FISCAL POLICY IN AN EXPECTATIONS DRIVEN LIQUIDITY TRAP*

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Abstract

We study the effects of fiscal policy interventions in a liquidity trap in a model with nominal rigidities and an interest rate rule. In a liquidity trap caused by a self-fulfilling state of low confidence, higher government spending has deflationary effects that reduce the spending multiplier when the zero lower bound is binding. Instead, cuts in marginal labor tax rates are inflationary and become more expansionary when the zero lower bound is binding. These findings contradict popular views about the effects of fiscal policy in a liquidity trap.

Keywords: Liquidity trap, zero lower bound, fiscal policy, confidence shocks

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1 Introduction

Aggressive central bank action during the early parts of the 2008 financial crisis led many developed economies into liquidity traps where the zero lower bound (ZLB) on nominal interest rates constrained - and continues to do so - conventional monetary policies. In the search for alternative stabilization tools, some analysts have aired strong views about the appropriate design of fiscal interventions in a liquidity trap. Romer and Bernstein’s (2009) analysis of the American Recovery and Reinvestment Act questions the effectiveness of tax cuts but argues that the government spending multiplier is likely to be large given the Fed’s policy of near zero interest rates. Similarly, the IMF’s 2012 World Economic Outlook partially blames ongoing fiscal consolidations for the unexpectedly poor growth performance of many European countries, citing the ZLB as a key reason for why it has underestimated spending multipliers. In this paper we question these views and show that the impact and relative effectiveness of spending and tax policies in a liquidity trap depend importantly on the underlying reasons for the crisis.

Our analysis builds on a dynamic rational expectations model with nominal rigidities in which monetary policy follows an interest rate rule that prescribes an aggressive response to deviations of inflation from a target. Such Taylor rules are widely viewed as empirically plausible descriptions of central bank policies in many countries over the last couple of decades. Taylor rules are also theoretically appealing because they eliminate many types of inefficient business cycle fluctuations. However, these rules do not prevent recessions caused by very large economic shocks that force the ZLB to bind. These not only include large shocks to fundamentals but, because the ZLB constraint generates multiple equilibria, also non-fundamental shocks to expectations. We show that fundamental and expectations driven liquidity trap equilibria have very different implications for the design of fiscal policies meant to stimulate aggregate activity.
Intuitively, multiple liquidity trap equilibria arise because the ZLB constraint in the Taylor rule induces a non-monotonic relationship between inflation and consumption growth. Households’ intertemporal allocation of consumption depends on real interest rates that are determined by how monetary policy adjusts nominal interest rates to inflation. When nominal interest rates are positive, lower inflation reduces real interest rates. When the ZLB binds, lower inflation instead leads to higher real interest rates. A state of low consumer confidence in which households expect a temporary but persistent drop in income can produce sufficient deflation to cause the ZLB to bind. Higher real interest rates encourage households to delay consumption and this leads to output drops that confirm households’ initial pessimism as a self-fulfilling rational expectations equilibrium. A liquidity trap can also occur because a large fundamental shock causes deflation, for instance when a shock to intertemporal preferences provides a strong incentive to delay consumption.

As in many countries post 2008, fundamental and confidence driven liquidity trap crises are both characterized by nominal interest rates at the ZLB and output below trend. Indeed, we show that, in the absence of policy interventions, the two types of liquidity traps can feature near-observationally equivalent paths of output, interest rates and inflation. However, under the assumption that fiscal or unconventional monetary policies do not succeed in shortening the duration of the liquidity trap, the impact of fiscal interventions is sharply different. In the confidence driven liquidity trap, increasing government purchases has deflationary effects. Higher real interest rates and crowding out of private consumption greatly limit the expansionary effects of higher government spending. A reduction in the marginal labor tax rate has inflationary effects, reduces real interest rates and crowds in private consumption. As a result, tax cuts become more expansionary at the ZLB. In a fundamental liquidity trap, increased government spending instead is inflationary and can have very large expansionary effects, whereas tax cuts are deflationary and become contractionary. The two different scenarios therefore lead to opposite conclusions about what fiscal instrument is effective at the ZLB.
Our results imply that the design of fiscal stimuli or austerity packages under current conditions may have to be more carefully conditioned on the underlying reasons for the continued weak growth performance and on how policies affect consumer confidence. Because confidence shocks can create an environment that in the short run is hard to distinguish from one caused by fundamental shocks, the best fiscal policy response in 2008/2009 may not have been obvious. However, the passing of time is informative. In our model, a fundamental liquidity trap only exists when its expected duration is sufficiently short whereas a confidence driven liquidity trap must have a relatively high expected duration. The very prolonged recent experience with near zero interest rates may therefore increasingly point to a role for self-fulfilling expectations that affect the impact of policy interventions.¹

Some researchers dismiss expectations driven fluctuations as mere theoretical curiosities because reasonable deviations from strict rationality often imply that non-fundamental rational expectations equilibria become unstable. Along these lines, Christiano and Eichenbaum (2012) reject the uncertainties regarding the impact of fiscal policies in a liquidity trap that arise because of equilibrium multiplicity. We extend our analysis to a setting with recursive learning and find that, when expectational errors are relatively small, learning dynamics are quantitatively not very important over relevant time horizons and do not alter the conclusions from the rational expectations model.

Our analysis contributes to a rapidly growing literature on the implications of the ZLB for stabilization policies. Fuhrer and Madigan (1997), Wolman (1998), Krugman (1998), and Orphanides and Wieland (1998, 2000) are among the first to study the ZLB and monetary policy in an intertemporal framework. Our paper is related to Benhabib, Schmitt-Grohé and Uribe (2001a,b), who point to the ZLB constraint as a source of global indeterminacy and discuss perfect foresight equilibria

¹In the US, the federal funds target has been 25 basis points or less since December 2008. The official bank rate in the UK has been 50 basis points since March 2009. The ECB’s main refinancing rate has not exceeded 150 basis points since March 2009. In Japan short term nominal interest rates have been near zero since the autumn of 1995.
converging to a liquidity trap steady state. Several studies have looked at the quantitative effects of fiscal interventions in a fundamental liquidity trap, e.g. Eggertsson and Woodford (2004), Cogan, Cwik, Taylor and Wieland (2010), Eggertson (2011), Woodford (2011), Christiano, Eichenbaum and Rebelo (2011) and Coenen et al. (2012). Aruoba and Schorfheide (2013) evaluate US fiscal policies during 2008/09 in the context of a non-fundamental liquidity trap and reach conclusions very similar to ours. Another strand of the literature replaces rational with learning agents and studies liquidity trap dynamics after expectational shocks, e.g. Evans and Honkapohja (2005), Evans, Guse and Honkapohja (2008) and Benhabib, Evans and Honkapohja (2012).

The remainder of the paper is structured as follows: Section 2 presents the model and characterizes the liquidity traps. Section 3 contains our analysis of the impact of fiscal policy. In Section 4, we replace rational expectations with recursive learning. Section 5 concludes.

2 The Model

2.1 Preferences, Technologies and Government Policies

There are four types of agents: A representative household; competitive final good producers; monopolistically competitive intermediate good firms that set prices subject to nominal rigidities; and a government that is in charge of fiscal and monetary policies.

Households A representative and infinitely-lived household with rational expectations has preferences

\[
\mathbb{E}_0 \sum_{t=0}^{\infty} \beta_t \omega_t \left( \frac{c_t^{1-\sigma} - 1}{1-\sigma} + \theta \frac{l_t^{1-\kappa} - 1}{1-\kappa} \right), \quad \sigma, \theta, \kappa \geq 0,
\]

where \( \mathbb{E}_s(x_t) \) denotes the mathematical expectation conditional on all information available at date \( s \), \( \beta \in (0,1) \) is the subjective discount factor, \( c_t \) and \( l_t \) denote consumption of a single final good and leisure, \( 1/\sigma \) is the intertemporal elasticity of substitution in consumption and \( \kappa \) determines the
Frisch labor supply elasticity. $\omega_t$ is an exogenous preference shock following a process to be specified later. As fluctuations in $\omega_t$ affect the household’s intertemporal preferences, we will refer to it as a discount factor shock.

