Univariate Time Series

Fall 2008

Time Series I

- A time series Y_t is a process (or data) observed in sequence over time, $t = 1, ..., T$.

Time Series I

- A time series Y_t is a process (or data) observed in sequence over time, $t = 1, ..., T$.
  - macroeconomic series: e.g., inflation rate, GDP, unemployment,....

Time Series I

- A time series Y_t is a process (or data) observed in sequence over time, $t = 1, ..., T$.
  - macroeconomic series: e.g., inflation rate, GDP, unemployment,....
  - temperature over time in global warming

Time Series I

- A time series Y_t is a process (or data) observed in sequence over time, $t = 1, ..., T$.
  - macroeconomic series: e.g., inflation rate, GDP, unemployment,....
  - temperature over time in global warming
  - monthly or annual returns of financial assets

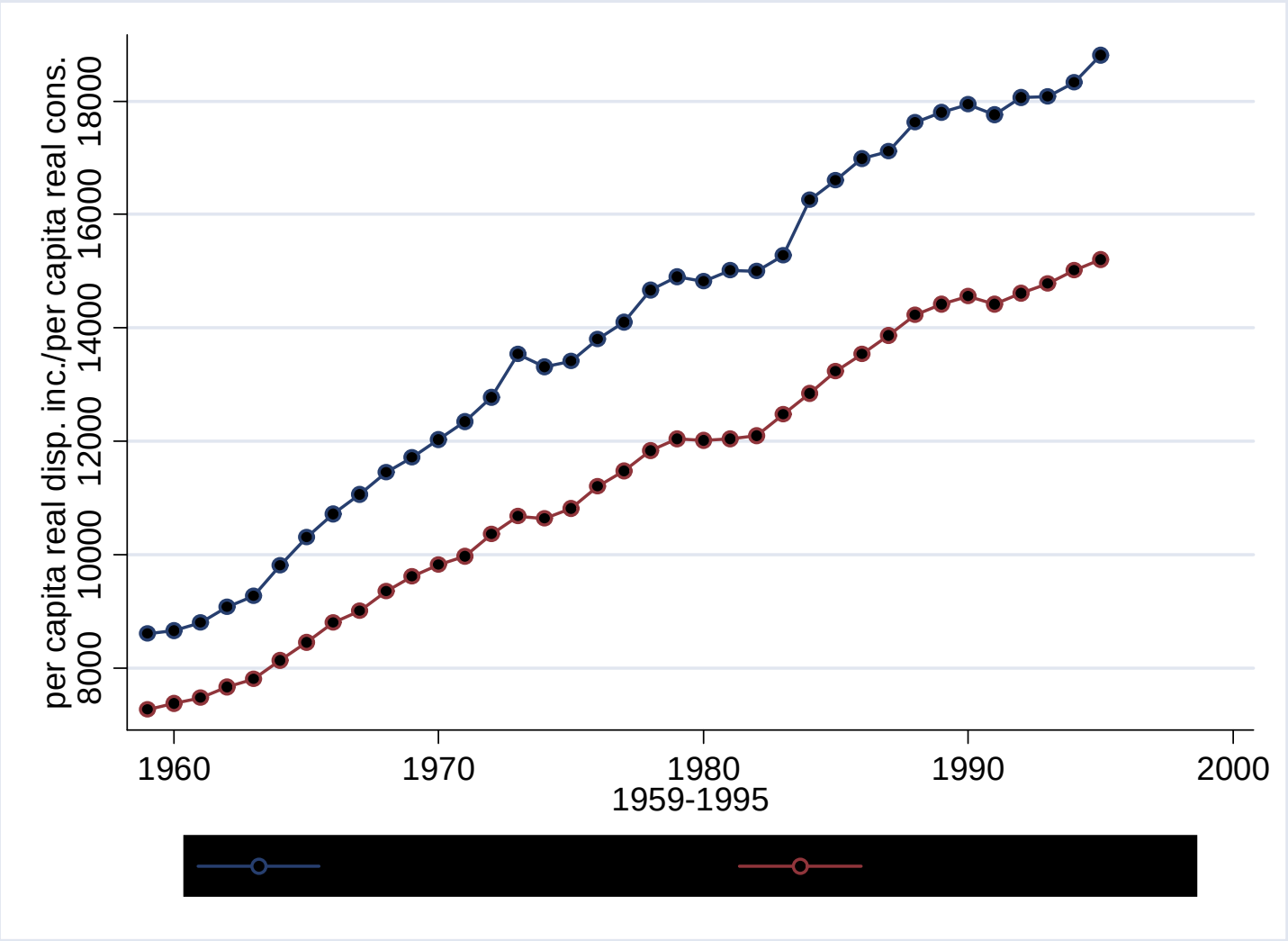
Time Series I

- A time series Y_t is a process (or data) observed in sequence over time, $t = 1, ..., T$.
  - macroeconomic series: e.g., inflation rate, GDP, unemployment,....
  - temperature over time in global warming
  - monthly or annual returns of financial assets
- The key issue in time series is **temporal dependence**.

