.

# ECONOMICS G003: MICROECONOMETRICS

Term 2, 2007

Time allowed: TWO hours

**Instructions:** Answer ALL questions from part A and TWO questions from part B. The questions from Part A count for 36% of the grade and those from Part B for 64%.

**Important Notes:** In cases where a student answers more questions than requested by the examination rubric, the policy of the Economics Department is that the students first set of answers up to the required number will be the ones that count (not the best answers). All remaining answers will be ignored.

## PART A

Answer all questions. 36 marks total.

1. Suppose that one has a random sample of observations $\{(x_i, y_i) : i = 1, ...n\}$ of outcome variables $y$ and covariates $x$. The linear least squares estimator for the conditional expectation of $y$ given x solves

$$\min_b \sum_{i=1}^{n} (y_i - x_i b)^2 .$$

Provide an analogous interpretation for the median regression estimator of $\beta$ in the linear model $Med(y|x) = x\beta$. That is, what sum does the median regression estimator minimize? [4]

2. Consider the conditional logit model of discrete choice, where

$$\Pr\{y = j|x = x_{ij}\} = \frac{\exp(x_{ij}\beta)}{\sum_{h=1}^{J} \exp(x_{ih}\beta)}, \ j = 0, ..., J.$$

The above expression is the probability that an outcome variable $y$ takes any of $J+1$ possible discrete values, conditional on covariates $x = x_{ij}$.

(a) What is the log-odds ratio for any two alternatives $j, k,\ j \neq k$? [4]

(b) What is meant by "independence of irrelevant alternatives"? [4]

3. Consider the following model:

$$y_i = x_i\beta + u_i \tag{1}$$

where $x_i$ is a $1 \times K$ vector of observable explanatory variables and $u_i$ is the unobservable shock.

(a) Under what conditions is the OLS estimator of $\beta$ consistent? [2]

(b) Prove consistency of the OLS estimator under your stated conditions. [2]

(c) Give 3 potential reasons for why $x$ and $u$ might be correlated. [2]

(d) Suppose the variance of $u$ is given by the model:

$$\text{var}(u_i|x_i) = \exp(x_i\gamma).$$

Discuss how you would estimate $\beta$ and $\gamma$. [2]

(e) Suppose equation (1) is a model of wages where

- $y_i$ is the log wage of worker $i$ and

- $x_i = [1 \;\; T_i]$ where $T_i$ is a dummy variable for whether the worker has obtained specialised training.

Suppose that workers with training find working in firms with new technology more attractive.

i. What problem could this create for the use of OLS? [2]

ii. What sort of information could help you identify the parameters of the model? [2]

4. What is an Euler equation and what is the usual estimation method for Euler equations? [3]

5. What does one mean by excess sensitivity of consumption to income? Suggest one possible source of excess sensitivity. [3]

6. Flinn and Heckman (82) say that a distribution $F(x)$ of a rv $X$ is recoverable from the truncated distribution $X$ given $X > r$ if the knowledge of $F(x|X > r)$ and $r$ imply that $F$ is uniquely determined. Why is that relevant for the job search model? [3]

7. What is a nonstationary search model? Give applications of this model. [3]

## PART B

Answer 2 questions. Each question carries 32 marks total.

8. Consider the Type I Tobit model:

$$\begin{aligned} y^* &= x\beta + u, \\ y &= 1\left[y^* > 0\right] \end{aligned}$$

where $u \sim N(0, \sigma^2)$, and $u$ is independent of $x$. Assume that $\{(x_i, y_i) : i = 1, ..., n\}$ constitute a random sample of observations of $(x, y)$. Note that neither $y_i^*$ nor $u_i$ are observed.

(a) Derive $\Pr\{y = 0|x\}$. [6]

(b) Derive the log-likelihood function for maximum likelihood estimation of this model. Be sure to show your work. [8]

(c) Suppose that one dispenses with the assumption that $u|x \sim N(0, \sigma^2)$, and instead asserts that Median$(u|x)$= 0. This model is the censored least absolute deviations model (CLAD). Can the model be estimated by maximum likelihood? Explain. [4]

(d) Compare the Tobit and CLAD model. Which model invokes stronger assumptions? [3]

(e) Suppose that one estimates the Tobit estimate, $\hat{\beta}_{Tobit}$ by maximum likelihood, and the CLAD estimate $\hat{\beta}_{CLAD}$. If each model is correctly specified, which of them provides more efficient estimates? [3]

(f) Suppose that one wishes to consistently estimate $\Pr\{y = 0|x\}$. In the Tobit model, can this be done using maximum likelihood estimates $\hat{\beta}_{Tobit}$ and $\hat{\sigma}_{Tobit}$? [4] In the CLAD model, can this be done using $\hat{\beta}_{CLAD}$? [4] For both models, explain whether one can consistently estimate $\Pr\{y = 0|x\}$, and if so, how.

