Multivariate Time Series

Fall 2008

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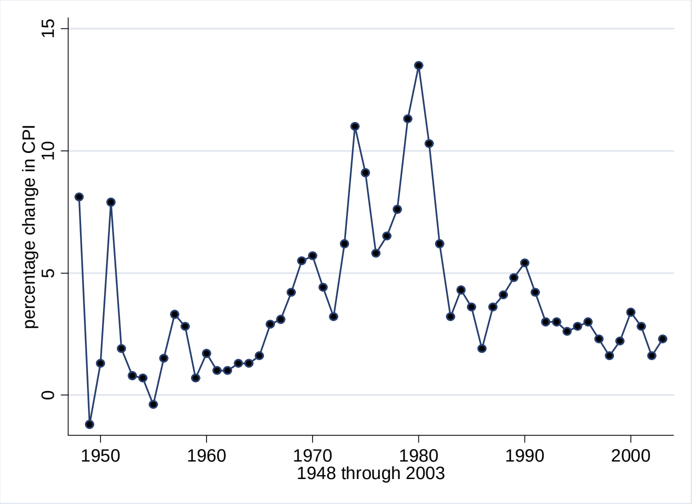
- Suppose the first difference of the time series follows a random walk:

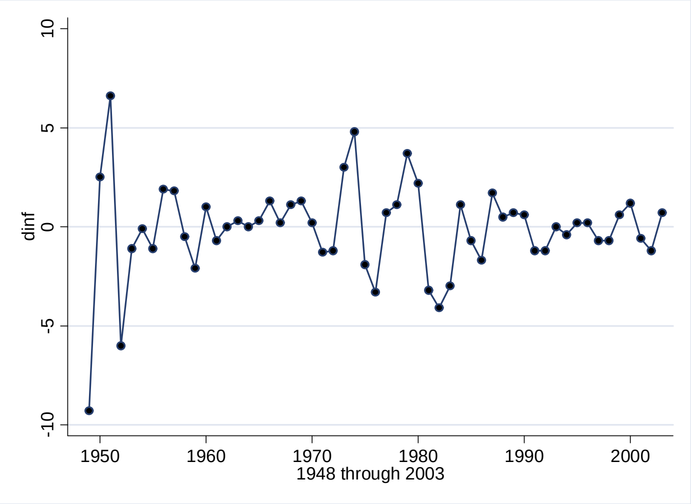
$$\Delta Y_t = \mu_0 + \Delta Y_{t-1} + u_t$$

  Again, in order to make it stationary, we need to take the differerence of the first difference, $\Delta Y_t - \Delta Y_{t-1}$, called the second difference of Y_t. This series is said to be **integrated of order** 2, or $\mathbf{I}(\mathbf{2})$.

Example: Inflation Rates in USA (1948~2003)



Example: First Differences of Inflation Rates



Testing for Unit Roots

- We want to test whether $AR(1)$ model has a unit root:

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- Dickey and Fuller (1979) obtained the asymptotic distribution of the t statistic under the null, called the Dickey-Fuller distribution.

Example: Testing whether Inflation Rates are I(1)

- $$Inf_t - Inf_{t-1} = \mu + \theta Inf_{t-1} + u_t$$

	Coeff.	t-stat.
Inf_1	-0.33	-3.31
Constant	1.17	2.40

- The asymptotic critical values for unit root t test is given

Sig. Level	1%	5%	10%
Critical Value	-3.43	-2.86	-2.57

- With 5% significance level, we reject the null since the t statistic (-3.31) is smaller than the critical value (-2.86).

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- A simple form is the vector autoregression model of two time series with order 1, called $VAR(1)$:

$$\begin{aligned} Y_t &= \mu_{10} + \rho_{11} Y_{t-1} + \rho_{12} X_{t-1} + u_{1t} \\ X_t &= \mu_{20} + \rho_{21} Y_{t-1} + \rho_{22} X_{t-1} + u_{2t} \end{aligned}$$

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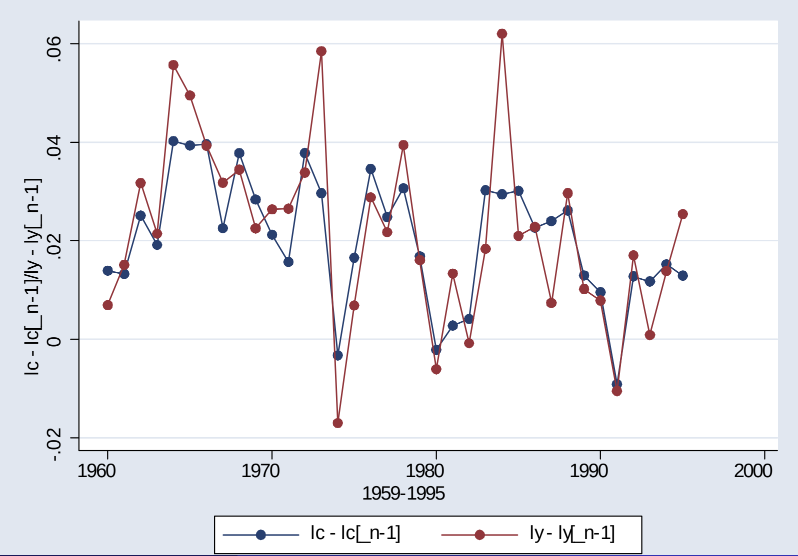
- Under the stationarity and ergodicity assumptions, we estimate the coefficients by using the OLS equation by equation.

