

## Some Questions on Moral Hazard

Suppose the agent has a utility function  $u(w)=w^{1/2}$  and initial wealth of 1 and an outside option that provides utility  $u$ . There are two effort levels for the agent  $e=0$  or  $e=1$ . When  $e=0$  the agent has no cost of effort and probability  $\pi$  of sustaining a loss 1 and probability  $1-\pi$  of sustaining a loss zero. When  $e=1$  the agent has probability  $\phi$  of a loss 1 and  $1-\phi$  of a loss zero (where  $\pi>\phi$ ), but this level of effort costs the agent  $c>0$ . Consider a firm that offers the agent an insurance policy at a price  $p$  that pays benefit  $B$  when loss one occurs and zero when no loss occurs.

- 1) Do the probabilities in this case satisfy the monotone likelihood ratio property?
- 2) Do the probabilities in this case satisfy First Order Stochastic Dominance?
- 3) Suppose the firm offers only one contract full insurance and the agent provides zero effort. Show that the agent will buy the contract provided  $p \leq 1-u^2$  and hence calculate the maximal profit the firm makes from zero effort.
- 4) Now suppose the firm want the agent to provide  $e=1$ . Write down the incentive compatibility constrain and the individual rationality constraint.
- 5) Assume that the constraints in Q.4 both bind. Show that when the constraints bind:

$$p=1-(u+c(1-\phi/\pi))^{-2}$$

$$B = p + (u - c(1-\pi)/(\pi-\phi))^2.$$

- 6) Write down the Lagrangean for the problem of maximizing the firms expected profit subject to the constraints in Q4. Differentiate this with respect to  $p$  and  $B$ . Hence show that the assumption in 5 is correct.
- 7) What is the firms profit when the agent provides high effort?