9. Consider the following model:

$$y_{it} = \alpha + x_{it}\beta + z_{it}\gamma + f_i + u_{it} \qquad \text{for } i = 1, \ldots, N \text{ and } t = 1, \ldots, T$$

where $x_{it}$ and $z_{it}$ are scalars, $f_i$ is a permanent unobserved effect and $u_{it}$ is the transitory shock. Furthermore, assume the $x_{it}$ is strictly exogenous:

$$E(u_{it}|x_{i1}, \ldots, x_{iT}) = 0 \tag{2}$$

(a) Suppose you can only establish the orthogonality condition (2) and you take first differences to attempt to estimate the model. What are the properties of OLS when used to estimate $\beta$ and $\gamma$ in the first differences model? [4]

(b) Continue considering the first differences model. Suppose T=2 and you decide to adopt the following reduced form model for $\Delta z_{i2}$:

$$\Delta z_{i2} = \eta_0 + \eta_1 \Delta x_{i2} + \eta_2 x_{i1} + v_{i2}$$

i. Under what condition(s) on the reduced form model are $\beta$ and $\gamma$ identified? [4]

ii. How can you test this (these) condition(s)? [4]

(c) Now suppose $T = 5$ and the additional assumption holds:

$$E(u_{it}|z_{i1}, \ldots, z_{it-1}) = 0 \tag{3}$$

i. How would you estimate $\beta$ and $\gamma$ now? [5]

ii. Derive the variance of your estimator. [5]

iii. How could you test for the fixed effects assumption? [5]

(d) Suppose $z_{it} = y_{it-1}$ and you know $\gamma = 1$. How would you estimate the model? [5]

10. A discrete time version of the job search model.

To make things simple, consider a cohort of workers entering the job market at the same age $t_0$, with the same education and retiring at age $T$. At the beginning of period $t \in [t_0, T]$, a worker can be employed ($S_t = e$) or unemployed ($S_t = u$). All workers start as unemployed ($S_{t_0} = u$). Let $X_t$ denote experience.

Define $U_t$ as the present value of unemployment and $V_t$ as the present value of employment. These present values satisfy the Bellman equations:

$$U_t(w_{t-1}, X_t) = rw_{t-1} + \beta \mathbb{E}_{w_{t+1}} \max\{U_{t+1}(rw_{t-1}, X_t), V_{t+1}(w_{t+1}, X_t)\}, \quad (4)$$

$$\begin{aligned} V_t(w_t, X_t) &= w_t + \beta[\delta U_{t+1}(w_t, X_t + 1) \\ &\quad + (1-\delta)\mathbb{E}_{w_{t+1}} \max\{U_{t+1}(w_t, X_t + 1), V_{t+1}(w_{t+1}, X_t + 1)\}], \end{aligned} \quad (5)$$

where $\mathbb{E}_w g(w)$ denotes the expectation of function $g(w)$ with respect to the random variable $w$. Assume that

$$\ln w_{t+1} = \alpha_0 + \alpha_1 X_{t+1} + u_{t+1}$$

where $u_{t+1}$ is iid and takes the value $\varepsilon$ with probability $p$ and $-\varepsilon$ with probability $1-p$. Assume also that $r \in [0, 1[$, $\beta \in ]0, 1[$, $X_{t_0} = 0$, and $w_{t_0 - 1} = 0$.

These two Bellman equations tell a story. Your goal in working through the following questions is to figure out what that story is.

(a) Determine $X_{t+1}$ given $X_t$ and occupational status $S_t$. [2.5]

(b) Interpret parameter $r$. Take the case of a worker who is employed in period $t$ at wage $w_t$ and who is unemployed in periods $t+1$ and $t+2$. What is his/her income in period $t+2$? [2.5]

(c) Define $R_t$ as the income in period $t$. Compute $R_t$ given $R_{t-1}$, $X_t$, $u_t$ and $S_t$. [2.5]

(d) Interpret equation (4). [2.5]

(e) Interpret equation (5). Interpret parameter $\delta$? [2.5]

(f) Unemployment is likely to be costly for two reasons. Which ones? [2.5]

(g) Give the values of $U_T(w_{T-1}, X_T)$ and $V_T(w_T, X_T)$. [2.5]

(h) Compute the expectation in equation (4) by its value given the assumption on the distribution of $u_{t+1}$. [4]

(i) How would you compute functions $U_t$ and $V_t$ at all points? [2.5]

(j) Suppose that you have computed $U_t$ and $V_t$ at all points. What is the distribution of $V_t(w_t, X_t)$ given $X_t$. [4]

(k) What is the probability of being employed in period $t$ given $w_{t-1}$ and $X_t$? [4]