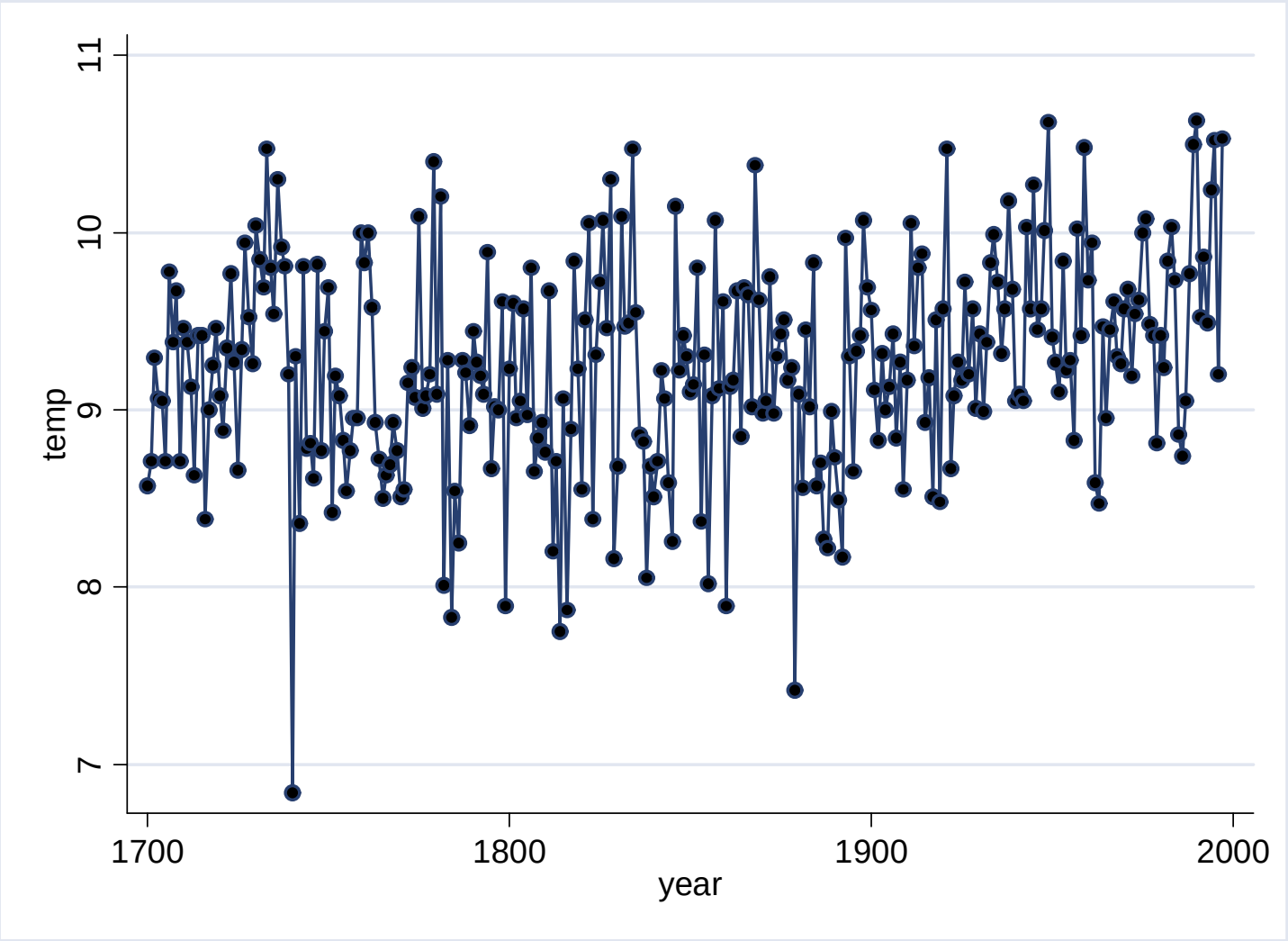
Time Series I

- A time series Y_t is a process (or data) observed in sequence over time, $t = 1, ..., T$.
  - macroeconomic series: e.g., inflation rate, GDP, unemployment,....
  - temperature over time in global warming
  - monthly or annual returns of financial assets
- The key issue in time series is **temporal dependence**.
- In order to have a meaningful regression analysis, we need some restrictions on temporal dependence.