In addition to a finite time endowment normalized to one and a no-Ponzi constraint, household choices are restricted by a sequence of budget constraints:

$$P_t c_t + \frac{B_t}{1 + i_t} \leq (1 - \tau_t) W_t (1 - l_t) + B_{t-1} + T_t + \Upsilon_t$$

(2)

where $P_t$ is the nominal price level, $B_t$ are holdings of one-period nominal bonds, $i_t$ is the nominal interest rate, $\tau_t$ is a proportional labor tax, $W_t$ is the nominal wage, $T_t$ are lump sum government transfers or taxes and $\Upsilon_t$ are firm profits. Utility maximization yields the first order necessary conditions:

$$\theta l_t^{-\kappa} c_t^\sigma = (1 - \tau_t) \frac{W_t}{P_t},$$

(3)

$$c_t^{-\sigma} = \beta (1 + i_t) \mathbb{E}_t \left[ \frac{\omega_{t+1} P_t}{\omega_t P_{t+1}} c_{t+1}^{-\sigma} \right].$$

(4)

Condition (3) determines labor supply by equating the marginal rate of substitution between leisure and consumption with the after tax real wage. Condition (4) determines consumption and saving decisions by equating the intertemporal marginal rate of substitution of consumption with the real interest rate. Finally, optimality requires the transversality condition $\lim_{s \to \infty} \mathbb{E}_t \left[ \frac{B_{t+s}}{(1 + i_t) \ldots (1 + i_{t+s})} \right] = 0$.

**Firms** A competitive sector of firms produces the final good $y_t$ using a continuum of intermediate inputs according to

$$y_t = \left( \int_0^1 y_{it}^{1-1/\eta} di \right)^{1/(1-1/\eta)},$$

(5)

5
where $\eta > 1$ is the elasticity of substitution between intermediate inputs $y_{it}$. Letting $P_{it}$ denote the price of $y_{it}$, cost minimization implies

$$
y_{it} = \left( \frac{P_{it}}{P_t} \right)^{-\eta} y_t , \quad (6)
$$

$$
P_t = \left( \int_0 P_{it}^{1-\eta} di \right)^{1/(1-\eta)} . \quad (7)
$$

Each intermediate input $y_{it}$ is produced by a monopolist using labor input $n_{it}$ according to

$$
y_{it} = n_{it} . \quad (8)
$$

Monopolists set prices taking into account the demand functions in (6). However, pricing decisions are constrained in the standard Calvo fashion: Any given period, an individual monopolist can adjust its price $P_{it}$ only with probability $0 < (1 - \xi) \leq 1$. The parameter $\xi$ indexes the degree of nominal rigidities with $\xi = 0$ corresponding to the case of perfect price flexibility. If it can adjust in period $t$, the monopolist chooses a new price $P_{it}^*$ that maximizes expected profits made at that price, given by

$$
\mathbb{E}_t \sum_{s=0}^{\infty} \xi^s Q_{t,t+s} \left( P_{it}^* - \left( 1 - \frac{1}{\eta} \right) W_{t+s} \right) \left( \frac{P_{it}}{P_{t+s}} \right)^{-\eta} y_{t+s} . \quad (9)
$$

The monopolist takes as given $Q_{t,t+s} = \beta^s (\omega_{t+s}/\omega_t) (P_t/P_{t+s}) (c_{t+s}/c_t)^{-\sigma}$, which is the period $t$ value to its owner, the representative household, of a profit made in period $t + s$. Firm profits in (9) incorporate a proportional labor subsidy $W_t n_{it}/\eta$ from the government. This corrective subsidy ensures that an efficient steady state exists that will act as a useful welfare benchmark in the analysis below.\textsuperscript{2} The first order necessary condition for the optimal reset price $P_{it}^*$ is:

$$
\mathbb{E}_t \sum_{s=0}^{\infty} \xi^s Q_{t,t+s} \left( P_{it}^* - W_{t+s} y_{it+s} \right) = 0 . \quad (10)
$$

\textsuperscript{2}We consider a cashless limit economy and ignore any real-balance effects.
Because the price setters’ decision problems are identical, we consider symmetric equilibria where $P_t^* = P_{it}^*$. Applying the law of large numbers to (7) then yields $P_t = \left( \xi P_{t-1}^{1-\eta} + (1 - \xi)(P_t^*)^{1-\eta} \right)^{1/\eta}$, such that newly adjusted relative prices $p_t^* \equiv P_t^*/P_t$ determine overall inflation $\pi_t \equiv P_t/P_{t-1}$ through

$$\xi \pi_t^{\eta} + (1 - \xi)(p_t^*)^{1-\eta} = 1. \quad (11)$$

Aggregating across producers by substituting (6) and (8) into (5) and using (11) yields

$$y_t = n_t/v_t \quad (12)$$

$$v_t = \xi \pi_t^{\eta} v_{t-1} + (1 - \xi)(p_t^*)^{-\eta} \quad (13)$$

where $n_t = \int_0^1 n_i d \bar{d}$ is aggregate labor input and $v_t \geq 1$ is the degree of price dispersion across goods that arises because of the price setting friction. Since every monopolist has the same technology, efficiency requires all monopolists charging the same price in which case there is no price dispersion and $v_t = 1$. If there is price dispersion and $v_t > 1$, labor is inefficiently allocated across firms and there is a wedge between total output $y_t$ and total labor input $n_t$. Labor market clearing requires

$$n_t = 1 - l_t. \quad (14)$$

**Government Policies** The government is in charge of monetary and fiscal policies. Monetary policy is specified by a Taylor rule for the nominal interest rate:

$$1 + i_t = \max \left( \frac{\pi_t}{\bar{\pi}}, \left( \frac{\pi_t}{\bar{\pi}} \right)^\phi, 1 \right), \quad (15)$$

where $\bar{\pi} \geq 1$ is the inflation target and $\phi > \phi$ measures the responsiveness of the interest rate to inflation. The interest rate rule implies (a) that the gross real interest rate is $1/\beta$ when inflation stays on target $\pi_t = \bar{\pi}$; and (b) that the nominal interest rate cannot be negative such that the ZLB may be a
binding constraint. We also assume that $\phi$ exceeds the lower bound $\overline{\phi}$ to guarantee local determinacy in a neighborhood of the inflation target $\overline{\pi}$.³

Fiscal policy involves choices of taxes and government purchases $g_t$ with debt $B_t$ evolving as

$$\frac{B_t}{1 + i_t} = B_{t-1} + P_t g_t + T_t + \frac{W_t n_t}{\eta} - \tau_t W_t (1 - l_t)$$

(16)

Clearing in the market for the final good requires

$$y_t = c_t + g_t .$$

(17)

We will later make precise assumptions on labor tax rates $\tau_t$ and government purchases $g_t$ and postpone the details for now. We assume throughout that fiscal policies are Ricardian: Lump sum transfers always adjust such that the present value of government debt converges to zero for all equilibrium and off-equilibrium paths.

### 2.2 Equilibrium

A monopolistically competitive rational expectations equilibrium consists of an allocation $(c_t, n_t, l_t, y_t)_{t=0}^{\infty}$, non-negative prices $(W_t/P_t, p_t, p_t^*, v_t)_{t=0}^{\infty}$, and monetary and fiscal policies $(i_t, g_t, \tau_t, B_t/P_t)_{t=0}^{\infty}$ such that (i) households maximize utility, (ii) firms maximize profits, (iii) monetary policy follows the interest rate rule, (iv) fiscal policies are consistent with the government budget constraint, and (vi) goods, asset and labor markets clear, given initial conditions $B_{-1}$, and $v_{-1} \geq 1$ and processes for $\omega_t$, $g_t$ and $\tau_t$.

³With a price stability target $\overline{\pi} = 1$, the condition is $\overline{\phi} = 1$ such that interest rates must respond more than one-for-one. Coibion and Gordonichenko (2011) show more generally that $\phi$ depends on the inflation target and is larger than one when $\overline{\pi} > 1$. 

8
In what follows, we study equilibrium sequences of output $y_t$, inflation $\pi_t$, reset prices $p_t^*$ and price dispersion $v_t$ that solve a system of non-linear expectational difference equations implied by the market clearing and optimality conditions. This system is given by

$$1 = \beta \left[ \max \left( \frac{\pi_t}{\bar{\beta} (\pi_t)} \phi, 1 \right) \right] E_t \left[ \frac{1}{\pi_{t+1}} \frac{\omega_{t+1, t}}{\omega_t} \left( \frac{y_{t+1} - g_{t+1}}{y_t - g_t} \right)^{-\sigma} \right],$$

$$p_t^* \pi_t = \frac{E_t \sum_{s=0}^{\infty} (\beta \xi)^s \omega_{t+s} \theta (1 - y_{t+s}/y_{t+s})^{-\kappa} \left( \prod_{j=0}^{s} \pi_{t+j} \right)^{\eta} y_{t+s}}{E_t \sum_{s=0}^{\infty} (\beta \xi)^s \omega_{t+s} (y_{t+s} - g_{t+s})^{-\sigma} \left( \prod_{j=0}^{s} \pi_{t+j} \right)^{\eta-1} y_{t+s}},$$

$$1 = \xi \pi_t^{\eta - 1} + (1 - \xi) (p_t^*)^{1-\eta},$$

$$v_t = \xi \pi_t^{\eta} v_{t-1} + (1 - \xi) (p_t^*)^{-\eta}, \quad v_{-1} \text{ given},$$

as well as the processes for the preference shock $\omega_t$ and fiscal policies $g_t$ and $\tau_t$. Equation (18) combines the consumption Euler condition, the interest rate rule and the final goods market clearing condition. Equations (19)-(21) jointly result from optimal price setting and labor and goods market clearing conditions.

