## MSC Macroeconomics ECONG022, Autumn 2009

## Problem Set 4: For tutorials in week 11. To be handed in to your tutor at the beginning of class.

**1.** Consumption: This question examines tests of the linear-quadratic version of the life-cycle Permanent Income Model of aggregate consumption. Suppose that labour income evolves as follows:

$$y_0 = 0$$
  

$$y_t = 0.9y_{t-1} + \varepsilon_t, t = 1, ..., 50$$
  

$$y_t = 0.5y_{t-1} + \varepsilon_t, t = 51, ..., 100$$

with  $\varepsilon_t iin(0, 0.05)$ . Notice that income becomes less persistent in the second half of the period. Suppose that the interest rate is constant and equal to 5 percent, r = 0.05. Let  $c_0 = 0$  and suppose that:

$$c_t = c_{t-1} + \frac{r}{1+r-\rho} \left( y_t - \rho y_{t-1} \right), \ t = 1, .., 100$$
(1)

where  $\rho$  denotes the persistence of the income process.

(a) On a computer, generate one replication of the labour income series.

(b) Using the  $\rho$  that applies for each time period, calculate the series for consumption.

(c) In this generated data, perform a Hall-type test of the random walk model of consumption, by regressing  $c_t - c_{t-1}$  on a constant and  $y_{t-1}$ .

(d) Now assume that an analyst does not realize that the labour income process has changed. Said analyst observes the income series you have generated in part (a) and the consumption series you have generated (with the appropriate shift in  $\rho$ ) in part (b). This person regresses  $y_t$  on  $y_{t-1}$  for the 100 observations to find an estimate of  $\hat{\rho}$  and knows that r = 0.05. He then generates an artificial series for consumption,  $\hat{c}_t$ , using the consumption function above in equation (1) using his estimated  $\hat{\rho}$  and r = 0.05. Plot the difference between the consumption series  $c_t$  and  $\hat{c}_t$ .

**2.** Labor Supply: Suppose that preferences are given by either one of the following three specifications:

$$u^{1}(c_{t}, h_{t}) = \log c_{t} + \theta \log (T - h_{t})$$
  

$$u^{2}(c_{t}, h_{t}) = \frac{c_{t}^{1-\sigma} - 1}{1-\sigma} - \theta h_{t}$$
  

$$u^{3}(c_{t}, h_{t}) = \frac{(c_{t} - \theta h_{t}^{1+\kappa})^{1-\sigma} - 1}{1-\sigma}$$

(a) Find the Frisch labor supply elasticities for each of these specifications.

(b) Find the labor supply relationships for each of these specifications. (Assume that households take wages for given and that they maximize utility subject to a sequence of budget constraints given as  $a_{t+1} = (1 + r_t) a_t + w_t h_t - c_t$ )

(c) Which one of these specifications would be consistent with common long run growth rates in wages and consumption while hours worked have remained trendless?

**3.** Solving Models I. Consider an optimal growth problem where the representative agent solves:

$$\max_{(c_t,k_{t+1})_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\sigma} - 1}{1 - \sigma}$$

subject to:

$$c_t + k_{t+1} = (1 + k_t^{1-\rho})^{1/(1-\rho)}, k_o > 0$$
 given

Assume also that  $\sigma = \rho$ .

- (a) Formulate Bellman's equation for this problem.
- (b) Guess that Bellman's equation is given as:

$$V\left(k_{t}\right) = F + Gk_{t}^{1-\sigma}$$

where F and G are unknown constants. Find the first-order conditions.

(c) Given this guess, derive the policy function for  $k_{t+1}$ 

(d) Verify the guess for the value function.

4. Solving Models II. Consider an optimal growth problem where the representative agent solves:

$$\max_{(c_t,k_{t+1},h_t)_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \log\left(c_t - \frac{\theta}{1+\kappa}h_t^{1+\kappa}\right)$$

subject to:

$$c_t + k_{t+1} = k_t^{\alpha} h_t^{1-\alpha}, \ k_0 > 0$$
 given

- (a) Derive the first-order conditions for  $c_t$ ,  $k_{t+1}$  and  $h_t$ .
- (b) Combine the first-order conditions for  $c_t$  and  $h_t$  to get that:

$$h_t = \left[\frac{(1-\alpha)}{\theta}\right]^{1/(\kappa+\alpha)} k_t^{\alpha/(\kappa+\alpha)}$$

and show from that expression that:

$$y_t = \left[\frac{(1-\alpha)}{\theta}\right]^{(1-\alpha)/(\kappa+\alpha)} k_t^{\alpha(1+\kappa)/(\kappa+\alpha)}$$

(c) (difficult) Now make the guess that

$$c_t = \gamma y_t$$
  
$$k_{t+1} = (1 - \gamma) y_t$$

Verify the guess from the combination of the first-order conditions for  $c_t$  and  $k_{t+1}$