive technology shock under sticky prices. Quantitatively, however, the differences are small. Next, we consider a version of the model with flexible prices (right panel). Effectively, this removes the amplification mechanism from the model. The right panel of the figure below (“complete markets”) again compares the versions with risk-averse and risk-neutral investors. The differences are similar to the complete markets case. Most importantly, differences are again small.

A4. The Euler equation at the ZLB

Consider the setup described in Section 5.3. For simplicity, we further assume that when the economy is in the depressed (ZLB) state, the households do not expect any further shock other than that the economy returns to the normal state with a probability p .

In the depressed state it holds, for $x = \{\eta, \Pi\}$, that $\mathbb{E}_s x_{s+1} = p\mathbb{E}_s x_{s+1}^{ZLB} + (1-p)x$, where x is the level at the intended steady state and a superscript ZLB indicates that the economy

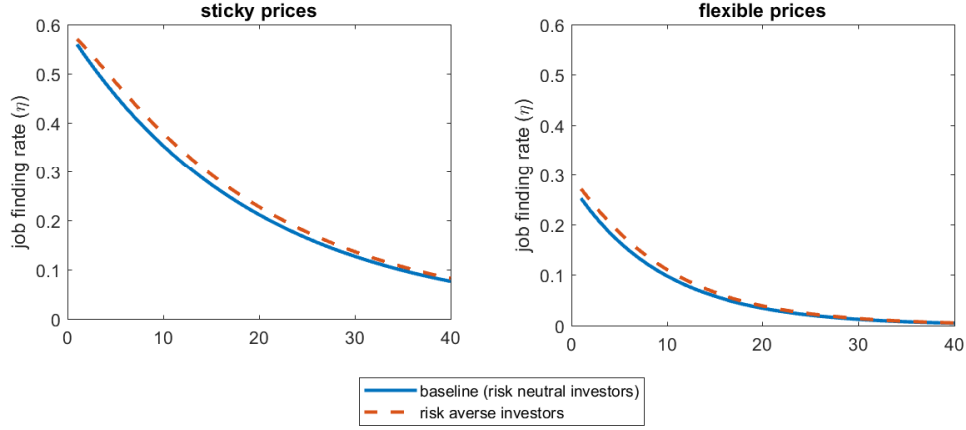


Figure 6: Responses to a positive technology shock.

remains in the depressed state. Log-linearization of this equation around the intended steady state gives $\mathbb{E}_s \hat{x}_{s+1} = p \mathbb{E}_s \hat{x}_{s+1}^{ZLB}$. Note further that at the ZLB, $R_s = 1$ and hence $\hat{R}_s = -\ln \bar{R}$.

Applying these results to the Euler equation, log-linearized around the intended steady state and as derived above, gives:

$$(\mu\chi (1 - \beta\bar{R}p) - \beta\bar{R}\Theta^F p) \hat{\eta}_s = \ln \bar{R} + p\hat{\Pi}_s.$$

Here we have used that if the ZLB binds in period s then $\mathbb{E}_s \hat{x}_{s+1} = p \mathbb{E}_s \hat{x}_{s+1}^{ZLB} = p\hat{x}_s$, exploiting the fact that variables remain constant as long as the depressed state persists. The Euler equation thus defines a linear relation between $\hat{\Pi}_s$ and $\hat{\eta}_s$, with a slope given by:

$$\frac{d\hat{\Pi}_s}{d\hat{\eta}_s} = \frac{\mu\chi}{p} (1 - \beta\bar{R}p) - \beta\bar{R}\Theta^F.$$

Applying the same logic, the log-linearized Phillips Curve at the ZLB can be written as:

$$\frac{\phi}{\gamma} (1 - \beta p) \hat{\Pi}_s = \left(w\chi + \frac{\kappa}{q} \frac{\alpha}{1 - \alpha} (1 - \beta(1 - \omega)p) \right) \hat{\eta}_s - wA_s,$$

which again defines a linear relation between $\hat{\Pi}_s$ and $\hat{\eta}_s$, conditional on the level of productivity A_s . The slope of the Phillips Curve is given by:

$$\frac{d\hat{\Pi}_s}{d\hat{\eta}_s} = \frac{w\chi + \frac{\kappa}{q} \frac{\alpha}{1 - \alpha} (1 - \beta(1 - \omega)p)}{\frac{\phi}{\gamma} (1 - \beta p)},$$

which is always positive. Note that an (unexpected) increase in productivity (A_s) shifts down the Phillips Curve, i.e. it reduces inflation, conditional on a certain level of the job finding rate.