### 2.3 Permanent Liquidity Traps

One possible equilibrium outcome is that the economy ends up in a permanent liquidity trap. Such an outcome can explain why, despite pegging its policy rate near zero for almost two decades, the Japanese central bank so far has failed to prevent continuing declines in nominal prices. To see this most clearly, suppose (i) there are no discount factor shocks, $\omega_t = 1$; and (ii) that government purchases and labor taxes are zero in every period, i.e. $g_t = 0$ and $\tau_t = 0$. Absent any uncertainty, constant steady state consumption requires through the Euler condition that the real interest rate $(1 + i)/\pi$ equals $1/\beta$. Using the Taylor rule, inflation is then determined by

$$\max \left( \frac{(\pi/\bar{\pi})^{\phi}}{\beta/\bar{\pi}}, 1 \right) \frac{1}{\pi} = \frac{1}{\bar{\beta}}.$$
Because of the ZLB constraint, there are two possible solutions: In an intended steady state $I$, inflation is on target $\pi_I = \bar{\pi} \geq 1$, and the nominal interest rate is positive; in an unintended steady state $U$, there is deflation $\pi_U = \beta < 1$ and the nominal interest rate is zero. Steady state output $y$ depends on inflation $\pi$ through

$$\theta (1 - y/v)^{-\kappa}/y^{-\sigma} = \frac{1 - \xi \beta \pi \eta}{1 - \xi \beta \pi \eta - 1} \left( \frac{1 - \xi}{1 - \xi \pi \eta - 1} \right)^{1/(\eta - 1)}$$

(23)

where $v = (1 - \xi)^{1/(1 - \eta)} (1 - \xi \pi \eta - 1)^{-\eta/(1 - \eta)} / (1 - \xi \pi \eta)$ is the steady state level of price dispersion.

The dynamics around the intended steady state explain why an active interest rate rule and an objective of price stability are widely considered to be desirable features of monetary policy. An active rule responds sufficiently aggressively to inflation ($\phi > \phi_0$) and ensures locally unique convergence to the intended steady state. A price stability target, i.e. $\bar{\pi} = 1$, implies that all relative price distortions disappear as the economy approaches the intended steady state and $p_I^*, v_I \to 1$. When $\pi_I = 1$, output is determined by the efficiency condition $\theta (1 - y_I)^{-\kappa}/y_I^{-\sigma} = 1$. But there also exist infinitely many perfect foresight equilibria that converge to the unintended steady state, see Benhabib et al. (2001a,b). In these equilibria, the economy ends up in a perpetual liquidity trap with deflation and zero nominal interest rates. In the presence of nominal rigidities, price dispersion never dissipates $v_U > 1$ and since $\theta (1 - y_U/v_U)^{-\kappa}/y_U^{-\sigma} < 1$ these outcomes are necessarily inefficient.

### 2.4 Confidence Shocks and Temporary Liquidity Traps

An important question is whether the recent ZLB episodes in the US and other countries should be viewed as part of fully anticipated trajectories towards a liquidity trap steady state. Based on observed inflation, output and interest rates since 1983, Aruoba and Schorfheide (2013) conclude that this is very unlikely for the US. The ZLB became binding after the financial crisis, a macroeconomic shock that almost nobody anticipated. Moreover, Gust et al. (2012) document that market expecta-
tions have been in line with a prolonged spell of near zero interest rates, but also with interest rates rising eventually in the long run.

The model permits equilibria featuring temporary and imperfectly anticipated liquidity traps that are not only more promising empirically, but also better justify the use of short run stabilization policies. For now we maintain the assumption that there are no discount factor or fiscal policy shocks. Although there is therefore no uncertainty regarding the economy’s fundamentals, there still exist equilibria in which outcomes are stochastic and depend on non-fundamental random variables, called sunspots.\footnote{See Shell (1977), Cass and Shell (1983). Benhabib and Farmer (2000) survey sunspots in macroeconomic models.} Denote such a sunspot by $\psi_t$. Defining $u_t \equiv [y_t, \pi_t, p^*_t]$, we are interested in equilibria for which the dynamics can be described by

$$u_t = f(v_t, \psi_t) \quad (24)$$

$$v_t = h(v_{t-1}, \psi_t), \quad v_{-1} \text{ given} \quad (25)$$

for a given stochastic process for $\psi_t$. The functions $f$ and $h$ generate equilibrium sequences if they solve (18)-(21). In a sunspot solution, agents rationally condition expectations on $\psi_t$ and output, inflation and the other variables fluctuate randomly.

Consider a sunspot that follows a two-state discrete Markov process, $\psi_t \in (\psi_O, \psi_P)$, with transition probabilities $\Pr(\psi_{t+1} = \psi_O | \psi_t = \psi_O) = 1$ and $\Pr(\psi_{t+1} = \psi_P | \psi_t = \psi_P) = q_\psi < 1$. The first state $\psi_O$ is absorbing, i.e. it is impossible to leave this state. Suppose $f(v_t, \psi_O)$ and $h(v_t, \psi_O)$ describe the unique equilibrium path to the intended steady state such that in the long run $u_t$ and $v_t$ converge almost surely to $v_I$ and $u_I \equiv [y_I, \pi_I, p^*_I]$. Since this outcome is desirable in terms of welfare, $\psi_O$ represent a state in which agents have ‘optimistic’ expectations. The economy starts in a second state $\psi_P$ with finite expected duration $1/(1 - q_\psi)$ in which agents instead have ‘pessimistic’ expec-
tations. While in this state, the economy converges to an inferior outcome, \( v_P \equiv \lim_{t \to \infty} h(v_t, \psi_P) \) and \( u_P \equiv [y_P, \pi_P, p^*_P] = f(v_P, \psi_P) \), that solves

\[
y^{-\sigma} = \beta \left[ \max \left( \frac{\pi}{\beta}, \left(\frac{\pi}{\beta} \phi, 1\right) \right) \right] \left[ \frac{q_{\psi}}{\pi} y^{-\sigma} + \frac{1 - q_{\psi}}{\pi_{O}(v)} (y_{O}(v))^{-\sigma} \right], \tag{26}
\]

\[
p^*_P = \frac{(1 - q_{\psi})^{\pi_{O}(v)-1}}{(1 - q_{\psi})^{\pi_{O}(v)}} \left( \Lambda(v) \theta (1 - yv)^{-\sigma} + (1 - \Lambda(v)) \pi_{O}(v) p^*_O(v) \right), \tag{27}
\]

\[
v = (1 - \xi)^{1/(1-\eta)} (1 - \xi \pi_{O}(v))^{\eta/(1-\eta)} / (1 - \xi \pi), \tag{28}
\]

\[
p^* = (1 - \xi)^{-1/(1-\eta)} (1 - \xi \pi_{O}(v))^{1/(1-\eta)} , \tag{29}
\]

where \( v_O(v) \equiv h(v, \psi_O) \), \( u_O \equiv [y_O(v), \pi_O(v), p^*_O(v)] = f(v, \psi_O) \) and \( \Lambda(v) \) are functions of \( v \). To see why the ZLB permits short run dynamics driven by pessimism despite long run convergence to the intended outcome, Figure 1 depicts the two key relationships that determine output and inflation while the \( \psi_P \) state lasts.\(^5\) The AS-schedule, obtained from (27) after appropriate substitutions, represents supply and price setting decisions and is positively sloped. The EE-schedule, based on (26) after substitutions, represents intertemporal consumption decisions as well as the interest rate rule and is kinked because of the ZLB constraint: With inflation or moderate deflation, the nominal interest rate is positive and sufficiently responsive such that more inflation leads to higher real interest rates and therefore lower consumption in the short run. With stronger deflation, the ZLB binds and higher inflation lowers real interest rates which encourages consumption in the short run. Because of this non-monotonicity, there are two \( AS/EE \) intersections. The solution where output and inflation converge to the first intersection to the northeast \((y_I, \pi_I)\) corresponds to the intended deterministic equilibrium. In the sunspot solution, output and inflation instead converge to the second intersection to the southwest \((y_P, \pi_P)\): The ZLB binds and there is deflation \( \pi_P < \beta < \pi_I \) and lower economic activity \( y_P < y_I \). For \( q_{\psi} \to 1 \) the liquidity trap becomes permanent and \((y_P, \pi_P) \to (y_U, \pi_U)\).

