Coordination Games

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Coordination and Multiple Equilibria

- A perennial question in economics concerns the conditions under which individuals cooperate to achieve an *efficient* outcome.
- This question is, especially, relevant to situations in which there are multiple equilibria.
- Consider the following examples:

	Left	*Right*
Top	80, 80	0, 0
Bottom	0, 0	100, 100

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Selection Criteria of Equilibrium

- Both game theorists and experimentalists develop possible selection criteria of equilibrium and test their empirical validity.
- Broadly speaking, selection criteria that have been suggested rely either on structural properties of games or on psychological frames.
- Criteria relying on structural properties of games are:
  - Payoff dominance
  - Risk dominance
  - History dependence
  - Information salience
- A criterion relying on psychological frames is "psychological salience" on the labeling of strategies, proposed by Schelling (1960) as part of his theory of focal equilibria.

Payoff Dominance

- Harsanyi and Selten (1988) introduced the notions of payoff and risk dominances based on the payoff structures of a game.
- A Nash equilibrium is *payoff-dominant* if it is Pareto-superior to all other Nash equilibria in a game, that is, there does not exist another equilibrium that yields greater payoffs to either player.
- An example is

	Left	*Right*
Top	80, 80	80, 0
Bottom	0, 80	100, 100

  An NE $(Bottom, Right)$ gives higher payoffs to both players than an NE $(Top, Left)$. Thus, $(Bottom, Right)$ is payoff-dominant.
- The notion of payoff dominance is based on *collective rationality*. The failure to obtain a Pareto-superior equilibrium is called *coordination failure*.

Risk Dominance

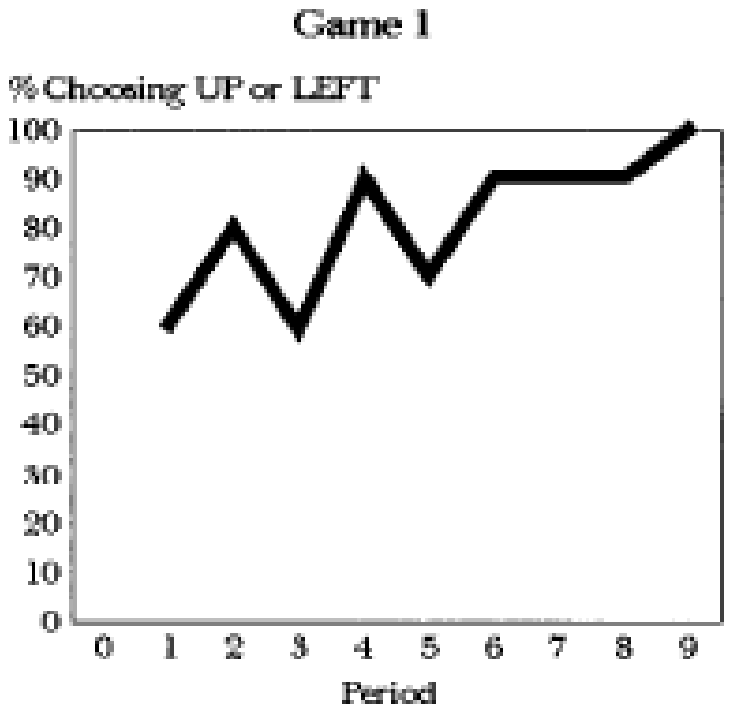
- Define the product of an equilibrium as the opportunity costs of unilaterally deviating from that equilibrium for each player.

	Left	*Right*
Top	80, 80	80, 0
Bottom	0, 80	100, 100

- The product of an NE $(Top, Left)$ is $(80-0)\times(80-0)=6400$. The product of an NE $(Bottom, Right)$ is $(100-80)\times(100-80)=400$.
- A (pure-strategy) equilibrium with the higher product is said to be *risk dominant* against another equilibrium with the lower product.
- In the above example, there is a conflict between payoff dominance and risk dominance: $(Bottom, Right)$ is payoff-dominant but $(Top, Left)$ is risk-dominant.

Experimental evidence on payoff and risk dominances I

- Straub (1995) ran a series of experiments testing on risk and payoff dominances, using 2×2 matrix games including the one we discussed above.
- With a matching protocol allowing each player to play the game against each other player only once, Straub found coordination failures due to the existence of the conflict between payoff and risk dominances.



Experimental evidence on payoff and risk dominances II

- Van Huyck, Battalio, and Beil (1990) ran an experiment on "minimum-effort" coordination games with 14 or 16 subjects in a group.

