MSC Macroeconomics ECONG022, Autumn 2009

Problem Set 2: For tutorials in week 9. To be handed in to your tutor at the beginning of class.

1. The Ramsey Model: An economy is inhabited by a large number of identical infinitely lived agents represented by a single stand-in representative agent. The preferences of the representative agent are given as:

$$V_0 = \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\sigma} - 1}{1 - \sigma}$$

where $0 < \beta < 1$ and $\sigma > 0$. In this equation c_t denotes consumption in period t. Output in the economy is produced by a large number of identical competitive firms according to the production function:

$$y_t = A_0 k_t^{\alpha}$$

where $0 < \alpha < 1$. In this equation y_t denotes output and k_t denotes the capital stock at the beginning of period t. Capital accumulates over time according to:

$$k_{t+1} = (1 - \delta) k_t + i_t$$

where i_t denotes investment and $0 < \delta < 1$ is the rate of depreciation. Finally, there is a resource constraint:

$$y_t = c_t + i_t$$

(a) Set up the social planning problem and derive the first-order necessary conditions. Discuss the first-order conditions and their interpretation.
(b) Derive the steady-state of the economy and analyze the dynamics of the economy.
(c) Suppose that productivity increases permanently from A_0 to A_1 at date t_1. Agents are told of this change at t_1. How will the economy adjust over time?
(d) Suppose as above that productivity increases permanently from A_0 to A_1 at date t_1. Agents, however, are now told of this change at date $t_0 < t_1$. In other words, the increase in productivity is anticipated. How will the economy adjust over time?
(e) Suppose now instead that productivity rises temporarily during the period $t_0 - t_1$ (ie. at t_0 productivity rises to A_1 and at t_1 it reverts to its original level A_0). The agents are informed about the temporary increase in productivity at date t_0. How will the economy adjust over time in response to this temporary increase in productivity?

2. Government Spending: Now extend the model above with government spending. We assume that government spending is financed by lump-sum taxation. Let G_0 denote the level of government spending.

(a) Analyze how the level of government spending affects the economy in the long run and how the economy responds to a permanent increase in government spending from G_0 to $G_1 > G_0$. Does the response of the economy correspond to how you think increases in government spending affects the economy?
(b) Now examine instead to impact of a temporary increase in government spending: At t_0 the level of government spending is announced to temporarily increase to G_1 until period $t_1$where it reverts to its original level. How does this affect the economy?
(c) Now consider the competitive equilibrium. In particular, suppose that the households are Robinson Crusoe households who both produce, consume, and accumulate capital. Households also pay lump sum taxes T_t in period t. Suppose that the government balances its budget period by period so that the government budget constraint is given as:

$$G_t = T_t$$

Argue that the planning solution above corresponds to a competitive solution. You do not need to go through all of the steps but do at least show that the government and household budget constraints when combined imply the resource constraint and argue that lump-sum taxes do not distort household behavior.

3. Preferences and Dynamics: Suppose that households have infinite planning horizons and that their preferences are given as:

$$V_0^1 = \sum_{t=0}^{\infty} \beta^t u(c_t)$$

where $u(c_t)$ is given by either of the following three specifications:

$$\begin{aligned} u^1(c_t) &= \frac{c_t^{1-\sigma} - 1}{1-\sigma}, \ \sigma \geq 0 \\ u^2(c_t) &= \left(-\frac{1}{\sigma}\right) \exp(-\sigma c_t), \ \sigma \geq 0 \\ u^3(c_t, c_{t-1}) &= \frac{(c_t - \theta c_{t-1})^{1-\sigma} - 1}{1-\sigma}, \ \sigma \geq 0, \ 0 < \theta < 1 \end{aligned}$$

(a) Define the intertemporal elasticity of substitution as:

$$\xi = -\frac{u'(c)}{u''(c)\,c}$$

Compute the intertemporal elasticity of substitution for each of the three specifications above. How does this elasticity depend on the level of consumption and why?
(b) Derive the Euler equations for each of the three specifications above. Interpret these conditions and discuss how they depend on the preference specification.

4. Time-inconsistent choices: Consider an individual who lives for three periods. In period 1, her objective function is $\log c_1 + \beta \log c_2 + \beta \log c_3$, where $0 < \beta < 1$. In period 2, the objective is $\log c_2 + \beta \log c_3$. (Since the individual's period-3 choice problem is trivial, the period-3 objective function is irrelevant.) The individual has wealth $W > 0$ and faces a real interest rate of 0.
(a) find the values of c_1, c_2, and c_3 under the following assumptions about how they are determined:

i. Commitment: The individual chooses c_1, c_2, and c_3 in period 1.

ii. No commitment, naïveté: The individual chooses c_1 in period 1 to maximize the period-1 objective function, thinking she will also choose c_2 to maximize this objective function. In fact, however, the individual chooses c_2 to maximize the period-2 objective function.

iii. No commitment, sophistication: The individual chooses c_1 in period 1 to maximize the period-1 objective function, realizing that she will choose c_2 in period 2 to maximize the period-2 objective function.

(b) Use your answers to parts 1 and 2 above to explain in what sense the individual's preferences are time-inconsistent
(c) Explain intuitively why sophistication does not produce different behavior than naïveté.