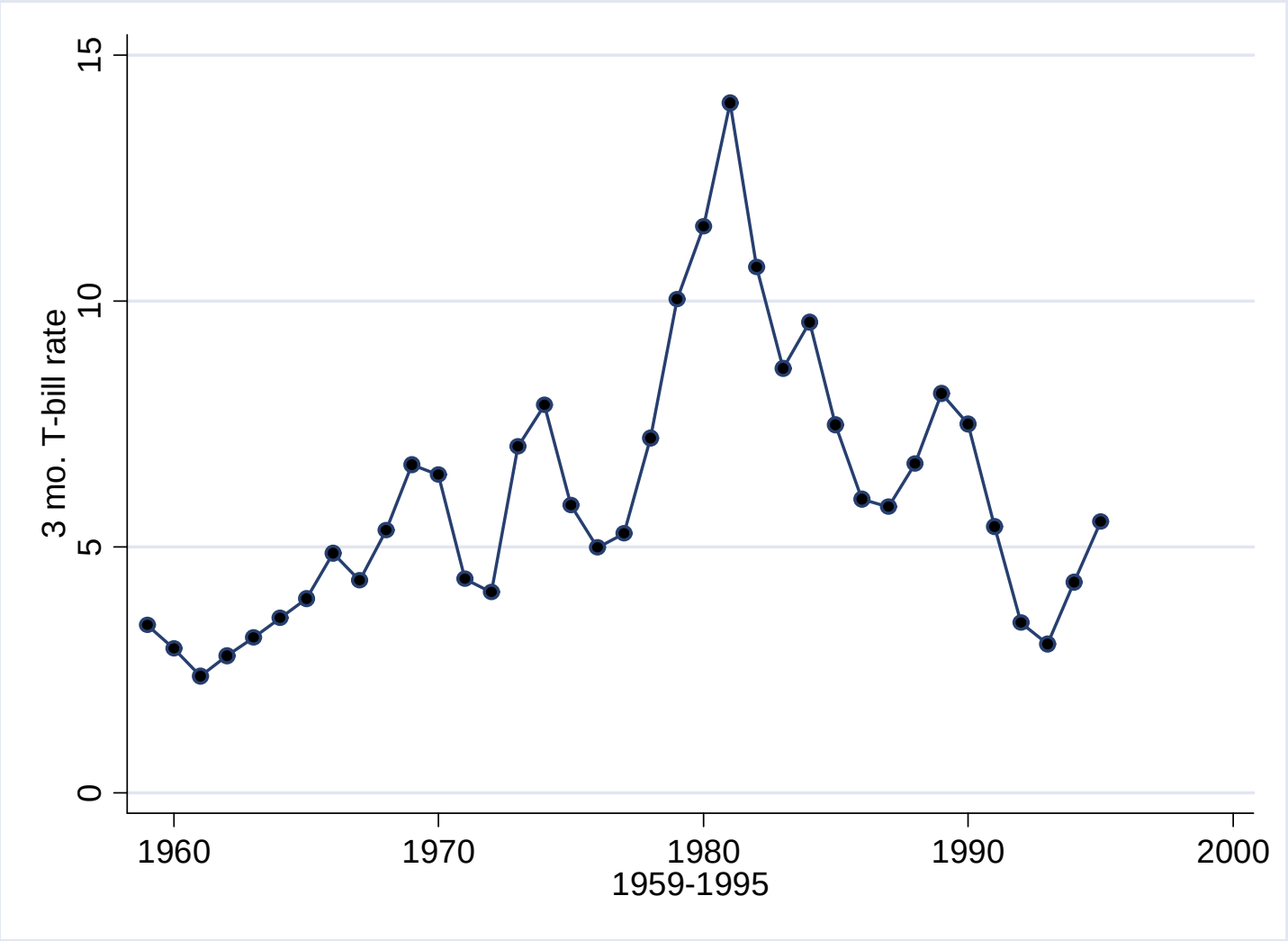
Example: Disposal income and consumption in USA



Example: Average temperature in Central London



Example: Return on three-month T-bills in USA



Time Series II

- The two main economic problems from time series are **dynamic causal effects** and **economic forecasting**.

Time Series II

- The two main economic problems from time series are **dynamic causal effects** and **economic forecasting**.
- (Dynamic causal effect) What is the effect of X on Y over time?

Time Series II

- The two main economic problems from time series are **dynamic causal effects** and **economic forecasting**.
- (Dynamic causal effect) What is the effect of X on Y over time?
  - the short-run/long-run effect of a change in an interest rate (by central bank) on inflation

Time Series II

- The two main economic problems from time series are **dynamic causal effects** and **economic forecasting**.
- (Dynamic causal effect) What is the effect of X on Y over time?
  - the short-run/long-run effect of a change in an interest rate (by central bank) on inflation
  - the effect of a decrease of carbon dioxide (by regulation) on global warming in 1/10/100 years later

Time Series II

- The two main economic problems from time series are **dynamic causal effects** and **economic forecasting**.
- (Dynamic causal effect) What is the effect of X on Y over time?
  - the short-run/long-run effect of a change in an interest rate (by central bank) on inflation
  - the effect of a decrease of carbon dioxide (by regulation) on global warming in 1/10/100 years later
- (Economic forecasting) What are predicted future values of Y, given available information?

Time Series II

- The two main economic problems from time series are **dynamic causal effects** and **economic forecasting**.
- (Dynamic causal effect) What is the effect of X on Y over time?
  - the short-run/long-run effect of a change in an interest rate (by central bank) on inflation
  - the effect of a decrease of carbon dioxide (by regulation) on global warming in 1/10/100 years later
- (Economic forecasting) What are predicted future values of Y, given available information?
  - average temperature in 2050?

