Questions on Auctions:

1) For the case where players' values are independently and uniformly distributed on the interval [0,1] calculate their symmetric equilibrium bids in a first price auction.

2) For the case where players' values have a density $f(v)=e^{-v}$ derive an expression for a player's symmetric equilibrium bids in a first price auction.

3) Consider a seller with 2 objects to sell to N buyers. Each buyer, i=1,2…,N, only wants to buy one object and has a value of v_i for that object. The seller decides to hold a third price auction. (The objects go to the two highest bidders and they pay the third highest price submitted.) (a) Show that bidding your value is a weakly dominant strategy, (b) If players' values are uniformly distributed on the interval [0,1], what is the seller's expected revenue from this auction?

4) Consider a first-price auction where there are two buyers. The buyers have either a zero value for the object or a value 1 for the object; their values are independent. With probability 1-p each buyer has a value zero and with probability p the buyer has a value 1 for the object.
a) If a buyer has a zero value what is their bid?
b) Is there an equilibrium where the buyers play pure strategies?
c) Let b(x) denote the probability a value 1 buyer bids x and let B(x) denote the probability the buyer bids less than or equal x. What is the probability x is a winning bid and what is the buyer's expected utility from bidding x?
d) If b(x) is an equilibrium mixed strategy the buyer must be indifferent from bidding x and x'. That is, your answer to part c is constant as x varies. This means that the derivative of your answer to part c is zero. Use this to derive a differential equation for B(x).
e) Solve this differential equation and find the equilibrium mixed strategy.
f) What is the seller's expected revenue at this auction?