ECONG022 *Revision Session*

Morten O. Ravn

May 2010

Revision Session

- I will do the following three things:

1. Quick Recap of my part of the course

Revision Session

- I will do the following three things:

1. Quick Recap of my part of the course
2. Exam format

Revision Session

- I will do the following three things:

1. Quick Recap of my part of the course
2. Exam format
3. Quick run through an example

Recap

We did the following topics:

- The Solow model: Determination of long-run income, the golden rule

Recap

We did the following topics:

- The Solow model: Determination of long-run income, the golden rule
- The Ramsey model: Analysis, dynamics and the steady-state, adjustment to shocks

Recap

We did the following topics:

- The Solow model: Determination of long-run income, the golden rule
- The Ramsey model: Analysis, dynamics and the steady-state, adjustment to shocks
- Determinants of long-run level of income and of growth rates

Recap

We did the following topics:

- The Solow model: Determination of long-run income, the golden rule
- The Ramsey model: Analysis, dynamics and the steady-state, adjustment to shocks
- Determinants of long-run level of income and of growth rates
- Consumption and labor supply

Recap

We did the following topics:

- The Solow model: Determination of long-run income, the golden rule
- The Ramsey model: Analysis, dynamics and the steady-state, adjustment to shocks
- Determinants of long-run level of income and of growth rates
- Consumption and labor supply
- Business Cycles

Recap

We did the following topics:

- The Solow model: Determination of long-run income, the golden rule
- The Ramsey model: Analysis, dynamics and the steady-state, adjustment to shocks
- Determinants of long-run level of income and of growth rates
- Consumption and labor supply
- Business Cycles
- and along the way we learned dynamic programming techniques for optimization in dynamic macroeconomic models

Solow model

- Dynamic model but with very little behavior

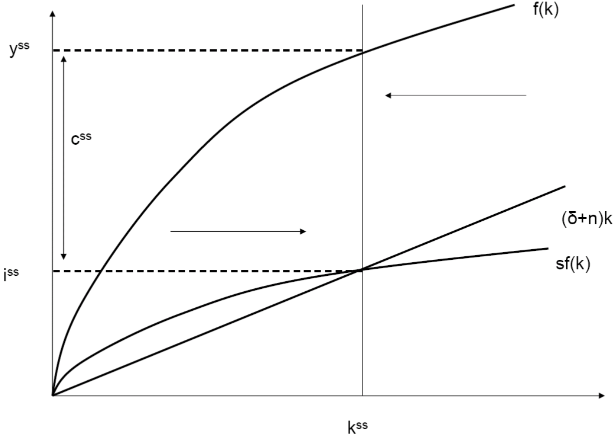
$$y_t = F(k_t)$$

$$\begin{aligned}
(1+n)k_{t+1} &= (1-\delta)\, k_t + i_t && (1)\\
s_t &= s y_t && (2)\\
y_t &= c_t + i_t && (3)\\
i_t &= s_t && (4)
\end{aligned}$$

Dynamics

- Combining the relationships, we then get that the general equilibrium is given by:

$$(1+n)\left(k_{t+1}-k_{t}\right)=sf\left(k_{t}\right)-(\delta+n)\,k_{t}$$



The Steady-State

- The Golden Rule: Which savings rate maximizes steady-state consumption?

$$F'\left(k^{GR}\right) = \delta \tag{5}$$

so that:

$$\begin{aligned} k^{GR} &= F'^{-1}(\delta) \\ c^{GR} &= F\left(k^{GR}\right) - \delta k^{GR} \end{aligned} \tag{6}$$

The Ramsey model

- Dynamic general equilibrium model with capital accumulation just like the Solow model
- But, it has behavior: Consumers and firms that maximize
- All agents behave competitively - they take all prices for given
- and there are no other distortions or externalities

Definition

A Competitive Equilibrium is a price system (p_1, p_2) (or r) and an allocation (c_1^I, c_2^I) such that (i) Households maximize their utility subject to their budget constraints (utility maximization), and (ii) Goods and asset markets clear (feasibility)

Definition

A Pareto Optimal allocation is an allocation (c_1^{PO}, c_2^{PO}) such that the allocation maximizes utility subject to the economy's resource constraint

The Ramsey model

Theorem

The First Fundamental Welfare Theorem: If every good is traded at publicly known prices, and if all agents act competitively taking all prices for given, then the market outcome is Pareto optimal.