\(^5\)Figure 1 is based on numerical evaluations using the parameter values discussed in Section 3. The functions \( v_O(v), u_O(v) \) and \( \Lambda(v) \) are not known analytically, but are approximated numerically, see also Section 3. The AS and EE schedules result from evaluating these functions as well as using (28) and (29) to substitute out \( v \) and \( p^* \).
By causing shifts from pessimism to optimism, sunspots can be viewed as exogenous shocks to confidence. Agents’ pessimism about income levels in the immediate future lead to lower desired consumption. Nominal rigidities imply that firms respond by lowering production as well as prices. If the confidence shock is sufficiently severe and monetary policy becomes constrained by the ZLB, falling prices result in a higher real interest rate which further reduces desired consumption. This in turn requires even stronger price declines, higher real interest rates that induce more saving, etc. Akin to the paradox of thrift, in equilibrium this vicious spiral ends only if income falls sufficiently to eliminate excess savings, which in turn validates the agents’ pessimism.\(^6\) The end result is a temporary liquidity trap with depressed economic activity.

2.5 Confidence versus Discount Factor Shocks

Temporary liquidity trap recessions can also occur because of shocks to the economy’s fundamentals. One short run deflationary shock that may cause the ZLB to bind is \(\omega_t\), which affects households’ time preferences.\(^7\) Consider equilibria characterized by

\[
\begin{align*}
  u_t &= f(v_t, \omega_t) \\
  v_t &= h(v_{t-1}, \omega_t), \; v_{-1} \text{ given}
\end{align*}
\]

where the functions \(f\) and \(h\) again solve (18)-(21), now for stochastic \(\omega_t\). For clarity, we now exclude all sunspots and focus on equilibria with the discount factor shock as the only source of random fluctuations. Suppose that \(\omega_t\) is generated by a two-state discrete Markov chain, \(\omega_t \in \{1, \omega\}\) where

\(^6\)A paradox of thrift occurs when higher desired savings lower aggregate demand and economic activity such that actual savings decrease in equilibrium. In our model, aggregate savings are not affected but the increase in desired savings leads to lower output.

\(^7\)Discount factor shocks often have similar implications as shocks to credit spreads or household borrowing constraints in more complicated models, e.g. Cúrdia and Woodford (2010).
$\omega < 1$, with transition probabilities $Pr(\omega_{t+1} = 1 | \omega_t = 1) = 1$ and $Pr(\omega_{t+1} = \omega | \omega_t = \omega) = q_{\omega} < 1$. State $\omega_t = 1$ is absorbing and the economy converges to the intended steady state in the long run. The economy starts in a state $\omega_t = \omega$ with expected duration $1/(1 - q_{\omega})$. For as long as $\omega_t = \omega$, the economy converges to a point, $v_L \equiv \lim_{t \to \infty} h(v_t, \omega)$ and $u_L \equiv [y_L, \pi_L, p^*_L] = f(v_L, \omega)$, that solves (28)-(29) as well as

$$y^{-\sigma} = \beta \left[ \max \left( \frac{\pi}{\beta} \left( \frac{\pi}{\pi^*} \right)^{\phi}, 1 \right) \right] \left[ \frac{q_{\omega}}{\pi} y^{-\sigma} + \frac{1 - q_{\omega}}{\omega} \pi_H(v)^{-\sigma} \right], \quad (32)$$

$$p^* = \frac{(1 - q_{\omega}) \beta \xi \pi^*}{(1 - q_{\omega}) \beta \xi \pi^*} \left( \Gamma(v) \theta (1 - \nu)^{-\xi} y^\sigma + (1 - \Gamma(v)) \pi_H(v) p^*_H(v) \right), \quad (33)$$

where $v_H(v) \equiv h(v, 1)$, $u_H \equiv [y_H(v), \pi_H(v), p^*_H(v)] = f(v, 1)$ and $\Gamma(v)$ are functions of $v$. After substitutions, these equations again provide AS/EE-schedules linking output and inflation, which are shown in Figure 2.\(^8\) There is a single intersection $(\pi_L, y_L)$ at which the ZLB binds, there is deflation and economic activity is depressed. Intuitively, for as long as $\omega_t = \omega < 1$, utility from current consumption is low relative to the expected utility of future consumption. In the face of temporary low demand for goods, firms decrease production as well as prices. If the shock to intertemporal preferences is large enough to constrain monetary policy at the ZLB, falling prices result in higher real interest rates, further reductions in consumption, even stronger price declines, higher real interest rates, etc. In equilibrium income must fall to eliminate excess savings and because of nominal rigidities the preference shock causes an inefficient liquidity trap recession.

Figures 1 and 2 reveal a key difference between confidence and discount factor shocks. In order to preserve a determinate outcome in which a fundamental shock causes the ZLB to bind, the Taylor rule must be expected to become active in the not too distant future. The discount factor shock can therefore not be too persistent. For an expectations driven liquidity trap to be possible, the Taylor

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\(^8\)Figure 2 is based on numerical evaluations, see in Section 3.
rule must be expected to be at the ZLB for a sufficiently long time and the state of low confidence instead must be relatively persistent. Graphically, an expectations driven liquidity trap \((y_p, \pi_p)\) only exists if the \(AS\) curve has a greater slope than the \(EE\) curve in the binding ZLB region. A fundamentals driven liquidity trap \((y_L, \pi_L)\) only exists under the exact opposite condition. Fixing other parameters, the key determinant of the slopes is the expected duration of the short run ZLB state. For longer durations (higher \(q_\psi\) or \(q_\omega\)), the \(AS\) schedule is steeper because price setters are more willing to adjust prices in response to weak demand as marginal costs are expected to remain low for a longer time. A longer expected spell of deflation and high short term real interest rates flattens the \(EE\) curve because deflation more strongly rewards delaying consumption.

3 A Quantitative Evaluation of Fiscal Interventions

We now turn to a quantitative evaluation of temporary government spending increases and labor tax cuts in a liquidity trap.

3.1 Calibration and Numerical Solution

One period corresponds to a quarter. We assume an annual real interest rate of 4 percent in the long run and set \(\beta = 0.99\). The intertemporal elasticity of substitution in consumption is \(1/\sigma = 1\), a value solidly in the range estimated in the consumption literature. \(\theta\) targets an average of one third of available time spent in the labor market. We set \(\kappa = 2.65\) to yield a Frisch labor supply elasticity of 0.75, which is in the range considered realistic by labor economists. The elasticity of substitution between intermediary inputs is \(\eta = 10\), which implies a markup of 11 percent in the long run. The degree of nominal rigidities is \(\xi = 0.65\), which implies that prices are adjusted approximately every three quarters, a value consistent with the empirical evidence of Nakamura and Steinsson (2008). For the interest rate rule, we assume a price stability target, \(\bar{\pi} = 1\), as well as \(\varphi = 1.5\), a conventional value that satisfies the Taylor principle for local determinacy.
We maintain the same stochastic processes for the shocks as in Sections 2.4 and 2.5, which require values for \(q_\psi\) and \(q_\omega\). For the case of a confidence driven liquidity trap, we set \(q_\psi = 0.70\) such that the expected duration is between 3 and 4 quarters. This in line with Gust et al. (2012), who document average expectations of 3 to 4 quarters of near zero short term interest rates in 2009Q1 and 2010Q2 based on Eurodollar contracts as well as professional forecasts. For the liquidity trap caused by the discount factor shock, the model forces us to set a lower value of \(q_\omega = 0.4\), implying an expected duration of under two quarters. As explained earlier, this lower value is needed to ensure existence of an equilibrium with a binding ZLB driven only by fundamentals.\(^9\) We set \(\omega = 0.9727\) such that the initial drop in output is the same for both types of liquidity traps. Finally, we assume that in the long run the tax rate is \(\tau = 0.20\) and that spending \(g\) is 20% of output, values that are similar to US postwar averages. The short run changes in the fiscal instruments are detailed later.