		Smallest Value of X Chosen						
		7	6	5	4	3	2	1
Your Choice of X	7	1.30	1.10	0.90	0.70	0.50	0.30	0.10
	6	–	1.20	1.00	0.80	0.60	0.40	0.20
	5	–	–	1.10	0.90	0.70	0.50	0.30
	4	–	–	–	1.00	0.80	0.60	0.40
	3	–	–	–	–	0.90	0.70	0.50
	2	–	–	–	–	–	0.80	0.60
	1	–	–	–	–	–	–	0.70

	Period									
	1	2	3	4	5	6	7	8	9	10
Experiment 1										
No. of 7's	8	1	1	0	0	0	0	0	0	1
No. of 6's	3	2	1	0	0	0	0	0	0	0
No. of 5's	2	3	2	1	0	0	1	0	0	0
No. of 4's	1	6	5	4	1	1	1	0	0	0
No. of 3's	1	2	5	5	4	1	1	1	0	1
No. of 2's	1	2	2	4	8	7	8	6	4	1
No. of 1's	0	0	0	2	3	7	5	9	12	13
Minimum	2	2	2	1	1	1	1	1	1	1
Experiment 2										
No. of 7's	4	0	1	0	0	0	0	0	0	1
No. of 6's	1	0	1	0	0	1	0	0	0	0
No. of 5's	3	3	2	1	0	0	1	1	0	1
No. of 4's	4	6	2	3	3	0	0	0	0	0
No. of 3's	1	4	2	5	0	1	1	0	1	0
No. of 2's	3	2	6	5	5	9	3	4	3	1
No. of 1's	0	1	2	2	8	5	11	11	12	13
Minimum	2	1	1	1	1	1	1	1	1	1
Experiment 3										
No. of 7's	4	4	1	0	1	1	1	0	0	2
No. of 6's	2	0	2	0	0	0	0	0	0	0
No. of 5's	5	6	1	1	1	0	0	0	0	0
No. of 4's	3	3	2	1	2	1	0	0	0	1
No. of 3's	0	0	7	6	0	2	3	0	0	0
No. of 2's	0	1	1	4	5	3	6	3	2	2
No. of 1's	0	0	0	2	5	7	4	11	12	9
Minimum	4	2	2	1	1	1	1	1	1	1
Experiment 4										
No. of 7's	6	0	1	1	0	0	1	0	0	0
No. of 6's	0	6	2	0	0	1	0	0	0	0
No. of 5's	8	5	5	5	0	1	0	0	0	0
No. of 4's	1	1	4	6	7	1	2	1	1	0
No. of 3's	0	2	3	2	4	3	2	2	1	0
No. of 2's	0	1	0	0	2	3	7	4	2	2
No. of 1's	0	0	0	1	2	6	3	8	11	13
Minimum	4	2	3	1	1	1	1	1	1	1

History Dependence

- In some cases, the selection of an equilibrium can be path-dependent and be sensitive to the initial conditions of histories.
- Van Huyck, Battalio, and Cook (1997) studied experimentally median-action games, so-called "Continental Divide" games, where players pick numbers from 1 to 14.
- The payoff for each player depends on his/her own choice of the numbers and the median choice in the group.
- A fixed group of seven players played this game together fifteen times. After each game, subjects learned about what the median was and their earnings.

Payoff Structures of Continental Divide Games

		Median choice													
		1	2	3	4	5	6	7	8	9	10	11	12	13	14
Your choice	1	45	49	52	55	56	55	46	−59	−88	−105	−117	−127	−135	−142
	2	48	53	58	62	65	66	61	−27	−52	−67	−77	−86	−92	−98
	3	48	54	60	66	70	74	72	1	−20	−32	−41	−48	−53	−58
	4	43	51	58	65	71	77	80	26	8	−2	−9	−14	−19	−22
	5	35	44	52	60	69	77	83	46	32	25	19	15	12	10
	6	23	33	42	52	62	72	82	62	53	47	43	41	39	38
	7	7	18	28	40	51	64	78	75	69	66	64	63	62	62
	8	−13	−1	11	23	37	51	69	83	81	80	80	80	81	82
	9	−37	−24	−11	3	18	35	57	88	89	91	92	94	96	98
	10	−65	−51	−37	−21	−4	15	40	89	94	98	101	104	107	110
	11	−97	−82	−66	−49	−31	−9	20	85	94	100	105	110	114	119
	12	−133	−117	−100	−82	−61	−37	−5	78	91	99	106	112	118	123
	13	−173	−156	−137	−118	−96	−69	−33	67	83	94	103	110	117	123
	14	−217	−198	−179	−158	−134	−105	−65	52	72	85	95	104	112	120

- If a player guesses that the median number is slightly below 7, his/her best response to that guess is to choose a number smaller than the guess itself.
- If a player guesses that the median number is 8 or above, they should choose nubmers that are higher their guesses.
- There are two (pure-strategy) NEs in which everybody chooses either 3 or 12.