Theorem

The Second Fundamental Welfare Theorem: In convex economies (economies with convex preferences and production sets), any Pareto optimal allocation can be achieved as competitive equilibrium subject to appropriate lump-sum transfers of wealth . The associated competitive equilibrium requires that all agents take prices for given and that every good is traded at publicly known prices.

The Ramsey model

- Households are identical, we will work with a representative stand-in agent
- The Central Planner's Problem

$$\max_{(c_s,k_{s+1})_{s=t}^{\infty}} \sum_{s=t}^{\infty} \beta^{s-t} u(c_s)$$

subject to:

$$\begin{aligned} c_s &= F(k_s) - (k_{s+1} - k_s) - \delta k_s,\ s = t, t+1, \ldots. \qquad (7) \\ k_t &> 0 \text{ given} \end{aligned}$$

- Transversality Condition:

$$\lim_{s\to\infty} \beta^s u'(c_s)\, k_{s+1} = 0$$

The Ramsey model

The first-order necessary conditions for this problem are given as:

$$\begin{aligned} c_s &: \quad \beta^{s-t} u'(C_s) = \beta^{s-t} \lambda_s \ \forall s \geq t \\ k_{s+1} &: \quad \beta^{s-t} \lambda_s = \beta^{s+1-t} \lambda_{s+1} \left(F'(k_{s+1}) + (1-\delta) \right) \ \forall s \geq t \\ \lambda_s &: \quad c_s = F(k_s) - (k_{s+1} - k_s) - \delta k_s \ \forall s \geq t \end{aligned}$$

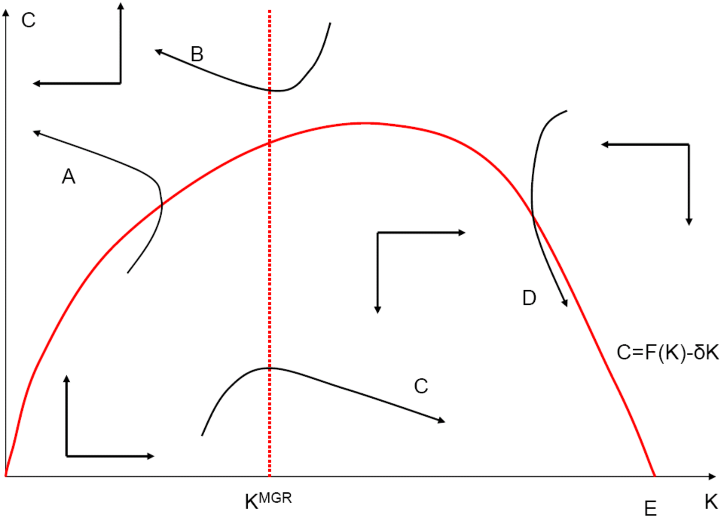
plus the transversality condition:

$$\lim_{s \to \infty} \beta^s u'(c_s) k_{s+1} = 0 \tag{8}$$

so we get he Euler equation:

$$u'(c_s) = \beta u'(c_{s+1}) \left(F'(k_{s+1}) + (1-\delta) \right) \tag{9}$$

Dynamics: Graphically



The Modified Golden Rule

- How does it compare to the Golden Rule? Compare the two conditions:

$$\begin{aligned} F'\left(k^{MGR}\right) &= \frac{1}{\beta} - (1-\delta) = \theta + \delta \\ k^{GR} &> k^{MGR} \end{aligned}$$

- Agents are impatient - the cost of having low consumption for a long time to get to k^{GR} does not fully make up for the benefit.
- The more impatient are the consumers, the lower will be the optimal capital stock

Dynamic Programming

- We can also formulate the problem recursively as

$$
\begin{aligned}
W(k) &= \max_{c,k'} \left[u(c) + \beta W(k') \right] \\
& \quad s.t. \\
c + k' &= F(k) + (1-\delta) k
\end{aligned}
$$