The equilibrium paths are obtained from numerical approximations of the functions in (24)-(25) for the case of a confidence shock, and in (30)-(31) for the case of a discount factor shock.\(^10\) The approximation in each case is by a piecewise linear function on a grid for \(v\) of 100 points for each of the two discrete states. Solutions are obtained by time iteration of a recursive formulation of the equilibrium system in (23)-(25). As discussed above, in the case of (24)-(25) there are two solutions given the stochastic process for \(\psi_t\): One describes the sunspot solution, the other describes the intended equilibrium. For an appropriate initial guess of the solution functions, the time iteration algorithm converges to the sunspot solution.

\(^9\)To ensure existence of an equilibrium with a binding ZLB, values for \(q_\omega\) higher than 0.5 are not possible unless we make substantial changes to other parameter values. For instance, increasing the degree of nominal rigidities \(\xi\) flattens the \(\text{AS}\) schedule and therefore permits higher values for \(q_\omega\).

\(^10\)Many papers in the literature instead rely on log-linear approximations. Notable exceptions include Wolman (1998), Judd, Maliar and Maliar (2012), Fernandez-Villaverde et al. (2012), Braun et al. (2012), Aruoba and Schorfheide (2013). Several of these papers document the dangers of relying on linear approximations to study liquidity trap dynamics.
3.2 Dynamics with Constant Fiscal Policies

We first establish the baseline scenarios in which tax rates and spending remain constant. As the initial condition for the dispersion of prices we set \( v_{-1} = 1 \). Time starts in period 0 and the economy remains at the ZLB until a stochastic date \( T \) at which the economy exogenously switches to a path towards the intended deterministic steady state.

The blue lines in Figure 3 depict the dynamics of output, price dispersion and the annualized levels of inflation and the nominal interest rate in an expectations driven liquidity trap. At time 0 pessimism prevails and the economy settles on a short run path towards \( (u_P, v_P) \). Output is about 1.6% below the long run level, the nominal interest rate is zero, prices fall by almost 9% at annual rates and become dispersed. As the economy approaches \( (u_P, v_P) \), output and inflation recover modestly. At date \( T \), equal to five in the figure, agents become optimistic and there is an immediate recovery: Output jumps close to long run levels, deflation turns to inflation, the nominal interest becomes positive and prices gradually become less dispersed. In the long run, the economy converges to the intended steady state.

The blue lines in Figure in 4 show the dynamics for a liquidity trap driven by the discount factor shock. At time 0, household preferences for future consumption are relatively high and the economy settles on a short run path towards \( (u_L, v_L) \). Because of our choice of \( \omega \), output is again about 1.6% below the long run level. The nominal interest rate is zero, prices fall by more than 5% and become dispersed. At date \( T \) agents’ intertemporal preferences change exogenously and there is an immediate recovery with gradual convergence to the intended steady state.

The dynamics of the key macroeconomic variables in the two types of liquidity traps are very similar: Real interest rates are high, economic activity is depressed and there is strong deflation. Because
of the longer expected duration, prices fall more in an expectations driven liquidity trap. By reducing the difference between \( g_\psi \) and \( q_\omega \), the model can generate near-observationally equivalent outcomes for nominal interest rates, inflation and output. In practice it may therefore be difficult to distinguish a confidence driven from a fundamental liquidity trap based only on inflation and output data.

### 3.3 The Effect of Spending Stimulus

We first examine the effect of an increase in government spending by assuming that, for as long as the ZLB is binding, \( g_t \) is temporarily increased from 20\% to 22\% of the long run output level. We also assume that increased spending does not affect the duration of the low confidence state, i.e. that \( q_\psi \) is invariant to the policy intervention. The red lines in Figures 3 and 4 depict the equilibrium paths with increased spending in a liquidity trap.

In an expectations driven liquidity trap (Figure 3), higher government spending leads only to a modest increase in economic activity and output remains more than 1.5\% below the long run level, compared to 1.6\% when \( g_t \) remains constant. Higher spending also leads to more deflation and price dispersion as well as higher real interest rates. In contrast, an increase in government spending of the same size in a fundamental liquidity trap (Figure 4) significantly mitigates the downturn. Output is now only 1\% below its long run level, and the spending stimulus instead reduces deflation and real interest rates. Even though without fiscal policy intervention both shocks yield very similar outcomes, the effects of the policy intervention differ significantly: A spending increase has significant expansionary effects if the ZLB is caused by a discount factor shock, but only minor expansionary effects if the ZLB is caused by low confidence.

The different effects of changes in government spending at the ZLB is not simply a feature of our calibration. Figure 5 reports marginal government spending multipliers for a wide range of parameter values. These marginal multipliers measure the change in output that results from an infinitesimal
increase in government spending, as a ratio of that change. Letting \((y_t)_{t=0}^\infty\) denote the output path with constant spending \(g_t = g\), and \((y_t(\delta))_{t=0}^\infty\) the path when \(g_t = g + \delta\) for as long as the ZLB binds, the marginal spending multiplier is

\[
\lim_{\delta \to 0} \frac{y_t(\delta) - y_t}{\delta}.
\]

(34)

As transitional dynamics are relatively unimportant, the blue lines in Figure 5 report only the multipliers in a neighborhood of the points to which the economy converges while the ZLB is binding, i.e. \((v_p, u_p)\) for a confidence shock and \((v_L, u_L)\) for a discount factor shock. The parameter range is determined by the requirement that there exists an equilibrium with a binding ZLB.\(^{11}\) For comparison, Figure 5 also depicts in red the ‘standard’ multipliers in a neighborhood of the long run steady state \((v_I, u_I)\), where nominal interest rates are positive. In this case there is no confidence (discount factor) shock, but \(g_t \in (g, g + \delta)\) is a two-state discrete Markov chain with the same transition probabilities as the confidence (discount factor) shock.

The right column in Figure 5 shows that government spending always becomes less effective as a stabilization tool in a liquidity trap recession driven by pessimism. The output effects of spending increases in an expectations driven liquidity trap are always smaller than those of equally persistent increases in an environment with positive nominal interest rates. For the benchmark parameter values, the ZLB multiplier is less than 0.2 compared to about 0.6 outside of the ZLB. Except for confidence shocks with very short expected durations (low \(q_\psi\)), government spending remains expansionary at the ZLB. However, the lower multipliers mean that larger spending increases are required to generate a significant impact on economic activity when nominal interest rates are zero. The left column in Figure 5 shows that government spending instead becomes more effective in a fundamental liquidity trap. The ZLB spending multipliers are in this case always larger than those associated with equally persistent spending changes away from the ZLB. In the benchmark calibration, the ZLB multiplier under a discount factor shock is around 1.5, which is similar to the assumptions made by

\(^{11}\)To expand the feasible range in the case of a discount factor shock, we lowered \(\omega\) to 0.965 in Figures 5 and 6.
Romer and Bernstein (2009) and more than twice as large as the multiplier when interest rates are positive. For some parameter values the ZLB multipliers exceed two or more, which means that relatively small expansions in government purchases can have very sizeable effects on economic activity. That spending multipliers can be large when the ZLB binds after a discount factor shock was shown before by Eggertson (2011) and Christiano et al. (2011).

The output effect of spending changes depends critically on the inflationary effects of the fiscal intervention as well as the monetary policy response. When nominal interest rates are positive, monetary policy responds to the inflationary effects of a fiscal stimulus by raising nominal interest rates more than proportionally in accordance with the Taylor rule. The resulting increase in real interest rates crowds out private consumption and multipliers are usually less than one, as is the case in Figure 5.12 In the case of a fundamental liquidity trap, increased spending is always inflationary and since the nominal interest rate remains at zero, real interest rates decrease and private consumption is crowded in. Multipliers are lower when the ZLB is caused by the confidence shock because increased spending instead is deflationary in equilibrium. Spending stimulus at the ZLB then leads to higher real interest rates and crowds out private consumption.