Time Series II

- The two main economic problems from time series are **dynamic causal effects** and **economic forecasting**.
- (Dynamic causal effect) What is the effect of X on Y over time?
  - the short-run/long-run effect of a change in an interest rate (by central bank) on inflation
  - the effect of a decrease of carbon dioxide (by regulation) on global warming in 1/10/100 years later
- (Economic forecasting) What are predicted future values of Y, given available information?
  - average temperature in 2050?
  - the forecasting is not necessarily based on the causality relationship.

Time Series II

- The two main economic problems from time series are **dynamic causal effects** and **economic forecasting**.
- (Dynamic causal effect) What is the effect of X on Y over time?
  - the short-run/long-run effect of a change in an interest rate (by central bank) on inflation
  - the effect of a decrease of carbon dioxide (by regulation) on global warming in 1/10/100 years later
- (Economic forecasting) What are predicted future values of Y, given available information?
  - average temperature in 2050?
  - the forecasting is not necessarily based on the causality relationship.
- We can separate time series into two categories: univariate (Y_t is scalar) and multivariate (Y_t is vector-valued).

Stationarity: future will be like past

- A time series $\{Y_t\}$ is *covariance stationary* if its mean and (co-)variances are constant across time periods:

$$E(Y_t) = \mu, \; Var(Y_t) = \sigma^2 \qquad \text{for all } t$$

$$Cov(Y_t, Y_{t+k}) = \gamma(k) \qquad \text{for all } t \text{ and } k$$

$\gamma(k)$ is called the *autocovariance* function and $\rho(k) = \gamma(k)/\gamma(0)$ is the *autocorrelation* function.

Stationarity: future will be like past

- A time series $\{Y_t\}$ is *covariance stationary* if its mean and (co-)variances are constant across time periods:

$$E(Y_t) = \mu, \; Var(Y_t) = \sigma^2 \qquad \text{for all } t$$

$$Cov(Y_t, Y_{t+k}) = \gamma(k) \qquad \text{for all } t \text{ and } k$$

  $\gamma(k)$ is called the *autocovariance* function and $\rho(k) = \gamma(k)/\gamma(0)$ is the *autocorrelation* function.

- $\{Y_t\}$ is said to be *strictly stationary* if the joint distribution of $(Y_t, ..., Y_{t+k})$ is independent of all t and k.

Stationarity: future will be like past

- A time series $\{Y_t\}$ is *covariance stationary* if its mean and (co-)variances are constant across time periods:

$$E(Y_t) = \mu, \ Var(Y_t) = \sigma^2 \qquad \text{for all } t$$

$$Cov(Y_t, Y_{t+k}) = \gamma(k) \qquad \text{for all } t \text{ and } k$$

  $\gamma(k)$ is called the *autocovariance* function and $\rho(k) = \gamma(k)/\gamma(0)$ is the *autocorrelation* function.
- $\{Y_t\}$ is said to be *strictly stationary* if the joint distribution of $(Y_t, ..., Y_{t+k})$ is independent of all t and k.
- Thus, a stationary time series is one whose probability distributions are stable over time.

Nonstationarity

- A time series process that is not stationary is called a *nonstationary* process.

Nonstationarity

- A time series process that is not stationary is called a *nonstationary* process.
  - changing mean:

$$Y_t = \beta_0 + \beta_1 t + u_t$$

Nonstationarity

- A time series process that is not stationary is called a *nonstationary* process.
  - changing mean:
    $$Y_t = \beta_0 + \beta_1 t + u_t$$
  - changing variance:
    $$Y_t = Y_{t-1} + u_t$$

    This process is called the *random walk*.

