G022 MSc Core Macroeconomics

You have THREE HOURS. Answer TWO of THREE questions in Part A, and ALL questions in Parts B and C. There are a total of 100 points on the exam.

In cases where a student answers more questions than requested by the examination rubric, the policy of the Economics Department is that the student's first set of answers up to the required number will be the ones that count (not the best answers). All remaining answers will be ignored.

PART A (20 points)

Answer **two** of the following **three** questions. Be concise in your answers; Irrelevant material will be penalized. Each question has equal weight.

1. Consider a dynamic economy in which each period one of n possible states of nature can occur. Let $q_{i,t}$, $i = 1, 2, \ldots, n$ be the prices of state contingent claims in period t that pay out one unit of the consumption good if the state is i in period $t+1$ and pay out zero otherwise. What is the period t price of a risk-free bond that pays out one unit of the consumption good in period $t+1$, regardless of the realized state?

2. The abstraction of an infinitely lived representative agent plays a prominent role in many macroeconomic models. Obviously agents are not infinitely lived. What justifies the use of this abstraction over explicitly modeling finitely lived agents who care about their descendants?

3. Consider the results of the following empirical studies. Souleles (1999) and Parker (1999) exploit the fact that the average US taxpayer stops paying Social Security (at rate 7%) in September of each year. They find a significant increase in some consumption expenditures in the months that income increases (mostly small durables) and that the households that respond do not appear to be liquidity constrained. Hsieh (2000) replicates this result for Alaska, and also looks at the consumption reaction to large, predicable, payments from the Alaska Permanent Fund. He finds that the same households who "overreact" to the income tax refund do not "overreact" to payments from the Alaska Permanent Fund. Browning and Collado (2001) note that a majority of Spanish workers receive double-pay in June and December. They find that the expenditure paths of "bonus" and "non-bonus" households for durable and non-durable goods over the 12 months of the year are indistinguishable. *Briefly* describe how Browning and Crossley (2001) reconcile these results within a permanent income/life-cycle model of consumption.

TURN OVER

Part B (40 points)

B.1 (20 points) Consider the following representation of the standard real business cycle model: The representative household has preferences over stochastic sequences of consumption $c_t \geq 0$ and leisure $0 \leq l_t \leq 1$, described by

$$U = \mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{(c_t^{\gamma} l_t^{1-\gamma})^{1-\theta} - 1}{1-\theta}, \qquad 0 < \gamma < 1, \qquad \theta > 0,$$

where E_0 is the expectations operator, conditional on time zero information, β is the discount factor, and $\beta \in (0,1)$. The household has one unit of time to divide between leisure and hours of work

$$l_t + h_t \;=\; 1, \qquad l_t \geq 0, \qquad h_t \geq 0.$$

There is a representative firm with a constant returns-to-scale Cobb-Douglas production function that uses capital k_t and labour hours h_t to produce output y_t

$$y_t = f(z_t, k_t, h_t) = \exp(z_t) k_t^{\alpha} h_t^{1-\alpha}, \qquad 0 < \alpha < 1,$$

where z_t is a stochastic term representing random technological progress. z_t evolves as

$$z_{t+1} = \rho z_t + \varepsilon_{t+1}, \qquad \mathrm{E}_t[\varepsilon_{t+1}] = 0, \qquad 0 < \rho < 1.$$

Capital evolves according to the law of motion

$$k_{t+1} = (1-\delta) k_t + i_t, \qquad k_0 \text{ is given},$$

where $0 < \delta < 1$ is the depreciation rate and i_t investment. The economy must satisfy the resource constraint

$$c_t + i_t = y_t.$$

1. Why are preferences of this form typically assumed in the real business cycle literature? Why is the Cobb-Douglas production function also typically assumed?
2. Let $V(k_0, z_0)$ be the value of $\mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{(c_t^{\gamma} l_t^{1-\gamma})^{1-\theta} - 1}{1-\theta}$ for the representative consumer who begins time 0 with capital stock k_0 and observes productivity shock z_0. Write down Bellman's equation for $V(k_t, z_t)$ at date t.
3. How well does this model do in accounting for the business cycle facts associated with consumption, investment, employment, and the real wage? [You do not need to solve the model.] How could the model be modified to match the facts better? **CONTINUED**

B.2 (20 points)

Suppose there are two countries, indexed by $i = 1,2$. In country i the utility of a representative agent is given by

$$U_i = \sum_{t=0}^{T} \mathrm{E}_0 \beta^t u(c_{it}), \qquad 0 < \beta < 1$$

with

$$u(c_{it}) = \log c_{it},$$

where $c_{it} \geq 0$ is the consumption of the representative agent in country i at time t. In each country there is an endowment of a non-storable commodity. The endowment in country i is given by

$$y_{it} = z_{it}, \qquad z_{it} > 0, \qquad z_{it} \sim iid, \qquad corr(z_{1t}, z_{2t}) = 0.$$