- Now substitute the constraint into the objective:

$$
W(k) = \max_{k'} \left[u\left(F(k) + (1-\delta) k - k'\right) + \beta W(k') \right]
$$

- First-order condition for k':

$$-\frac{\partial u\left(c\right)}{\partial c}+\beta\frac{\partial W\left(k'\right)}{\partial k'}=0$$

and the derivative of the value function follows from the envelope condition:

$$\frac{\partial W\left(k\right)}{\partial k}=\frac{\partial u\left(c\right)}{\partial c}\left(\frac{\partial F\left(k\right)}{\partial k}+\left(1-\delta\right)\right)$$

- Combining these two equations gives us:

$$\frac{\partial u\left(c\right)}{\partial c}=\beta\frac{\partial u\left(c'\right)}{\partial c'}\left(\frac{\partial F\left(k'\right)}{\partial k'}+\left(1-\delta\right)\right)$$

- which is the Euler equation that we analyzed earlier but derived in a much simpler way from a two-period problem!!!

Economic Development

1. What determines the level of income?
2. What determines the growth rate of the economy?

Three models:

1. Solow model

Economic Development

1. What determines the level of income?
2. What determines the growth rate of the economy?

Three models:

1. Solow model
2. Ramsey model

Economic Development

1. What determines the level of income?
2. What determines the growth rate of the economy?

Three models:

1. Solow model
2. Ramsey model
3. Endogenous growth models

The Solow model

- Without technological progress: No long run growth. But there is transitional growth
- with technological progress: labor augmenting technological progress:

$$\begin{aligned} Y_t &= F\left(K_t, A_t N_t\right) \\ A_{t+1} &= (1+g) A_t \end{aligned}$$

- If we define $k_t^e = K_t / \left(N_t A_t\right)$ as the amount of capital per worker measured in efficiency units, we then get that:

$$(1+g)(1+n)\left(k_{t+1}^e - k_t^e\right) \simeq s\left(k_t^e\right)^\alpha - (\delta + n + g) k_t^e$$

- This model has a unique stable steady-state

The Solow Model with Technological Progress

In this economy there exists a balanced growth path along which:

$$k^e, y^e, c^e, i^e \text{ are constant}$$
$$k, y, c, i \text{ grow at the rate of } g$$

The balanced growth path is consistent with Kaldor's growth facts 1-5:

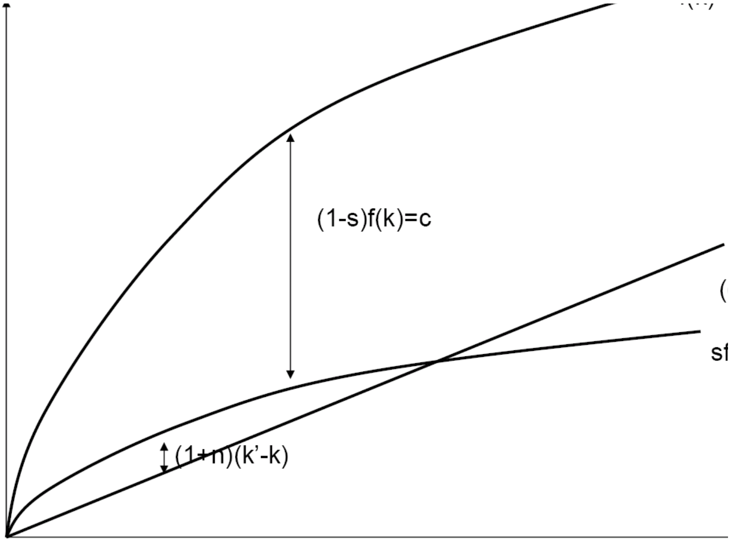
Productivity Growth in the Ramsey Model

$$
\begin{aligned}
Y_t &= K_t^{\alpha} (A_t N_t)^{1-\alpha} \\
u(c_t) &= c_t^{1-\sigma} / (1-\sigma)
\end{aligned}
$$