Nonstationarity

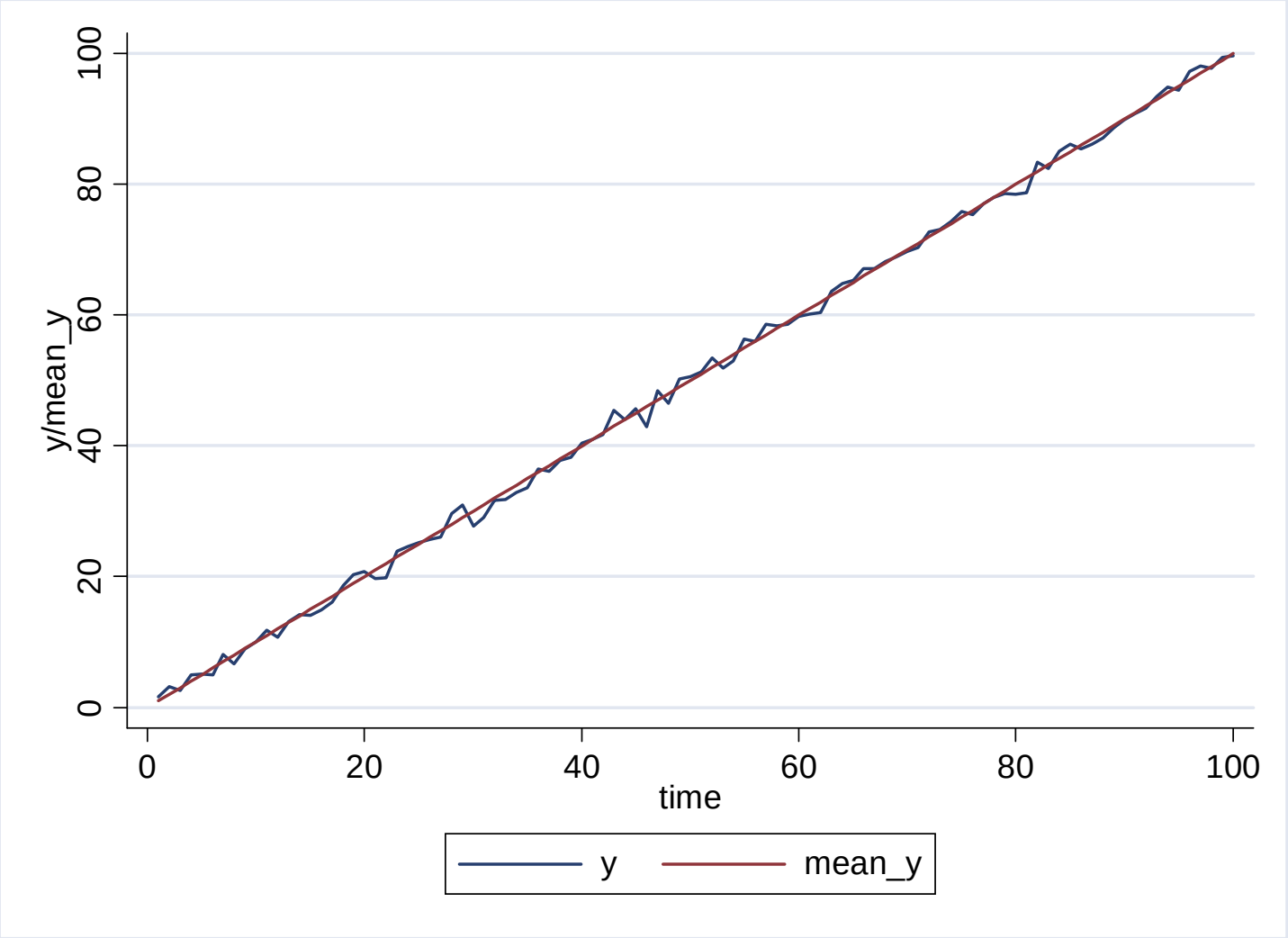
- A time series process that is not stationary is called a *nonstationary* process.
  - changing mean:
    $$Y_t = \beta_0 + \beta_1 t + u_t$$
  - changing variance:
    $$Y_t = Y_{t-1} + u_t$$
    This process is called the *random walk*.
  - changing mean and variance:
    $$Y_t = \beta_0 + Y_{t-1} + u_t$$
    This process is called the *random walk with drift*.

Nonstationarity

- A time series process that is not stationary is called a *nonstationary* process.
  - changing mean:
    $$Y_t = \beta_0 + \beta_1 t + u_t$$
  - changing variance:
    $$Y_t = Y_{t-1} + u_t$$
    This process is called the *random walk*.
  - changing mean and variance:
    $$Y_t = \beta_0 + Y_{t-1} + u_t$$
    This process is called the *random walk with drift*.
- It is easier to spot certain nonstationary series rather than stationary ones.