To better understand why the policy effects are so different, Figure 7 shows the impact of increased spending on the AS/EE curves that determine where the economy converges to in the short run. The curves are based on numerical evaluations under the benchmark calibration. For an expectations driven liquidity trap (left panel), these curves are now defined by (28)-(29) and

\[
(y - g - \delta)^{-\sigma} = \beta \left[ \max \left( \frac{\pi}{\beta} \left( \frac{\pi}{\pi} \right)^\varphi, 1 \right) \right] \left[ \frac{q_y}{\pi}(y - g - \delta)^{-\sigma} + \frac{1 - q_y}{\pi_o(v)}(y_o(v) - g)^{-\sigma} \right],
\]

\[
p^* = \frac{(1 - q_y \beta^\xi^\eta^{-1})}{(1 - q_y \beta^\xi^\eta)} \left( \Lambda(v) \frac{\theta}{1 - \tau} (1 - vy)^{-k} (y - g - \delta)^{\sigma} + (1 - \Lambda(v)) \pi_o(v)p_o^*(v) \right). \tag{36}
\]

---

12See Woodford (2011) for a careful analysis of the determinants of the spending multiplier in models with nominal rigidities.
where $\delta$ is the short run increase in spending and $g$ and $\tau$ are the long run spending and tax levels. For the fundamental liquidity trap (right panel), the curves are similarly defined by (28)-(29) and

\[
(y - g - \delta)^{-\sigma} = \beta \left[ \max \left( \frac{\pi}{\bar{\pi}}, 1 \right) \right] \left[ \frac{q_0}{\pi} (y - g - \delta)^{-\sigma} + \frac{1 - q_0}{\bar{\pi}} (y_H(v) - g)^{-\sigma} \right], \tag{37}
\]

\[
p^* = \frac{(1 - q_0 \beta^2 \pi^{-1})}{(1 - q_0 \beta^2 \pi^{-1})} \left( \Gamma(v) \frac{\theta}{1 - \tau} (1 - vv)^{-\kappa} (y - g - \delta)^{\sigma} + (1 - \Gamma(v)) \pi_H(v) p_H^*(v) \right). \tag{38}
\]

The full lines in Figure 7 are the curves defining $(y_P, \pi_P)$ and $(y_L, \pi_L)$ for the case where there is no short run increase in spending, $\delta = 0$. The curves with broken lines define the new convergence points $(y'_P, \pi'_P)$ and $(y'_L, \pi'_L)$ when $\delta > 0$ and spending is increased in the short run. The stimulus is identical in both panels and equals 2% of the long run output level.\(^\text{13}\)

Higher government spending implies that total demand for final goods is higher and the EE curve shifts to the right. Higher government spending also has a positive wealth effect on labor supply which shifts the AS curve to the right. As explained earlier, in a confidence driven liquidity trap (left panel) the EE curve is positively sloped but less steep than the AS curve: The relatively high expected duration of the state of low confidence means that price setters are more inclined to respond to increased public demand by adjusting prices. But the high expected duration also means that lower deflation strongly encourages private consumption. As a result, excess demand for the final good is increasing in inflation. Absent the wealth effect on labor supply, an increase in public demand leads to a drop in inflation which increases real interest rates and discourages private consumption. The positive wealth effect on labor supply is what ultimately makes the stimulus expansionary, but the deflationary effect of the demand stimulus in equilibrium limits the output expansion. In a fundamental liquidity trap (right panel), excess demand instead depends negatively on inflation because the EE curve is steeper than the AS curve. Since an increase in public spending leads to higher inflation and crowds in private consumption, demand policies can in this case become very expan-

\(^{13}\)The transitional dynamics associated with this policy experiment are shown in Figures 3 and 4.
sionary. Thus, the size of the spending multiplier at the ZLB depends on whether additional public sector spending is inflationary or deflationary, which in turn depends on the expected duration and the source of the liquidity trap.

3.4 The Effect of A Tax Cut

Next we examine the output effects of a temporary cut in the marginal labor tax rate. We now assume that government spending remains constant but that the labor tax \( \tau_t \) is temporarily lowered from 0.20 to 0.19 for as long as the ZLB is binding. As before, by assumption the policy change does not affect the duration of the state of low confidence. The black lines in Figures 3 and 4 depict the equilibrium paths when the tax rate is cut by one percentage point in a liquidity trap.

In an expectations driven liquidity trap (Figure 3), the tax cut mitigates the downturn considerably and the output drop is limited to about 0.9%, compared to 1.6% when the tax rate is constant. The tax stimulus also reduces deflation and real interest rates. In sharp contrast, an identical decrease in the labor tax rate in a fundamental liquidity trap (Figure 4) worsens the recession. Output is now more than 2% below its long run level and the temporary reduction in the tax rate leads to more deflation and higher real interest rates. Again, there is a large difference in policy outcomes: Tax cuts have very large expansionary effects if the ZLB is caused by a confidence shock, whereas the same tax cut is contractionary if the liquidity trap is caused by a discount factor shock.

These differences in the effects of tax changes at the ZLB are also robust. Figure 6 reports, for a wide range of parameters, marginal tax multipliers measuring the percentage change in output that results from an infinitesimal decrease in tax rate, as a ratio of the change. If \((y_t)_{t=0}^{\infty}\) is the output path with constant tax rates \( \tau_t = \tau \), and \((y_t(\delta))_{t=0}^{\infty}\) is the output path when \( \tau_t = \tau - \delta \) for as long as
the ZLB binds, then the marginal tax multiplier is computed as

$$\lim_{\delta \to 0} \frac{y_t(\delta) - y_t}{y_t \delta}.$$  \hspace{1cm} (39)

Blue lines in Figure 6 are the multipliers in a neighborhood of the short run convergence points and red lines are the ’standard’ multipliers in a neighborhood of the long run steady state. In the latter case, $\tau_t \in (\tau, \tau - \delta)$ is a two-state discrete Markov chain with the same transition probabilities as the confidence (resp. discount factor) shock.

The right column in Figure 6 shows that cutting taxes consistently becomes more effective as a stabilization tool in an expectations driven liquidity trap recession. The ZLB tax multipliers are always larger compared to an environment where nominal interest rates are positive. In the benchmark parameter calibration, the ZLB tax multiplier is about 0.8, more than twice as large as when the ZLB does not bind. The left column in Figure 6 shows that cutting taxes in a fundamental liquidity trap instead always becomes contractionary at the ZLB. For the benchmark calibration, the ZLB multiplier is -0.5 compared to around 0.3 otherwise. For some parameter values the ZLB tax multipliers are very negative. In a liquidity trap recession caused by a discount factor shock, stimulating economic activity therefore requires higher marginal tax rates, which was shown previously by Eggertson and Woodford (2004) and Eggertson (2011).

As with spending changes, the output effect of a tax change depends importantly on the inflationary effect and the monetary policy response. The increase in the after-tax real wage following a tax cut leads to an increase in labor supply and lowers the marginal cost of production. Price setters respond with lower prices. When nominal interest rates are positive, monetary policy accommodates the effects of the tax cut by reducing nominal interest rates in response to lower inflation. The real interest rate decreases, which crowds in private consumption and leads to an output expansion. When the
nominal interest rate remains at zero, the deflationary effect of the tax cut raises real interest rates and crowds out private consumption. Tax cuts in a confidence driven liquidity trap instead lead to output expansions because they are inflationary and encourage private consumption in equilibrium.

Figure 8 depicts the impact of a one percentage point labor tax cut on the $AS/EE$ curves, which are analogous to (35)-(38) but now with constant spending and allowing for short run changes in the tax rate.\textsuperscript{14} In both the expectations (left panel) and fundamental (right panel) liquidity trap, the reduction in the marginal labor tax rate leads to an outward shift of the $AS$ curve. Without a change in inflation, there is excess supply in the economy. In case of a confidence shock with high expected duration, price setters are more willing to lower prices to draw in additional demand but deflation strongly discourages consumption. Since excess supply is decreasing in inflation, cutting taxes has inflationary effects and is expansionary. In a fundamental liquidity trap, excess supply instead is increasing in inflation, tax cuts are deflationary in equilibrium and the tax multiplier is negative. Therefore, the sign of the tax multiplier at the ZLB depends on the shock that drives the economy into a liquidity trap.

### 3.5 Discussion

Our analysis shows that conclusions about the effectiveness of fiscal instruments at the ZLB depend generally on the nature of the liquidity trap and its expected duration. Because the ZLB constraint generates equilibrium multiplicity, the outcome of policy interventions can be very different in more persistent expectations driven liquidity traps driven by self-fulfilling expectations. Our choice of stochastic processes for the shocks makes the contrast between a fundamental and expectations driven liquidity trap particularly stark by imposing that the expected duration of the liquidity trap is unaffected by fiscal interventions. In practice, policy changes may also have direct effects on

\textsuperscript{14} Transitional dynamics resulting from this policy experiment are depicted in Figures 3 and 4.
consumer confidence and expectations. Equilibria selected by more complicated sunspot processes can capture these effects and generate spending and tax multipliers that are both smaller or larger. Nonetheless, confidence and fundamental shocks initially can lead to very similar output and inflation dynamics in a liquidity trap and designing the appropriate fiscal response is challenging without a deeper understanding of the nature of the underlying shocks.