Impulse Response Function

- It is often the case that VAR results can be difficult to interpret.
- What are the short-run/long-run effects of variables?
- What is the dynamics of Y_t after a unit-shock in X_t at time period t?
- A impluse response function measures this kind of dynamics: the predicted response of one of the dependent variables to a shock through time.

Example: Growth rates of consumption and income in USA (1959 - 1995)

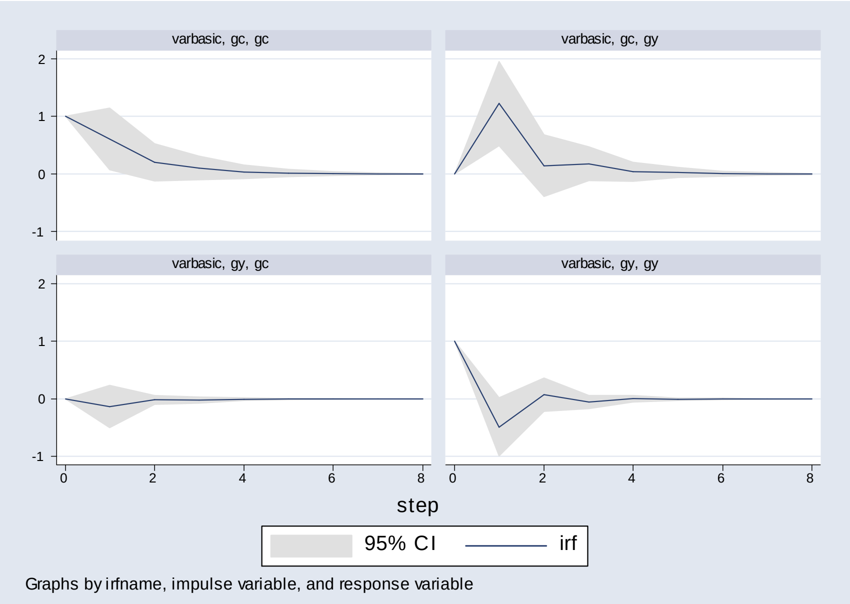


Example: VAR(1)

$$
\begin{aligned}
gc_t &= \mu_{10} + \rho_{11} gc_{t-1} + \rho_{12} gy_{t-1} + u_{1t} \\
gy_t &= \mu_{20} + \rho_{21} gc_{t-1} + \rho_{22} gy_{t-1} + u_{2t}
\end{aligned}
$$

	Cons. growth (gc)		Inc. growth (gy)	
	Coeff.	Std. Err.	Coeff.	Std. Err.
gc_1	0.61	0.27	1.23	0.37
gy_1	-0.13	0.19	-0.49	0.26
constant	0.01	0.00	0.01	0.01

Example: Impulse Response Function from VAR(1)



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- The Granger causality test is the F-statistic or the *Chi*-square statistic testing the null hypothesis that $\gamma_{11} = \gamma_{12} = 0$.

Example: Consumption and income growth rates - VAR(1)

Granger causality Wald tests			
Equation	Excluded	Chi2 (df)	p-value
gc	gy_1	0.517 (1)	0.47
gy	gc_1	10.73 (1)	0.00

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- (Spurious regression) Because the two series have a stochastic trend, OLS often picks up this apparent correlation.
- Thus, it is risky to use OLS when the both series are nonstationary.

Stationarizing Variables I

- Suppose we still want to estimate the relationship between two variables:

$$Y_t = \beta_0 + \beta_1 X_t + \varepsilon_t,$$

where Y_t and X_t are nonstationary.

- To see whether these variables are correlated, we need to make them stationary before regressing two variables.
- Examples:
  - deterministic trend

$$\begin{aligned} Y_t &= \alpha_{10} + \alpha_{11} t + u_t \\ X_t &= \alpha_{20} + \alpha_{21} t + v_t \end{aligned}$$

  - stochastic trend

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  - get the residuals $\widehat{v}_t$ and $\widehat{u}_t$
  - regress $\widehat{u}_t$ on $\widehat{v}_t$
- (First-differencing) Taking first differences gives a stationary process on each side:

$$Y_t - Y_{t-1} = \beta_1 \left(X_t - X_{t-1} \right) + \varepsilon_t - \varepsilon_{t-1}.$$