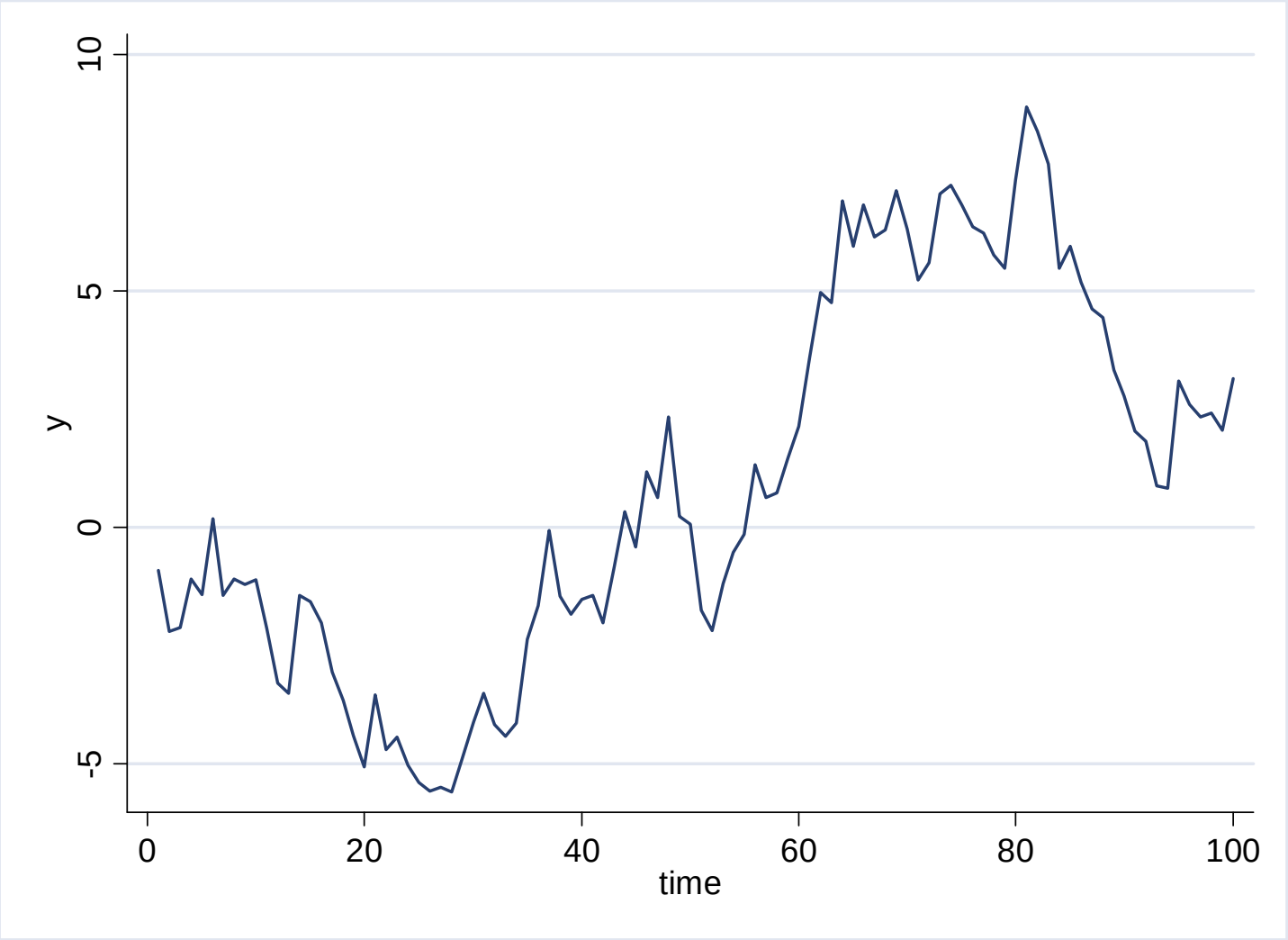
Example 1: Changing mean

- $Y_t = \beta_0 + \beta_1 t + u_t$, where $\beta_0 = 0$, $\beta_1 = 1$ and $u_t \sim$ *iid* $N(0,1)$