A5. Pricing Risky Assets

This section explores asset pricing implications of the model. We show that the model generates a positive risk premium, but only if markets are incomplete. Intuitively, agents dislike asset with returns that co-move negatively with the probability of becoming unemployed, and hence require a discount relative to asset with acyclical returns.

For simplicity, consider the model with sticky wages ($\chi = 0$) and no sunspots. We focus on equilibria around the intended steady state. The stochastic discount factor of an employed household is given by $\Lambda_{e,s,s+1} = \beta\omega(1 - \eta_{s+1})(\vartheta/w)^{-\mu} + \beta(1 - \omega(1 - \eta_{s+1}))$. Note that the period- s conditional correlation between $\Lambda_{s,s+1}$ and η_{s+1} (and hence between $\Lambda_{s,s+1}$ and A_{s+1}) is perfectly negative, due to the fact that $\vartheta < w$. The appendix shows that the conditional variance of the stochastic discount factor is given by:

$$Var_s \{\Lambda_{e,s,s+1}\} = \beta^2 (\Theta^F)^2 \Gamma_\eta^2 \sigma_A^2.$$

Note that under complete markets ($\Theta^F = 0$), we obtain $Var_s \{\Lambda_{e,s,s+1}\} = 0$, i.e. the stochastic discount factor is constant. Intuitively, when agents' income is fully insured against unemployment risk and wages are sticky, their income, and hence their desire to save, is completely constant. When markets are incomplete, the precautionary savings motive emerges and fluctuates with the cycle since the amount of unemployment and wage risk varies over the business cycle.

Exogenous payoffs: We now use the model to price risky assets with simple payoff structures. First, consider a risky asset that pays off $1 + A_{s+1} - \rho A_s$ in period $s + 1$. We choose this payoff structure as it has the simplifying property that the expected payoff is one, while at the same time payoffs increase after an expansionary shock to productivity.

To obtain analytical tractability, we again assume that the asset is in zero net supply and that households cannot go short in the asset. As a result, the employed asset-poor households are the ones pricing the asset at the margin, whereas the other two types of households are in equilibrium at the no-short sale constraint. Krusell, Mukoyama and Smith (2011) exploit a similar setup to price risky asset under incomplete markets, but in an economy with exogenous

endowments. Here, we analyze the importance of the endogenous feedback mechanism created by HANK and SAM, and study the effects of monetary policy on asset prices.

Below we show that the employed households' stochastic discount factor and the solution of the log-linearized model imply that the price of the risky asset, denoted z_s , is given by:

$$z_s = \mathbb{E}_s \Lambda_{e,s,s+1} - \beta \Theta^F \Gamma_\eta \sigma_A^2.$$

In the above equation, the term $\beta \Theta^F \Gamma_\eta \sigma_A^2$ is the discount relative to a riskless asset. To see this, consider a riskless asset that pays out one unit of goods in the next period regardless of the state of the world (i.e. a real bond). Again imposing the no-shortsale constraint, it follows immediately from the households' discount factor that the price of the riskless asset is given by $\mathbb{E}_s \Lambda_{e,s,s+1}$.

The above equation thus makes clear that if the endogenous earnings risk is countercyclical, i.e. $\Theta^F > 0$, there is a risk premium, which emerges despite the fact that the above equation is based on the solution of the log-linearized model.³³ Further, recall that Γ_η is the response of the job finding rate to a productivity shock. The magnitude of Γ_η depends on the strength of the endogenous interaction between HANK and SAM, as well as on the monetary policy rule. By responding more aggressively to economic shocks, the central bank stabilizes the economy, reducing the strength of the precautionary savings mechanism and thereby the risk premium. Finally, note that without shocks, i.e. $\sigma_A = 0$, there is no risk premium.

Endogenous payoffs: Consider now another risky asset with an payoff equal to $1 + \widehat{\eta}_{s+1} - \rho \widehat{\eta}_s$. Note that, again, the expected payoff is one and that the payoff is increasing in next period's job finding rate. Again, we impose the no-shortsale constraint. Below we show that the price of the asset is given by:

$$z_s = \mathbb{E}_s \Lambda_{e,s,s+1} - \beta \Theta^F \Gamma_\eta^2 \sigma_A^2.$$

Note that in the return of the risky asset we now observe Γ_η^2 rather than Γ_η . This reflects the fact that the payoff of the asset is now endogenous. As a result, market frictions and monetary policy affect the risk premium via two channels: through the households' stochastic discount factor (via their unemployment risk) and through the asset payoff (via the equilibrium effects of household demand).

