MSC Macroeconomics ECONG022, Autumn 2009

Problem Set 1: For tutorials in week 8. To be handed in to your tutor at the beginning of class.

1. The Solow Model: Suppose that we are looking at an economy described by the following equations:

$$\begin{aligned} y_t &= k_t^\alpha \\ k_{t+1} &= (1-\delta)\, k_t + i_t \\ s_t &= s y_t \end{aligned}$$

where y_t denotes output per capita in period t, k_t is the capital stock per capita at the beginning of period t, i_t denotes investment per capita during period t, s_t denotes savings during period t, $\alpha \in (0,1)$ is the elasticity of output to capital, $\delta \in (0,1)$ is the depreciation rate, and $s \in (0,1)$ is the savings rate.

(a) Illustrate the economy graphically and discuss the determination of the steady-state.
(b) Suppose $\delta = 0.1$ and $\alpha = 0.5$. Find the Golden Rule capital stock, savings rate and level of consumption.
(c) Suppose now that α increases to 0.75. Repeat question (b) and discuss your results.
(d) The economy above implicitly does not have any population growth. Suppose now that population grows at the constant rate n. Repeat questions (a) and (b) and discuss the results. According to this, which economy would you expect to have higher per capita income - an economy with high or low population growth?

2. Savings in a 2-period Model: A consumer lives for two periods. She has an endowment x_1 in period 1 and x_2 in period 2. She can borrow and lend freely at the interest rate $r > 0$. She is born with no wealth and must leave period 2 with no debt. Her preferences are given as:

$$U = \frac{c_1^{1-\sigma} - 1}{1-\sigma} + \beta \frac{c_2^{1-\sigma} - 1}{1-\sigma}, \ \sigma \geq 0, \ \beta \in (0,1)$$

(a) Formulate the household's optimization problem and find the first-order conditions.
(b) Derive the Euler equation and discuss its interpretation.
(c) Suppose now that period 1 endowment increases. How does this affect first and second period consumption. What is the effect on savings?
(d) Repeat question (c) in the case of an increase in period 2 endowment.
(e) Repeat question (c) for an increase in the interest rate.

3. General Equilibrium in a 2-period model. Consider a two-person two-period model. Agent 1 consumes quantities c_1^1 and c_2^1 in periods 1 and 2 while agent 2 consumes quantities c_1^2 and c_2^2 in the two periods. Agent 1 is endowed with $\lambda \in (0,1)$ units of the good in the first period, and λg units in the second period where $g > 1$. Agent 2 gets $1-\lambda$ and $(1-\lambda)\,g$ units, resp. Each consumer has utility function $u^i = \log c_1^i + \log c_2^i$, $i = 1, 2$.
(a) Solve for the central planning solution (the Pareto Optimum) assuming that the planner's welfare function is given as:

$$W = \lambda u^1 + (1-\lambda)\, u^2$$

Recall that the planner maximizes utility subject to the feasibility constraints which are:

$$\begin{aligned} c_1^1 + c_1^2 &= 1 \\ c_2^1 + c_2^2 &= g \end{aligned}$$

How do the allocations look like across consumers and across time?
(b) Define the shadow prices of the goods as the implied marginal rates of substitution. Show that the marginal rates of substitution are equalized across the two consumers. Compute these shadow prices. What is the implied interest rate?
(c) Define a competitive equilibrium as an allocation $(c_1^1, c_1^2, c_2^1, c_2^2)$ and a non-negative price system (p_1, p_2) such that (i) households maximize their utility subject to their budget constraints, and (ii) supply equals demand in each period. Verify that the solution derived in questions (a) and (b) fulfills each of the criteria for a competitive equilibrium when the relative price equals the relative shadow price that you found in question (b).

4. Risk: As in question 2, consider a two-person two-period model but now extended with uncertainty. As above agent 1 consumes quantities c_1^1 and $c_2^1(s_2)$ in periods 1 and 2 while agent 2 consumes quantities c_1^2 and $c_2^2(s_2)$ in the two periods and each consumer has utility function $u^i = \log c_1^i + \beta E_1 \log c_2^i(s_2)$. E_1 denotes the mathematical expectations operator.

The variable s_2 is the "state of nature" in period 2 and captures the uncertainty. We assume that both agents have an endowment of 1 in period 1. Period two endowments are stochastic. s_2 can take on two values, $s_2 = 1$ or $s_2 = 2$. Let the second period endowment of agent i be denoted $x^i(s_2)$. Then the endowment process in the second period is given as:

$$\begin{aligned} s_2 &= 1 : \left\{ \begin{array}{l} x^1(s_2 = 1) = \mu g \\ x^2(s_2 = 1) = (1-\mu)\, g \end{array} \right. \\ s_2 &= 2 : \left\{ \begin{array}{l} x^1(s_2 = 2) = (1-\mu)\, g \\ x^2(s_2 = 2) = \mu g \end{array} \right. \end{aligned}$$

where $\mu \in (0.5, 1)$. Thus, agent 1 is lucky in state 1 and unlucky in state 2, and vice versa for agent 2. We assume that the two states of nature are equally likely.

(a) How many goods are there in this economy?

(b) Suppose that there is a central planner who maximizes an equally weighted sum of the two agents' expected utility. Formulate the central planner's maximization problem.

(c) Find the consumption allocation that solves the central planner's problem. How does each consumer's consumption in period 2 depend upon her endowment in that period?

(d) (DIFFICULT) Speculate on how the central planning solution can be derived as a competitive equilibrium. In particular, try and formulate a situation in which the agents at the beginning of period 1 trade contracts on delivery of goods in both periods 1 and 2. Which type of contracts need to be allowed for?