Experimental Result

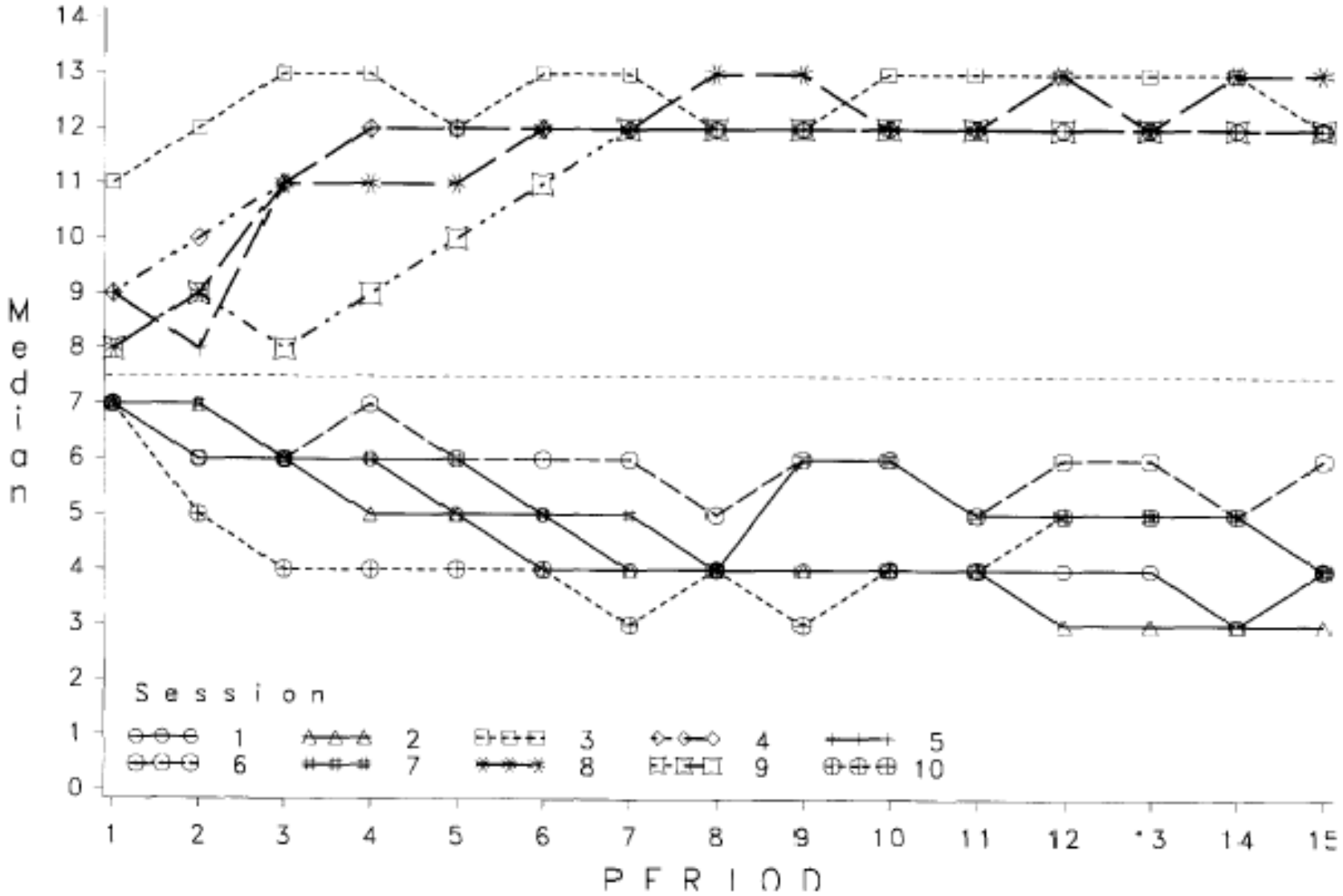


Fig. 3. Median choice in sessions 1 to 10 by period.

Interpretations of the result

- Medians below 7 are a "basin of attraction" for convergence toward 3. Similarly, medians above 8 are a basin of attraction for convergence toward 12.
- Subjects do not always gravitate toward the high-payoff equilibrium even though players who end up at low numbers earn half as much.
- The currents of history are strong, creating "extreme sensitivity to initial conditions."

Schelling's salience of decision labels

- Thomas Schelling (1960) asked subjects to choose independently and without communication where they would try to meet one another in New York City.
- Those who chose the same meeting location as their partner would receive a positive (hypothetical) payoff, equal to that of their partner's and independent of the specific location. Those who didnot would receive a zero payoff.
- Despite the plethora of possible meeting locations, a majority of subjects chose Grand Central Station, which was the most salient traffic hub in New York at the time.
- Schelling concluded that even though traditional game theory allows no role for the salience of decision labels, there are many situations where decision labels serves as a focal point.

Symmetric pure coordination with labelling salience

	Trafalgar Square	BT Tower
Trafalgar Square	100, 100	0, 0
BT Tower	0, 0	100, 100

A (minutely) asymmetric pure coordination with labeling salience

	Trafalgar Square	BT Tower
Trafalgar Square	100, 100	0, 0
BT Tower	0, 0	110, 110

Fragility of Labeling Salience

- Crawford, Gneezy, and Rottenstreich (2007) found evidence that labeling salience plays a significant role of selecting an equilibrium when in the symmetric pure coordination game.
- However, they found evidence that this labeling salience lose its power of selecting an equilibrium and miscoordination abounds even in a minutely asymmetric coordination game.