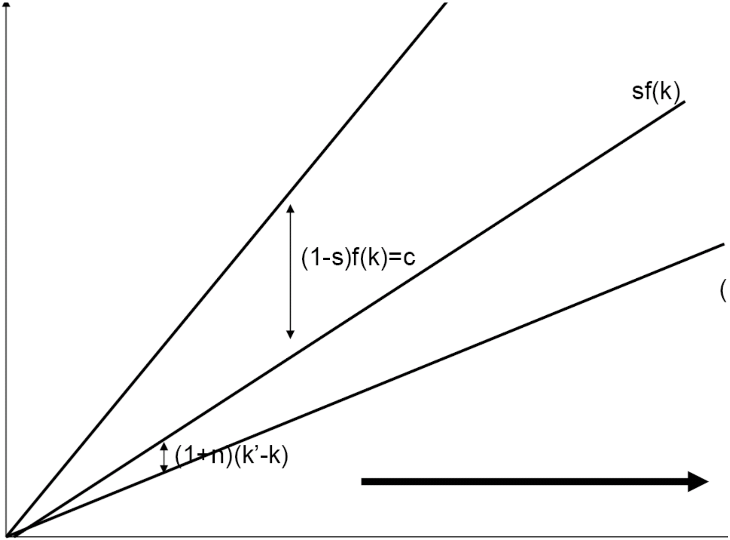
- Questions:
  - Is there a balanced growth path?
  - How does growth affect the economy?
- Answers:
  - Yes
  - small modification of parameters

Endogenous Growth

Diminishing Marginal Returns to Accumulable Factor



Constant Returns to the Accumulable Factor



Endogenous Growth

So how may we have constant returns to factors that can be accumulated?

- The "AK" model - here the production function is simply assumed to be linear in capital
- Models with human capital as well as physical capital
- Models with externalities across firms
- Models with R&D or other sources of growth

Consumption

- Consumption makes up for around 60 percent of total aggregate spending
- The household's problem implies the Euler equation:

$$u'(c_t) = \mathrm{E}_t \beta (1 + r_{t+1}) u'(c_{t+1})$$

What does this imply for:

- evolution of consumption over time and over the life-cycle?
- relationship between consumption and income?
- relationship between consumption and real interest rates?

The Random Walk Theory (Permanent Income)

Hall, 1978, derived a famous theory of consumption. Assume that

1. The real interest rate is constant and equal to r
2. $\beta = 1/(1+r)$
3. Quadratic preferences:

   $$c_t = \mathrm{E}_t c_{t+1}$$

   which implies that:

   $$\begin{aligned} c_{t+i} &= c_t + \upsilon_{t+i} \\ \mathrm{E}_t \upsilon_{t+i} &= 0 \end{aligned}$$

- Consumption should behave like a random walk
- Given c_t, no other information available at date t should be helpful for forecasting future consumption
- Consumption changes only when new information arrives, but this is unforecastable but lagged income has predictive power!

Labor Supply

- The household's optimization with labor supply is:

$$
\begin{aligned}
& \max \mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t, h_t) \\
a_{t+1} &= (1+r_t) a_t + w_t h_t - c_t, \ t \geq 0
\end{aligned}
$$

which implies

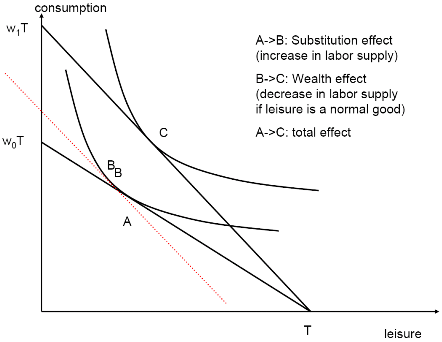
$$
\begin{aligned}
-\frac{u_h(c_t, h_t)}{u_c(c_t, h_t)} &= w_t \\
u_h(c_t, h_t) &= \beta u_h(c_{t+1}, h_{t+1}) \frac{w_t}{w_{t+1/(1+r_{t+1})}}
\end{aligned}
$$

- The household will set the marginal rate of substitution between consumption and work equal to the real wage
- and the intertemporal marginal rate of substitution between work today and tomorrow equal to the inverse of wage growth in present value terms

Labor Supply

How will an increase in the real wage affect labor supply?