Unfortunately, empirical evidence that helps discriminate between fundamental and non-fundamental liquidity traps so far is scarce. Almunia et al. (2010) find large multipliers associated with US defense spending in the 1930s. Ramey (2011) instead finds no evidence that the multiplier was larger during 1939-1949 when interest rates were near zero. Brückner and Taludhar (2011) exploit cross-regional differences in Japan to estimate the effectiveness of fiscal policies during the 1990s. They find that government spending did not have multiplier effects that are on average larger than one and that government personnel expenditures had significant negative output effects. Aruoba and Schorfheide (2013) estimate a medium-scale model using pre-2008 US data and, allowing for both discount factor and confidence shocks, extract the model-implied shocks during 2000-2012. Depending on the inflation measure used, they find that confidence shocks are relatively important to account for the Great Recession and can explain the limited expansionary effects of the American Recovery and Reinvestment Act. Wieland (2012) empirically rejects the prediction of the discount factor shock model that negative supply shocks, such as labor tax increases, are expansionary at the ZLB. Gust et al. (2012) argue that an estimated model can account reasonably well for US data up to 2011 by only allowing for fundamental shocks. However, by not allowing for confidence shocks, their model relies on an increasingly unlikely sequence of large fundamental shocks to explain the long duration of the ZLB episode in the US.
One possible objection to the possibility of expectations driven fluctuations is that their existence too strongly depends on the assumption of rational expectations. When there is equilibrium multiplicity and the impact of policy changes is uncertain, some have argued for dismissing equilibria that yield unstable dynamics under alternative assumptions about the formation of expectations, see for instance McCallum (2003). On these grounds, Christiano and Eichenbaum (2012) argue that the uncertainty surrounding the effects of fiscal interventions in a liquidity trap can perhaps be dismissed. In this section we discuss the stability of the confidence shock equilibria under learning and verify that our results on the effects spending and tax policies are robust to reasonable deviations from rational expectations.

4.1 A Linearized Model

To facilitate the analysis, we linearize the equilibrium conditions preserving only the ZLB non-linearity. Linearizing (18)-(21) around the intended steady state and assuming that $\sigma = 1, \bar{\pi} = 1$ and that there are no discount factor shocks, yields

$$\hat{y}_t - s_g \hat{g}_t = \hat{y}_{t+1} - s_g \hat{g}_{t+1} - (1 - s_g) \left( \beta \max \left( r + \frac{\hat{\pi}_t}{\beta}, 0 \right) - r \right) - \hat{\pi}_{t+1}^e, \quad (40)$$

$$\hat{\pi}_t = \frac{(1 - \beta \xi)(1 - \xi)}{\xi} \left( \frac{\kappa y}{1 - y_I} + \frac{1}{1 - s_g} \right) \hat{y}_t - \frac{s_g}{1 - s_g} \hat{g}_t + \frac{\tau - \tau}{1 - \tau} + \beta \hat{\pi}_{t+1}^e, \quad (41)$$

where hatted variables are in deviation of the intended steady state value, $r = 1/\beta - 1$ is the steady state real interest rate and $s_g = g/y_I$. The notation $x_{t+1}^e$ denotes the period $t$ expectation of $x_{t+1}$. The linearization eliminates all transitional dynamics and the sunspot equilibrium conditions in (26)-(29).
become

\[
\hat{y} = q_{\psi}(\hat{y} + \hat{\pi}) + \beta r - \beta \max \left( r + \frac{\phi \hat{\pi}}{\beta}, 0 \right) \tag{42}
\]

\[
\hat{\pi} = \rho \hat{y} + \beta q_{\psi} \hat{\pi} \tag{43}
\]

where \( \rho = \frac{(1 - \beta \xi)(1 - \xi)}{\xi} \left( \frac{\kappa_{yI}}{1 - \gamma_I} + 1 \right) > 0 \). For any \( q_{\psi} \in (q^*, 1] \) this system has a solution given by

\[
\hat{\pi}_P = -\frac{1 - \beta}{\Delta} < 0 \quad , \quad \hat{y}_P = -\frac{(1 - \beta q_{\psi})(1 - \beta)}{\rho \Delta} < 0 \tag{44}
\]

where \( \Delta = q_{\psi} - (1 - q_{\psi}) \frac{1 - \beta q_{\psi}}{\rho} < 1 \). For the ZLB to bind, it is required that \( 0 < \Delta < \phi \). Since \( \phi > 1 \) and \( \Delta < 1 \) the second inequality is redundant and the critical value \( q^* \) is the smallest root of \( q^* - (1 - q^*) \frac{1 - \beta q^*}{\rho} = 0 \). As in the non-linear model, \( (\hat{y}_P, \hat{\pi}_P) \) corresponds to an intersection of the now linear AS/EE schedules in (42) and (43) that lies to the southwest of the intended steady state.

The condition \( \Delta > 0 \) imposes a sufficiently high expected duration and is equivalent to the condition on the slopes of the AS/EE curves discussed before. Thus, the expectation driven liquidity traps in the non-linear model are also present in the linearized system.

### 4.2 Fiscal Multipliers with Adaptive Expectations

Now suppose that expectations are not strictly rational. More specifically, we assume that agents perfectly know the long run state but make small expectational errors in the short run. Output and inflation in the short run must be consistent with

\[
\hat{y}_t = q_{\psi}(\hat{y}_{t+1} + \hat{\pi}_{t+1}) + \beta r - \beta \max \left( r + \frac{\phi \hat{\pi}_t}{\beta}, 0 \right), \tag{45}
\]

\[
\hat{\pi}_t = \rho \hat{y}_t + \beta q_{\psi} \hat{\pi}_{t+1} \tag{46}
\]
where $x_{t+1}^{e|P}$ now denotes the expectation conditional on state $\psi_P$. Rational expectations solutions to systems as (45)-(46) are sometimes judged by their ‘learnability’. A common criterion in this regard is the expectational (E-) stability of the underlying system of expectational difference equations. This is because E-stability is closely related to dynamic stability under simple recursive learning schemes, such a least squares or (small) constant gain learning.\textsuperscript{15} For (45)-(46), the E-stability condition around $(\hat{y}_P, \hat{\pi}_P)$ is that the eigenvalues of

$$q_\psi \begin{bmatrix} 1 & 1 \\ \rho & \rho + \beta \end{bmatrix}$$

are less than one in absolute value. The largest eigenvalue is given by $q_\psi/q^*$ and since $q_\psi \in (q^*, 1]$, the sunspot equilibria are never E-stable. In contrast, the E-stability condition in a small neighborhood of the intended steady state is the Taylor principle: $\phi > 1$.\textsuperscript{16} The fact that the rational expectations solution $(\hat{y}_P, \hat{\pi}_P)$ is not E-stable means that under certain learning schemes, the slightest expectational errors imply that output and inflation diverge away from $(\hat{y}_P, \hat{\pi}_P)$. Depending on the errors made, output and inflation either converge towards the E-stable intended steady state or do not converge at all.

Christiano and Eichenbaum (2012) use E-stability results to argue that non-fundamental liquidity traps should perhaps be viewed as curiosities. However, as shown by Evans, Guse and Honkapohja (2008), the presence of the permanent liquidity trap rational expectations equilibria of Benhabib et al. (2001a,b) can have profound implications for output and inflation dynamics even with recursive learning. This is also true for the case of temporary liquidity traps. Consider the following constant

\textsuperscript{15}See Marcet and Sargent (1989) and Evans and Honkapohja (2001). E-stability has been used to evaluate policy rules, e.g. Bullard and Mitra (2002) and for selecting among multiple RE equilibria, e.g. McCallum (2003)

\textsuperscript{16}These findings are closely related to Bullard and Mitra (2002). Evans and Honkapohja (2005) and Evans, Guse and Honkapohja (2008) show that the permanent liquidity traps of Benhabib et al. (2001a,b) (i.e. the case where $q_\psi \to 1$) are not E-stable.
gains learning rules:

\[
\hat{\pi}^{cP}_{t+1} = \hat{\pi}^{cP}_t + \gamma (\hat{\pi}_{t-1} - \hat{\pi}^{cP}_t), \quad (48)
\]

\[
\hat{y}^{cP}_{t+1} = \hat{y}^{cP}_t + \gamma (\hat{y}_{t-1} - \hat{y}^{cP}_t), \quad (49)
\]

where \(0 < \gamma < 1\) is a gain parameter and \(\hat{\pi}_{-1}, \hat{y}_{-1}, \hat{\pi}^{cP}_0\) and \(\hat{y}^{cP}_0\) are given. In our setting, constant gains learning is identical to classical adaptive expectations. The dynamics under learning are given by a sequence of temporal equilibria determined by (45)-(46), the learning rules (48)-(49), and the following initializations:

\[
\hat{y}^{cP}_0 = \hat{y}_P(1 + \varepsilon_y), \quad \hat{\pi}^{cP}_0 = \hat{\pi}_P(1 + \varepsilon_\pi), \quad \hat{\pi}_{-1} = \hat{y}_{-1} = 0 \quad (50)
\]

where \(\varepsilon_\pi\) and \(\varepsilon_y\) are initial expectational errors.