1. Solve for the consumption allocation in a competitive equilibrium, by solving a social welfare maximization problem with equal weights on the two countries.
2. What is the correlation between c_{1t} and c_{2t}?
3. Now, modify the environment in two ways. First, suppose that

   $$y_{it} = n_{it}^{\alpha} z_{it}, \qquad 0 < \alpha < 1,$$

   where $n_{it} \geq 0$ is labour supply in country i at time t. The exogenous component of output, z_{it}, is still *iid* and has zero correlation across countries. Second, labour supply now enters the utility function. Specifically, the utility of a representative agent is given by

   $$U_i = \sum_{t=0}^{T} \mathrm{E}_0 \beta^t u(c_{it}, n_{it}),$$

   with

   $$u(c_{it}, n_{it}) = \log(c_{it} - n_{it}).$$

   Again, by solving a Pareto planning problem, find the optimal n_{it}.
4. Do these modifications affect the prediction for the cross-country, consumption correlation? Why or why not?
5. Would your answer to question B.2.4 change if the period utility function were specified as $u(c_{it}, n_{it}) = \log(c_{it}) - \gamma \log(n_{it})$, $\gamma > 0$?

TURN OVER

PART C: Temporary equilibrium, production and the quantity theory of money (40 points)

Consider an economy made up of generations of identical consumers-workers, each with a two period horizon.

The economy has a non durable consumption good, produced with labour, and a durable financial asset, which serves as the numeraire. The typical consumer produces when young and consumes when old. The old consumer enters period t with asset B_{t-1}. All consumers have the same utility function

$$U(L,C) = -\frac{L^2}{2} + \frac{C^{1-\sigma}}{1-\sigma},$$

where L and C denote respectively the supply of labour when young and the consumption of the non durable good when old, both being non negative, and σ is a positive parameter different from 1. Production is homemade and exhibits constant returns, with one unit of work producing one unit of the non durable good.

The price of the non durable good in terms of the numeraire is p_t, and carries a superscript e when it is expected by the consumer born at date $t-1$.

C.1: Temporary equilibrium (20 points)

In all of part C.1, the stock of financial asset stays constant over time at a value $B = B_0 > 0$.

1. Discuss the structure of the model in comparison with what has been studied in this course.

2. Compute the demand for the non durable good by the old consumer at date t as a function of the current price.

3. Compute the labor supply of the young consumer at date t, supposing that her expectation p_{t+1}^e is non random, so that the consumer's behavior is described by the following program

$$\begin{cases} \max \;\; U(L_t, C_{t+1}) \\ B_t = p_t L_t \\ p_{t+1}^e C_{t+1} = B_t \end{cases}$$

where the maximization takes place with respect to the variables (L_t, C_{t+1}, B_t).

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4. Define a temporary competitive equilibrium at date t. Write the equation characterizing the temporary equilibrium price. Show that it can be written as

$$\ln\left(\frac{p_t}{B}\right) = \alpha \ln\left(\frac{p^e_{t+1}}{B}\right),$$

for an appropriately chosen α. Give the expression of α as a function of σ and show that the absolute value of α is smaller than 1 if and only if $\sigma < 3$.

5. Suppose that the consumers base their expectations on the past price, according to the formula $p^e_{t+1} = p_{t-1}$. Given an initial value of p_0, study the price dynamics along a sequence of temporary equilibria starting at date 1. Does it always converge? Describe the limit allocation when σ is smaller than 3.

6. What is the economic intuition for the results of the previous question? How do they compare with the results seen in the lectures?

C.2 Interest rate policy (20 points)

In all of part C.2, it is assumed that $\sigma < 3$.

1. Describe the *quantity theory of money*.

2. Show that the quantity theory holds in the *long run* in the economy studied here, under the conditions of Question C.1.5.

3. Again in the context of Question C.1.5, starting from the initial condition $p_0 = B_0$ associated with a stationary equilibrium, suppose that the Central Bank multiplies the nominal holdings by a factor $(1+r)$, $r \geq 0$, where r is the nominal interest rate at date 1. This is a one shot monetary injection: the nominal interest rate is kept equal to zero afterwards and $B = (1+r)B_0$ from date 1 onwards.

   Let $\{p_t(r), L_t(r)\}_{t=1}^{\infty}$ be the temporary equilibrium price and output trajectory of the economy which receives the monetary injection r at date 1. Give the analytical expression of the changes of the logarithms of price and output induced by the monetary policy, $\Delta \ln p_t = \ln p_t(r) - \ln p_t(0)$ and $\Delta \ln L_t = \ln L_t(r) - \ln L_t(0)$. What are the effects of monetary policy over time?

4. What happens when expectations take the form $p^e_{t+1} = (1+r_t)p_{t-1}$? Compare the expectations errors here and in the preceding question. Discuss.

END OF EXAM