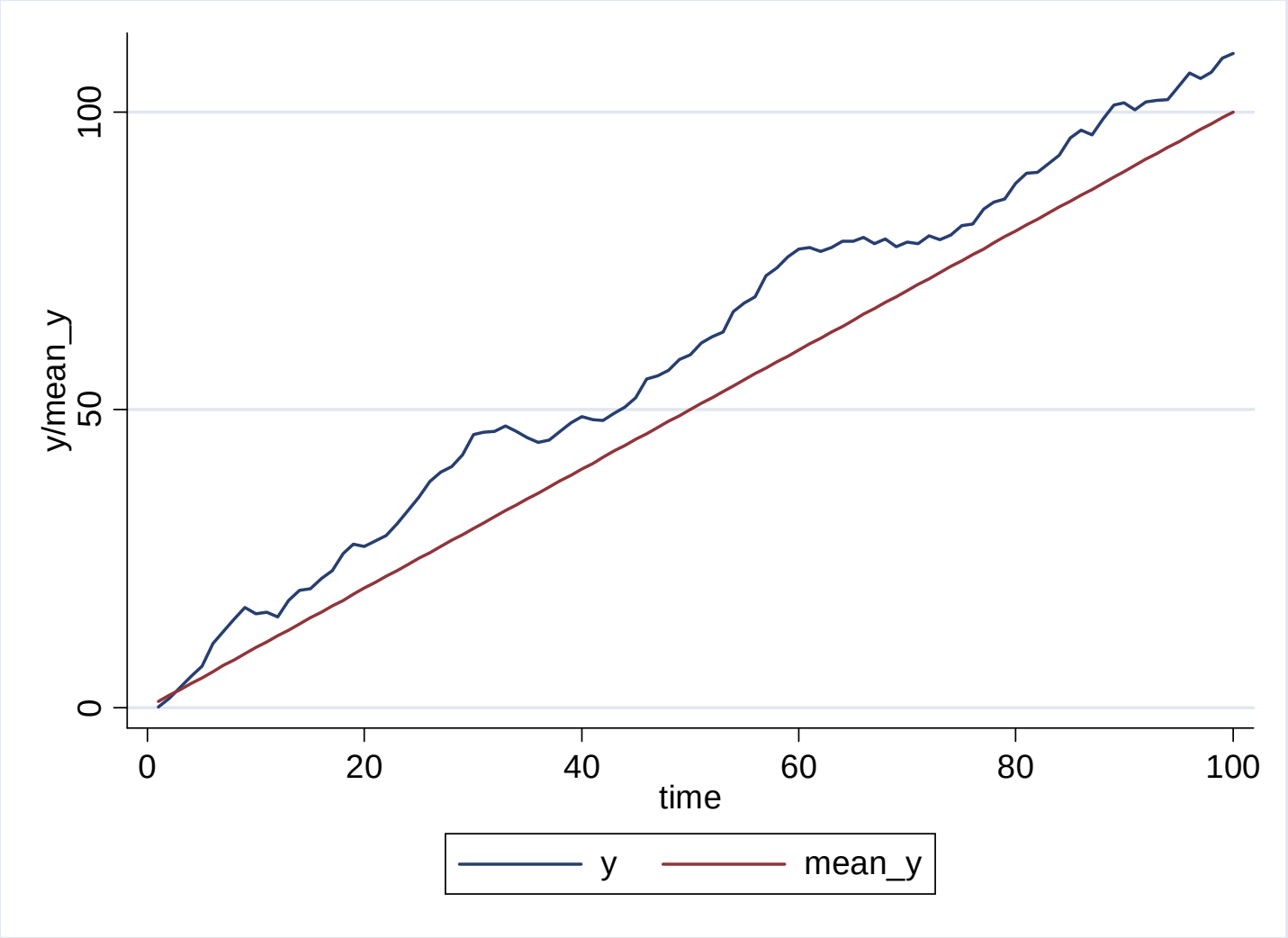
Example 2: Random walk

- $Y_t = Y_{t-1} + u_t$, where $Y_0 = 0$ and $u_t \sim$ *iid* $N(0,1)$



Example 3: Random walk with drift

- $Y_t = \beta_0 + Y_{t-1} + u_t$, where $\beta_0 = 1$, $Y_0 = 0$ and $u_t \sim$ *iid* $N(0,1)$



Ergodicity

- In most of *stationary* time series analysis, we will still use the OLS estimation method.

Ergodicity

- In most of *stationary* time series analysis, we will still use the OLS estimation method.
- In order to have properties of consistency and asymptotic normality of estimators, we need another property of stationary time series.

Ergodicity

- In most of *stationary* time series analysis, we will still use the OLS estimation method.
- In order to have properties of consistency and asymptotic normality of estimators, we need another property of stationary time series.
- A stationary time series is (loosely) said to be **ergodic** if $\gamma(k) \to 0$ as $k \to \infty$. Sometime a similar property is called *weakly dependent*.

Ergodicity

- In most of *stationary* time series analysis, we will still use the OLS estimation method.
- In order to have properties of consistency and asymptotic normality of estimators, we need another property of stationary time series.
- A stationary time series is (loosely) said to be **ergodic** if $\gamma(k) \to 0$ as $k \to \infty$. Sometime a similar property is called *weakly dependent*.
- (Ergodic Theorem) If Y_t is strictly stationary and ergodic and $E|Y_t| < \infty$, then as $T \to \infty$,

$$\frac{1}{T}\sum_{t=1}^{T} Y_t \to_p E(Y_t)$$

Ergodicity

- In most of *stationary* time series analysis, we will still use the OLS estimation method.
- In order to have properties of consistency and asymptotic normality of estimators, we need another property of stationary time series.
- A stationary time series is (loosely) said to be **ergodic** if $\gamma(k) \to 0$ as $k \to \infty$. Sometime a similar property is called *weakly dependent*.
- (Ergodic Theorem) If Y_t is strictly stationary and ergodic and $E|Y_t| < \infty$, then as $T \to \infty$,

$$\frac{1}{T}\sum_{t=1}^{T} Y_t \to_p E(Y_t)$$

- Using the Ergodic theorem, we can establish the consistency and asymptotic normality of OLS estimatiors in the regression with stationary and ergodic time series.

Autoregressions

- We first focus on the case with univariate time series, $\{Y_t\}_{t=1}^{T}$. Let $I_{t-1} = \{Y_{t-1}, Y_{t-2}, ...\}$ denote the past history of the series.

Autoregressions

- We first focus on the case with univariate time series, $\{Y_t\}_{t=1}^{T}$. Let $I_{t-1} = \{Y_{t-1}, Y_{t-2}, ...\}$ denote the past history of the series.
- The primary example in the univariate time series is a model of autoregression specifying that only a finite number of past lags matter:

$$E\left(Y_t | I_{t-1}\right) = E\left(Y_t | Y_{t-1}, ..., Y_{t-k}\right).$$

Autoregressions

- We first focus on the case with univariate time series, $\{Y_t\}_{t=1}^{T}$. Let $I_{t-1} = \{Y_{t-1}, Y_{t-2}, ...\}$ denote the past history of the series.
- The primary example in the univariate time series is a model of autoregression specifying that only a finite number of past lags matter:

$$E\left(Y_t | I_{t-1}\right) = E\left(Y_t | Y_{t-1}, ..., Y_{t-k}\right).$$

- An autoregressive process of order k, called $AR\left(k\right)$, is

$$Y_t = \mu + \rho_1 Y_{t-1} + ... + \rho_k Y_{t-k} + u_t,$$

where

$$E\left(u_t | I_{t-1}\right) = 0.$$

AR(1) Model

- AR(1) Model:

$$Y_t = \mu + \rho Y_{t-1} + u_t$$

  - if $|\rho| < 1$, then

$$E(Y_t) = \frac{\mu}{1-\rho}$$

$$Var(Y_t) = \frac{\sigma^2}{1-\rho^2}$$

$$Cov(Y_t, Y_{t-k}) = \sigma^2 \frac{\rho^k}{1-\rho^2}$$

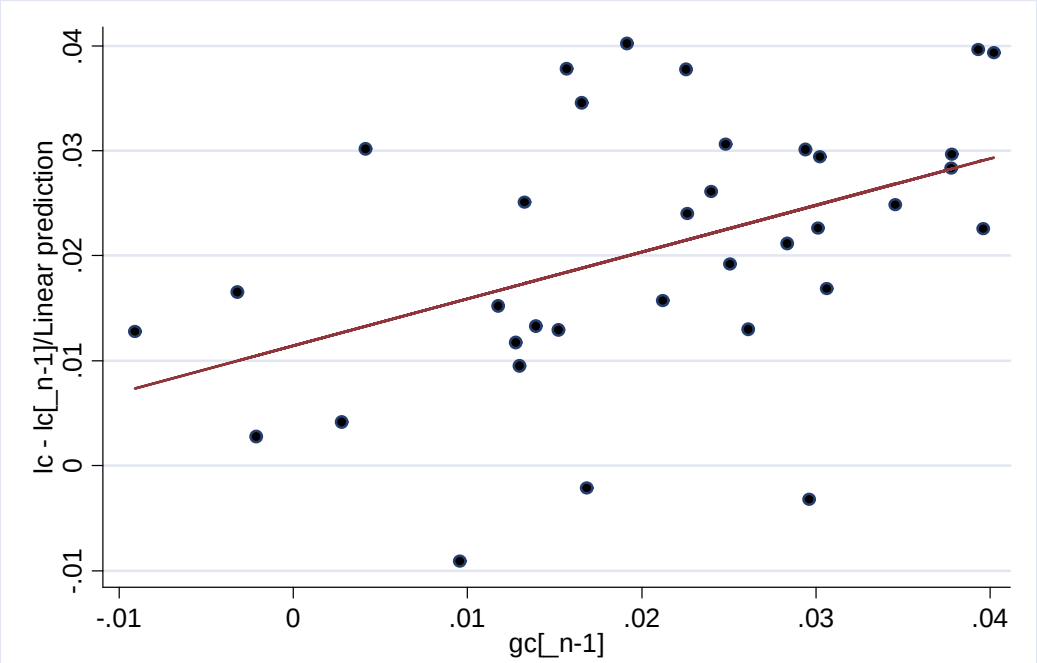
- If $|\rho| < 1$, the process $\{Y_t\}$ is stationary and ergodic.
- (Estimation) In order to estimate μ and ρ, we just need to run the OLS regression of Y_{t-1} on Y_t.

Example: Consumption growth in USA

- We model the consumption growth series in USA as AR(1):

$$gc_t = \mu + \rho gc_{t-1} + u_t$$

	Coeff.	Std. Err.
gc_1	0.45	0.156
constant	0.01	0.003



Moving Average

- Another example is a model of moving average process of order q, $\{Y_t\}$, that is a weighted sum of lagged i.i.d. shocks:

$$Y_t = \mu + u_t + \lambda_1 u_{t-1} + ... + \lambda_q u_{t-q}$$

Moving Average

- Another example is a model of moving average process of order q, $\{Y_t\}$, that is a weighted sum of lagged i.i.d. shocks:

$$Y_t = \mu + u_t + \lambda_1 u_{t-1} + ... + \lambda_q u_{t-q}$$

- The simplest case is a moving average process of order 1, called $MA(1)$:

$$Y_t = \mu + u_t + \lambda u_{t-1}$$

Moving Average

- Another example is a model of moving average process of order q, $\{Y_t\}$, that is a weighted sum of lagged i.i.d. shocks:

$$Y_t = \mu + u_t + \lambda_1 u_{t-1} + ... + \lambda_q u_{t-q}$$

- The simplest case is a moving average process of order 1, called $MA(1)$:

$$Y_t = \mu + u_t + \lambda u_{t-1}$$

-

$$\begin{aligned} E(Y_t) &= \mu, \ Var(Y_t) = \left(1+\lambda^2\right)\sigma^2 \\ Cov(Y_t, Y_{t-1}) &= \gamma(1) = \lambda\sigma^2 \\ Cov(Y_t, Y_{t-k}) &= \gamma(k) = 0 \quad \text{for } k \geq 2 \end{aligned}$$

Moving Average

- Another example is a model of moving average process of order q, $\{Y_t\}$, that is a weighted sum of lagged i.i.d. shocks:

$$Y_t = \mu + u_t + \lambda_1 u_{t-1} + ... + \lambda_q u_{t-q}$$

- The simplest case is a moving average process of order 1, called $MA(1)$:

$$Y_t = \mu + u_t + \lambda u_{t-1}$$

-

$$\begin{aligned} E(Y_t) &= \mu, \ Var(Y_t) = \left(1+\lambda^2\right)\sigma^2 \\ Cov(Y_t, Y_{t-1}) &= \gamma(1) = \lambda\sigma^2 \\ Cov(Y_t, Y_{t-k}) &= \gamma(k) = 0 \quad \text{for } k \geq 2 \end{aligned}$$

  - Thus, an MA(1) process is stationary and ergodic.