Figure 9 illustrates the expectational dynamics under learning based on the benchmark calibration as well as setting \(\gamma = 0.10\).\(^{17}\) The figure also shows trajectories for three different initializations of expectations in the neighborhood of \((\hat{y}_P, \hat{\pi}_P)\) (blue circle). All three trajectories converge asymptotically to the intended steady state outcome (red circle). The left panel depicts the case with a ZLB constraint (blue when it is binding, red when not). For comparison, the right panel shows the dynamics for the same initial conditions but permitting negative nominal rates. The dynamics around \((\hat{y}_P, \hat{\pi}_P)\) are saddle path stable, whereas the dynamics around the intended steady state follow a stable spiral. The existence of \((\hat{y}_P, \hat{\pi}_P)\) due to the ZLB has important effects on the dynamics with adaptive expectations. Expectations converge only very slowly to the intended steady state and can initially even be attracted to \((\hat{y}_P, \hat{\pi}_P)\). In our calibration, it takes approximately 100 quarters before

\(^{17}\)The figure is based on the model that includes (constant) fiscal policies using the parameter values described in Section 3. Our value for \(\gamma\) is conservative as the learning literature typically assumes lower values, e.g. Evans et al. (2008) assume \(\gamma = 1/30\) while Eusepi and Preston (2011) calibrate \(\gamma = 0.002\) based on data from the Survey of Professional Forecasters.
learning dynamics lead to an endogenous exit from the ZLB. Given that the low confidence state has an expected duration of less than 4 quarters, the unstable transitional learning dynamics are not very relevant in the short run. With no ZLB but the same initializations, expectations spiral towards the intended steady state much faster. As Evans et al. (2008), we conclude that the potential for destabilizing expectational shocks remains present with learning dynamics.

We can also examine the impact of fiscal policies when agents have adaptive expectations. Figure 10 depicts standard and ZLB spending and tax multipliers both in case of rational and adaptive expectations for an example trajectory with $\varepsilon_y = \varepsilon_\pi = -0.03$. More precisely, we first fix initial expectation errors $(\varepsilon_y, \varepsilon_\pi)$, and compute the output path conditional on agents being pessimistic and for a constant level of government spending and taxes. Next, for the same initial expectation errors, we compute the output path when short run spending (tax rates) are marginally higher (lower). We then compute multipliers as in Section 3.

By construction, the rational expectations multipliers at the intended steady state in the linearized model are identical to those of the non-linear model (squares in the left columns in Figure 5 and 6). The multipliers in the rational expectations liquidity trap are quantitatively different from the non-linear model because of the linearization. But as in the non-linear model, the rational expectations spending multiplier is much smaller at the ZLB, whereas the tax multiplier is larger. The multipliers under adaptive expectations change over time because of transitional learning dynamics. Initially the multipliers are in a neighborhood of those that occur in the liquidity trap with rational expectations. As agents update expectations, the multipliers under learning diverge slowly from their rational expectations liquidity trap values and when nominal interest rates become positive (around the 100th quarter after the initial shock), the multipliers jump discretely to a neighborhood of their values at the intended steady state. In the long run the multipliers converge to the standard (intended steady state) rational expectations values.
Hence, our conclusions about fiscal multipliers hold true for local deviations of rational expectations: When the liquidity trap is generated by an expectational shock, spending multipliers are smaller than in normal times while the opposite is true for tax multipliers. The main new insight from the introduction of learning is that as long as pessimism persists the economy may eventually endogenously exit a liquidity trap induced by an expectational shock. However, the learning process is very slow. For time horizons that are relevant given the transitory nature of agents’ pessimism, our conclusions based on rational expectations remain intact.

5 Conclusions and Future Extensions

As the US and many European countries enter a fifth year of near zero short term nominal interest rates, the effects of the 2008 global financial crisis are proving to be very persistent. We have shown that when a liquidity trap is caused by a long-lasting state of low consumer confidence, government spending stimulus has deflationary effects and becomes relatively ineffective at the ZLB. In contrast, cuts in marginal labor tax rates are inflationary and generate much larger expansionary effects than when interest rates are positive. These results flat out contradict widely held views of the effects of fiscal policies in a liquidity trap as well as recent conclusions drawn from macroeconomic models with nominal rigidities. The key policy implication of our analysis is that in the current macroeconomic environment, the design of fiscal stimulus or austerity packages should be more considerate of the reasons for the continued economic weakness.

Our analysis can be extended in several ways. First, an important assumption we made is that the duration of the state of low confidence is not changed by the fiscal intervention. Policy changes may in practice have direct effects on consumer confidence that can affect fiscal multipliers. Bachmann and Sims (2012) provide some evidence for such confidence effects of government spending
in recessions and more complicated sunspot processes can capture these in the model. Second, our analysis can be repeated in larger models that capture more features of real economies. Several larger scale studies, e.g. Cogan et al. (2010), Erceg and Lindé (2010), Drautzburg and Uhlig (2011) and Coenen et al. (2012), discuss many factors that determine fiscal multipliers in a fundamental liquidity trap, such as the financing or timing of the fiscal stimulus. Similarly, Fernandez-Villaverde et al. (2012) show that long run supply stimuli remain expansionary in a fundamental liquidity trap. All of these factors are also relevant in an expectations driven liquidity trap. The potential for confidence driven liquidity traps remains present also in larger models with richer specifications for the interest rate rule or the confidence process, see for instance Schmitt-Grohé and Uribe (2012) and Aruoba and Schorfheide (2013). In Mertens and Ravn (2011a,b) we extend the model to incorporate housing and financial constraints.

Finally, our analysis can be extended to include additional or more sophisticated government policies. A growing literature considers alternative policy instruments or more generally optimal policy responses in a fundamental liquidity trap, e.g. Reifschneider and Williams (2000), Eggertsson and Woodford (2003), Coenen, Orphanides and Wieland (2004), Adam and Billi (2006) or Werning (2011). Another important question is which policies can eliminate unintended equilibrium outcomes. Benhabib et al. (2002) propose monetary and fiscal policies that violate the households’ transversality conditions along candidate deflationary equilibrium paths. Atkeson, Chari and Kehoe (2010) describe sophisticated monetary policies that implement the intended equilibrium uniquely in a linear model by switching to an appropriate monetary growth rule. Correia, Fahri, Nicolini and Teles (2012) demonstrate how distortionary taxes can be used to replicate the effects of negative nominal interest rates and circumvent the zero bound problem. Cochrane (2011) discusses and criticizes various proposals to eliminate indeterminacies. Schmitt-Grohé and Uribe (2012) have recently proposed interest rate policies that generate an exit from a confidence driven liquidity trap after it has occurred. We leave these and other extensions for future work.
References


Figure 1 A Temporary Liquidity Trap Driven by a Confidence Shock

Figure 2 A Temporary Liquidity Trap Driven by a Discount Factor Shock
Figure 3 Dynamics in a Liquidity Trap Driven by a Confidence Shock

Figure 4 Dynamics in a Liquidity Trap Driven by a Discount Factor Shock
Figure 5 Marginal Spending Multipliers. Boxes indicate values in the benchmark calibration.
Figure 6 Marginal Tax Multipliers. Boxes indicate values in the benchmark calibration.
Figure 7 A Spending Increase in a Liquidity Trap.

Figure 8 A Tax Cut in a Liquidity Trap.
Figure 9 Expectational Dynamics with Adaptive Expectations. Circles denote rational expectations equilibria. Blue indicates the ZLB is binding, red indicates the Taylor rule is active.

Figure 10 Fiscal Multipliers under Rational and Adaptive Expectations