³³In representative agent models risk premia typically vanish after log-linearization since in the steady state there is no risk. Recall that in our model, by contrast, there is still idiosyncratic risk in the steady state.

Derivations: Consider the stochastic discount factor of the employed, asset-poor households:

$$\Lambda_{e,s,s+1} = \beta\omega (1 - \eta_{s+1}) (\vartheta/w)^{-\mu} + \beta (1 - \omega (1 - \eta_{s+1})).$$

Given the solution, the job finding rate is –up to a first-order approximation– given by $\eta_s = \eta + \eta\Gamma_\eta A_s$. We exploit this to write the period- s conditional expectation and variance of $\Lambda_{e,s,s+1}$, respectively, as:

$$\begin{aligned} \mathbb{E}_s \Lambda_{e,s,s+1} &= \beta\omega (1 - \mathbb{E}_s \eta_{s+1}) (\vartheta/w)^{-\mu} + \beta (1 - \omega (1 - \mathbb{E}_s \eta_{s+1})), \\ &= \beta\omega (1 - \eta - \rho_A \eta \Gamma_\eta A_s) (\vartheta/w)^{-\mu} + \beta (1 - \omega (1 - \eta - \rho_A \eta \Gamma_\eta A_s)), \end{aligned}$$

and

$$\begin{aligned} Var_s \{\Lambda_{e,s,s+1}\} &= \beta^2 \omega^2 (1 - (\vartheta/w)^{-\mu})^2 Var_s \{\eta_{s+1}\}, \\ &= \beta^2 \omega^2 (1 - (\vartheta/w)^{-\mu})^2 \eta^2 \Gamma_\eta^2 Var_s \{\rho_A A_s + \sigma_A \varepsilon_{s+1}^A\}, \\ &= \beta^2 \omega^2 (1 - (\vartheta/w)^{-\mu})^2 \eta^2 \Gamma_\eta^2 \sigma_A^2, \\ &= \beta^2 (\Theta^F)^2 \Gamma_\eta^2 \sigma_A^2. \end{aligned}$$

Exogenous payoffs: The pricing equation for the asset that pays off $1 + A_{s+1} - \rho_A A_s$ in period $s + 1$ reads:

$$\begin{aligned} z_s &= \mathbb{E}_s \{\Lambda_{e,s,s+1} (1 + A_{s+1} - \rho_A A_s)\} \\ &= \mathbb{E}_s \Lambda_{e,s,s+1} \mathbb{E}_s (1 + A_{s+1} - \rho_A A_s) + Cov_t(\Lambda_{e,s,s+1}, 1 + A_{s+1} - \rho_A A_s) \\ &= \mathbb{E}_s \Lambda_{e,s,s+1} - \sqrt{Var_s \{\Lambda_{e,s,s+1}\} Var_s \{1 + A_{s+1} - \rho_A A_s\}} \\ &= \mathbb{E}_s \Lambda_{e,s,s+1} - \beta \Theta^F \Gamma_\eta \sigma_A^2 \end{aligned}$$

where we exploited the fact that the $Cor_s \{\Lambda_{e,s,s+1}, A_{s+1}\} = -1$, that $1 + \mathbb{E}_s A_{s+1} - \rho_A A_s = 1$, and that $Var_s \{1 + A_{s+1} - \rho_A A_s\} = \sigma_A^2$.

Endogenous payoffs: Consider now another risky asset with an payoff equal to $1 + \hat{\eta}_{s+1} - \rho \hat{\eta}_s$.

The pricing equation for this asset reads:

$$\begin{aligned}
z_s &= \mathbb{E}_s \left\{ \Lambda_{e,s,s+1} \left(1 + \widehat{\eta}_{s+1} - \rho_A \widehat{\eta}_s \right) \right\}, \\
&= \mathbb{E}_s \left\{ \Lambda_{e,s,s+1} \left(1 + \Gamma_\eta A_{s+1} - \rho_A \Gamma_\eta A_s \right) \right\}, \\
&= \mathbb{E}_s \Lambda_{e,s,s+1} - \sqrt{\text{Var}_s \left\{ \Lambda_{e,s,s+1} \right\} \text{Var}_s \left\{ 1 + \Gamma_\eta A_{s+1} - \rho_A \Gamma_\eta A_s \right\}}, \\
&= \mathbb{E}_s \Lambda_{e,s,s+1} - \beta \Theta^F \Gamma_\eta^2 \sigma_A^2.
\end{aligned}$$