Answers to the Moral Hazard Questions

1. Yes if $\pi > \phi$ then
$$\frac{\pi}{\phi} > \frac{1-\pi}{1-\phi}$$

2. Yes.

3. The agent's utility from full insurance at the price p is $(1-p)^{1/2}$. Full insurance is better than none if $(1-p)^{1/2} \geq u$. The largest price for which this holds is $p = 1 - u^2$ and the firm's expected profit from this is $1 - u^2 - \phi B$

4. The incentive compatibility constraint is
$$(1-\pi)(1-p)^{1/2}+\pi(1-p+B-1)^{1/2} \leq (1-\phi)(1-p)^{1/2}+\phi(1-p+B-1)^{1/2}-c.$$
The individual rationality constraint s
$$(1-\phi)(1-p)^{1/2} + \phi(1-p+B-1)^{1/2} - c \geq u.$$

5. If both constraints bind
$$(1-\phi)(1-p)^{1/2} + \phi(B-p)^{1/2} - c = (1-\pi)(1-p)^{1/2} + \pi(B-p)^{1/2} = u$$
Now we must solve these for B and p. The first equality gives
$$\begin{aligned} (1-\phi)(1-p)^{1/2} + \phi(B-p)^{1/2} - c &= (1-\pi)(1-p)^{1/2} + \pi(B-p)^{1/2} \\ -\phi(1-p)^{1/2} + \phi(B-p)^{1/2} - c &= -\pi(1-p)^{1/2} + \pi(B-p)^{1/2} \\ (\pi-\phi)(1-p)^{1/2} - (\pi-\phi)(B-p)^{1/2} &= c \\ (1-p)^{1/2} &= (B-p)^{1/2} + \frac{c}{\pi-\phi} \end{aligned}$$
If this is substituted into the second equality we get
$$(1-\pi)(B-p)^{1/2} + \frac{c(1-\pi)}{\pi-\phi} + \pi(B-p)^{1/2} = u$$
So
$$(B-p)^{1/2} = u - \frac{c(1-\pi)}{\pi-\phi}$$

This solves to give the B value in the question. If this is substituted into $(1-p)^{1/2} = (B-p)^{1/2} + \frac{c}{\pi-\phi}$ we then get

$$(1-p)^{1/2} = u + \frac{c\pi}{\pi - \phi}$$

or

$$p = 1 - \left(u + \frac{c\pi}{\pi - \phi}\right)^2$$

(There is a 1 that should be a π in the question.)

6. The Lagrangean for this question is

$$\begin{aligned} L &= p - B\phi \\ &\quad +\lambda\left((1-\phi)(1-p)^{1/2} + \phi(B-p)^{1/2} - c - (1-\pi)(1-p)^{1/2} - \pi(B-p)^{1/2}\right) \\ &\quad +\mu\left((1-\phi)(1-p)^{1/2} + \phi(B-p)^{1/2} - c - u\right) \end{aligned}$$

This can be tidied up to give

$$\begin{aligned} L &= p - B\phi \\ &\quad +\lambda\left((\pi-\phi)(1-p)^{1/2} - (\pi-\phi)(B-p)^{1/2} - c\right) \\ &\quad +\mu\left((1-\phi)(1-p)^{1/2} + \phi(B-p)^{1/2} - c - u\right) \end{aligned}$$

The first order conditions for a maximum profit $0 = dL/dp$ and $0 = dL/db$ are

$$1 = \lambda\left(\frac{\pi-\phi}{2(1-p)^{1/2}} + \frac{\phi-\pi}{2(B-p)^{1/2}}\right) - \mu\left(\frac{1-\phi}{2(1-p)^{1/2}} + \frac{\phi}{2(B-p)^{1/2}}\right)$$

and

$$\phi = -\lambda\frac{\pi-\phi}{2(B-p)^{1/2}} + \mu\frac{\phi}{2(B-p)^{1/2}}$$

Remember that Lagrange multipliers must be positive (when you've written the Lagrangean correctly).

When neither constraint binds $\lambda = \mu = 0$: it is impossible for the first equation to hold.

When $\lambda = 0$ **and** $\mu > 0$: the right of the first equation is negative and cannot be equal 1.

When $\mu = 0$ **and** $\lambda > 0$: the right of the second equation is negative and cannot be equal ϕ.

This it is impossible for either constraint to be slack.

7. Profit $= p - \phi B$ from our solution above we then get

$$\begin{aligned}
p - \phi B &= p - \phi\left(p + \left(u - \frac{c(1-\pi)}{\pi - \phi}\right)^2\right) \\
&= p(1-\phi) - \phi\left(u - \frac{c(1-\pi)}{\pi - \phi}\right)^2 \\
&= \left(1 - \left(u + \frac{c\pi}{\pi - \phi}\right)^2\right)(1-\phi) - \phi\left(u - \frac{c(1-\pi)}{\pi - \phi}\right)^2 \\
&= 1 - \phi - (1-\phi)\left(u + \frac{c\pi}{\pi - \phi}\right)^2 - \phi\left(u - \frac{c(1-\pi)}{\pi - \phi}\right)^2
\end{aligned}$$