MSC Macroeconomics ECONG022, Autumn 2009

Problem Set 3: For tutorials in week 10. To be handed in to your tutor at the beginning of class.

1. Technological Progress: Suppose that we are looking at a Solow growth model. The production function in the economy is given as:

$$Y_t = K_t^{\alpha} N_t^{1-\alpha}, \ \alpha \in (0,1)$$

where K_t denotes the capital stock and N_t is employment. Employment is exogenous and grows at the constant rate $n > 0$ per period. Savings in the economy are given as:

$$S_t = sY_t, \ s \in (0,1)$$

where s is the savings rate. The capital accumulation equation is given as:

$$K_{t+1} = (1-\delta) K_t + I_t, \ \delta \in (0,1)$$

where I_t denotes investment.

(a) Demonstrate that the only permissable long-run growth rate of the per-capita output, investment, and capital stock is 0.
(b) Now suppose that the production function is given as:

$$\begin{aligned} Y_t &= K_t^{\alpha} (A_t N_t)^{1-\alpha} \\ A_{t+1} &= (1+g) A_t, \ g > 0 \end{aligned}$$

Is there growth in the long run in this model? At which rate does per-capita output, investment and capital stock grow in the long-run?
(c) Now set A_t constant again. Instead the capital accumulation equation is given as:

$$\begin{aligned} K_{t+1} &= (1-\delta) K_t + X_t I_t, \ \delta \in (0,1) \\ X_{t+1} &= (1+m) X_t, \ m > 0 \end{aligned}$$

Repeat question (b). Will the macroeconomic aggregates grow at the same rate in the long run?
(d) In national accounts data, the price of capital goods (in particular equipment and durables) has dropped very considerably over the last few decades. Is this more consistent with growth in A or growth in X?

2. Growth with Externalities: In an an economy there is a large number of identical infinitely lived households and a large number of identical firms. Both firms and households act competitititively taken prices for given. The households' preferences are given as:

$$U_0 = \sum_{t=0}^{\infty} \beta^t \log c_t$$

The capital accumulation equation is given as:

$$k_{t+1} = (1-\delta)\,k_t + i_t$$

Firms produce output using the following technology:

$$y_{i,t} = Ak_{i,t}^{\alpha}K_t^{1-\alpha}$$

where i refers to a single firm and K_t denotes the aggregate per-firm capital stock. Individual firms take the latter for given. Output is used for either consumption or investment.

(a) Formulate the central planner's problem and derive the first-order necessary conditions.
(b) Derive the balanced growth path rate of consumption growth from the central planning problem.
(c) Now assume that households own the capital stock which they rent out to the firms. Find the first-order conditions for firms and households.
(d) Derive the growth rate of consumption along the balanced growth path for the competitive equilibrium. Do you get the same as in question (b)? Why?
(e) Now suppose that the government levies lump sum taxes on households. It uses the proceeds of these to provide a subsidy to firms for the use of capital at the rate of s_k. How does this subsidy affect the long run growth rate of the economy?

3. Public Infrastructure: Output in an economy is produced by a large number of identical competitive firms. Suppose that the production function is given as:

$$Y_{it} = AK_{it}^{\alpha}N_{it}^{1-\alpha}G_t^{1-\alpha}$$

where G_t denotes aggregate public spending on infrastructure which we assume is provided free of charge by the government, Y_{it} denotes output of firm i, K_{it} the capital services used by firm i, and N_{it} is employment in firm i. The government finances G by taxing output:

$$G_t = \tau Y_t$$

where τ is a proportional tax rate and Y_t is aggregate output.

(a) Show that G can be expressed as

$$G = (\tau AN)^{1/\alpha}\,k$$

where $k = K/N$.
(b) Formulate the profit maximization problem of a representative firm and derive the first-order necessary conditions.
(c) Using the result from question (a) show that, in equilibrium, the return on capital per worker does not depend on k.

(d) Suppose that the firms and the capital stock are owned by households with the following preferences:

$$U_0 = \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\theta}}{1-\theta}$$

Derive the Euler equation and from this, using the results derived earlier, show that this model allows for long-run growth.

4. Savings and uncertainty: Consider an individual who lives for two periods and has constant absolute-risk-aversion utility:

$$U = -\exp(\gamma c_1) - \exp(\gamma c_2)$$

The agent can borrow and lend freely at a zero real interest rate, is born without assets and must leave the second period without debt. Hence, she is faced with the following life-time budget constraint:

$$c_1 + c_2 = y_1 + y_2$$

where y_i denotes income in period i.

(a) Suppose that y_1 and y_2 are known at the beginning of period 1. Solve the consumer's maximization problem. Under which conditions would she save in the first period?
(b) Given the instantaneous utility function u(c)) $= -\exp(\gamma c)$ find the signs of u', u'', and u'''.
(c) Suppose now that y_2 is stochastic, in particular it is assumed to be Normally distributed with mean $\overline{y}_2$ and variance σ^2. Recall that for a normally distributed variable x, it is the case that $E\exp(x) = \exp(Ex)\exp\left((x - Ex)^2/2\right)$. Using this, find an expression for the individual's expected life-time utility as a function of first period consumption and income and of the parameters $\overline{y}_2$, σ^2, and γ.
(d) Derive the optimal choice of c_1. How is it affected by uncertainty? Does the principle of question (a) still hold true?