- **Substitution effect**: An increase in wage makes leisure more expensive to the agent will work harder
- **Wealth effect**: Higher wage means - for unchanged labor supply - higher wealth. If consumption and leisure are both normal goods, labor supply must fall



Labor Supply

The Frisch elasticity: An important determinant for the behavior of labor supply is the Frisch labor supply elasticity which is defined as the elasticity of labor supply for a constant level marginal utility of wealth. This is the labor supply elasticity that enters the first-order condition for labor supply:

$$\begin{aligned} -u_h(c_t, h_t) &= \lambda_{c,t} w_t \\ \zeta^h &= \frac{dh_t / h_t}{dw_t / w_t}\Big|_{\lambda_{c,t}} = \frac{u_h(c_t, h_t)}{h_t u_{hh}(c_t, h_t)} \end{aligned}$$

- This parameter determines, for given wealth, the elasticity of the labor supply response to changes in wages and is a key parameter in many macroeconomic theories

How do we solve Dynamic General Equilibrium Models?

1. Guess and verify - method of undetermined coefficients - with or without log-linearizing

Example:

1. Cobb-Douglas Production Function:

$$y_t = k_t^{\alpha} h_t^{1-\alpha}$$

2. Log-log utility function:

$$u(c_t, h_t) = \theta \log c_t + (1-\theta) \log (T - h_t)$$

3. Complete depreciation:

$$k_{t+1} = i_t$$

How do we solve Dynamic General Equilibrium Models?

1. Guess and verify - method of undetermined coefficients - with or without log-linearizing
2. Dynamic programming

Example:

1. Cobb-Douglas Production Function:

$$y_t = k_t^{\alpha} h_t^{1-\alpha}$$

2. Log-log utility function:

$$u(c_t, h_t) = \theta \log c_t + (1-\theta) \log (T - h_t)$$

3. Complete depreciation:

$$k_{t+1} = i_t$$

How do we solve Dynamic General Equilibrium Models?

The first-order conditions are given as:

$$\begin{aligned}
\frac{(1-\theta)\, c_t}{\theta\,(T-h_t)} &= (1-\alpha)\, k_t^{\alpha} h_t^{-\alpha} = (1-\alpha)\,\frac{y_t}{h_t} \\
\frac{1}{c_t} &= \beta \frac{1}{c_{t+1}} \alpha k_{t+1}^{\alpha-1} h_{t+1}^{1-\alpha} = \beta\alpha \frac{y_{t+1}}{k_{t+1}} \frac{1}{c_{t+1}} \\
c_t + k_{t+1} &= k_t^{\alpha} h_t^{1-\alpha}
\end{aligned}$$

- Informed guess:

$$\begin{aligned}
h_t &= \overline{h} \\
c_t &= \gamma y_t \\
k_{t+1} &= (1-\gamma)\, y_t
\end{aligned}$$

Dynamic Programming

Bellman's equation for the social planner's problem:

$$V(k_t) = \max_{k_{t+1}, h_t} \left(\theta \log\left(k_t^{\alpha} h_t^{1-\alpha} - k_{t+1}\right) + (1-\theta)\log(T - h_t) + \beta V(k_{t+1})\right)$$

I could think about writing the Bellman equation as:

$$\begin{aligned} & V_{i+1}(k_t) \\ = \quad & \max_{k_{t+1}, h_t} \left(\theta \log\left(k_t^{\alpha} h_t^{1-\alpha} - k_{t+1}\right) + (1-\theta)\log(T - h_t) + \beta V_i(k_{t+1})\right) \end{aligned}$$

I could then iterate on the Bellman equation as follows:

1. Make a guess on V_0
2. Given the guess solve the maximization problem.
3. Find V_1
4. Return to step 1 unless $V_1 = V_0$

- stochastic models
- trends vs. business cycles

US Facts and Figures

Table 1
Business Cycle Statistics for the U.S. Economy

	Standard Deviation	Relative Standard Deviation	First Order Auto-correlation	Contemporaneous Correlation with Output
Y	1.81	1.00	0.84	1.00
C	1.35	0.74	0.80	0.88
I	5.30	2.93	0.87	0.80
N	1.79	0.99	0.88	0.88
Y/N	1.02	0.56	0.74	0.55
w	0.68	0.38	0.66	0.12
r	0.30	0.16	0.60	-0.35
A	0.98	0.54	0.74	0.78

Households

$$\max \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{\left(c_t^{\theta} l_t^{1-\theta}\right)^{1-\kappa} - 1}{1-\kappa}$$

$$\begin{aligned}
c_t + i_t &= w_t h_t + r_t k_t + \pi_t \\
k_{t+1} &= (1-\delta) k_t + i_t \\
l_t + h_t &= T
\end{aligned}$$

- with first-order necessary conditions:

$$\begin{aligned}
\frac{1-\theta}{\theta} \frac{c_t}{T-h_t} &= w_t \\
c_t^{\theta(1-\kappa)-1} \left(T-h_t\right)^{(1-\theta)(1-\kappa)} &= \beta \mathbb{E}_t \big[c_{t+1}^{\theta(1-\kappa)-1} \left(T-h_{t+1}\right)^{(1-\theta)(1-\kappa)} \\
&\quad \left(r_{t+1} + (1-\delta)\right)\big] \\
c_t + k_{t+1} &= w_t h_t + r_t k_t + (1-\delta) k_t + \pi_t
\end{aligned}$$

Firms

$$\max_{k_t, h_t} \pi_t = A_t k_t^{\alpha} h_t^{1-\alpha} - r_t k_t - w_t h_t$$

with first-order conditions:

$$\begin{aligned} r_t &= \alpha A_t k_t^{\alpha-1} h_t^{1-\alpha} \\ w_t &= (1-\alpha) A_t k_t^{\alpha} h_t^{-\alpha} \end{aligned}$$

Productivity shocks:

$$\log A_t = \rho \log A_{t-1} + \varepsilon_t$$

Equilibrium

$$
\begin{aligned}
\frac{1-\theta}{\theta}\frac{c_t}{T-h_t} &= (1-\alpha)\,A_t k_t^{\alpha} h_t^{-\alpha} \\
c_t^{\theta(1-\kappa)-1}\,(T-h_t)^{(1-\theta)(1-\kappa)} &= \beta\mathbb{E}_t[c_{t+1}^{\theta(1-\kappa)-1}\,(T-h_{t+1})^{(1-\theta)(1-\kappa)} \\
&\quad (\alpha A_{t+1} k_{t+1}^{\alpha-1} h_{t+1}^{1-\alpha} + (1-\delta))] \\
c_t + k_{t+1} &= A_t k_t^{\alpha} h_t^{1-\alpha} + (1-\delta)\,k_t
\end{aligned}
$$

1. Find the deterministic steady-state as the equilibrium of the model if $A = 1$ forever
2. Log-Linearize the first-order necessary conditions around the steady-state

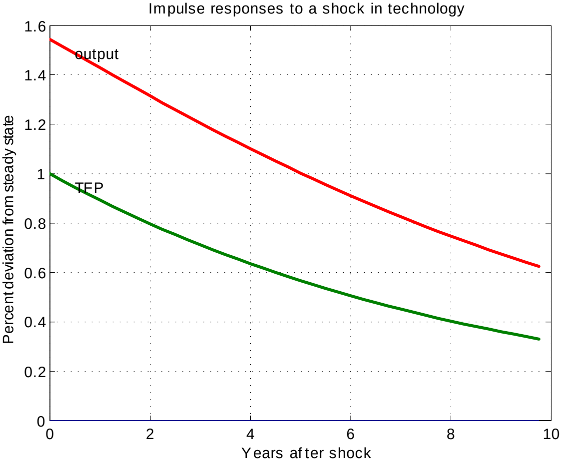
Solution:

$$\begin{aligned} \widehat{c}_t &= \gamma_c \widehat{k}_t + \mu_c \widehat{A}_t \\ \widehat{k}_{t+1} &= \gamma_k \widehat{k}_t + \mu_k \widehat{A}_t \\ \widehat{h}_t &= \gamma_h \widehat{k}_t + \mu_h \widehat{A}_t \end{aligned}$$

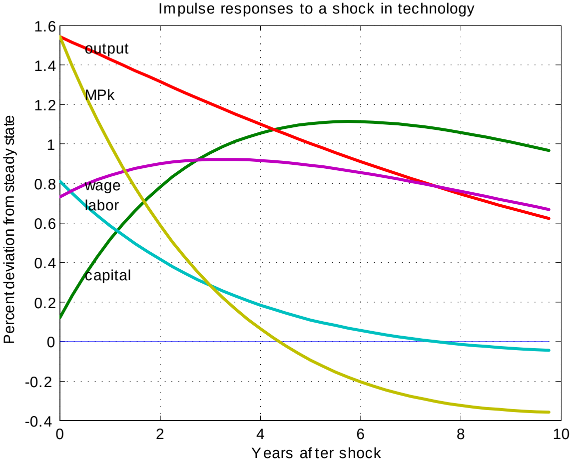
Calibration

parameter	interpretation	type
β	subjective discount factor	share parameter
θ	utility weight	share parameter
$1/\kappa$	Intertemp. elasticity of substitution	curvature parameter
α	capital share of income	share parameter
δ	depreciation rate	share parameter
ρ	persistence of TFP shock	driving process parameter
σ_ε^2	volatility of TFP innovations	driving process parameter

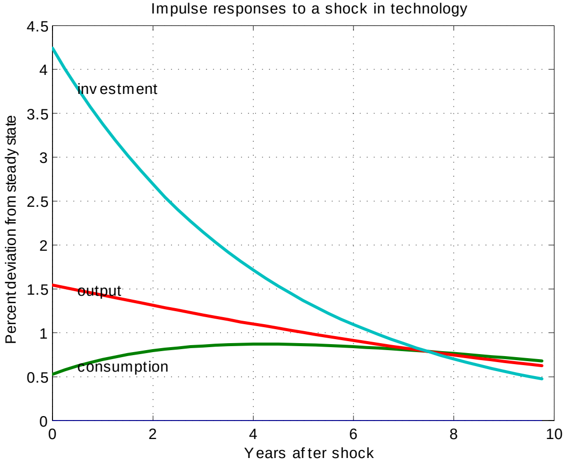
The Impact of Technology shocks



The Impact of Technology shocks



The Impact of Technology shocks



The Impact of Technology shocks

Table 3
Business Cycle Statistics for Basic RBC Model[35]

	Standard Deviation	Relative Standard Deviation	First Order Auto-correlation	Contemporaneous Correlation with Output
Y	1.39	1.00	0.72	1.00
C	0.61	0.44	0.79	0.94
I	4.09	2.95	0.71	0.99
N	0.67	0.48	0.71	0.97
Y/N	0.75	0.54	0.76	0.98
w	0.75	0.54	0.76	0.98
r	0.05	0.04	0.71	0.95
A	0.94	0.68	0.72	1.00

Note: All variables have been logged (with the exception of the real interest rate)and detrended with the HP filter.

Exam Format

Three Parts:

A. Short questions with short answers required - answer 2 out of 4 (**20 points**)

B. Long questions on my part of the course - answer 2 out of 2 (with sub-questions) (**2 times 20 points**)

C. Long question on Guy's part of the course - answer 1 out of 1 (with sub-questions) (**40 points**)

Exam Format

A. Short questions with short answers required - answer 2 out of 4 (**20 points**)

- you do NOT get extra points for answering more than 2 questions
- questions are not analytical
- give short and concise answers

B. Long questions on my part of the course - answer 2 out of 2 (with sub-questions) (**2 times 20 points**)

- Analytical questions, similar to the type of questions you have seen in exercises
- make sure that you do not waste too much time if you get stuck
- make sure that you explain your answers rather than just stating them
- questions may be difficult but not “tricky”