# *MSC Macroeconomics G022, 2009*

## Lecture 5: (Real) Business Cycles

Morten O. Ravn

University College London

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# In this lecture

- Business cycles
- Empirics: “Stylized Facts”
- Theory: Dynamic Stochastic General Equilibrium Theory
- Theory vs. Data

# Business Cycles

So far in this course:

- Determination of long run income
- Determination of equilibrium growth rates

Now:

- Fluctuations in aggregate activity and its components at the business cycle frequencies
- Fluctuations in the economy with a periodicity typically between 2 and 10 years (although there are exceptions)
- To investigate this we will first look at "facts" and then at theory

# Business Cycle "Facts"

- What are business cycles?
- Here we will use the view that business cycles refer to **fluctuations in main macroeconomic aggregates around their trend**. This is a statistical definition. It does not *necessarily* relate directly to NBER definitions of business cycles or other definitions used more popularly. We will need to make precise what we mean by trend and this will correspond to a view upon the average length of business cycles.
- The NBER definition is "The NBER does not define a recession in terms of two consecutive quarters of decline in real GDP. Rather, a recession is a significant decline in economic activity spread across the economy, lasting more than a few months, normally visible in real GDP, real income, employment, industrial production, and wholesale-retail sales". This is a nice definition but not very applicable for practical purposes. Its main strength is in terms of communicating to the public the state of the economy

**Detrending: The Hodrick-Prescott filter**

Suppose we observe a time-series $\{y_t\}$ for the sample period $t = 1, .., T$. We wish to decompose this into a trend component and a "deviations from trend":

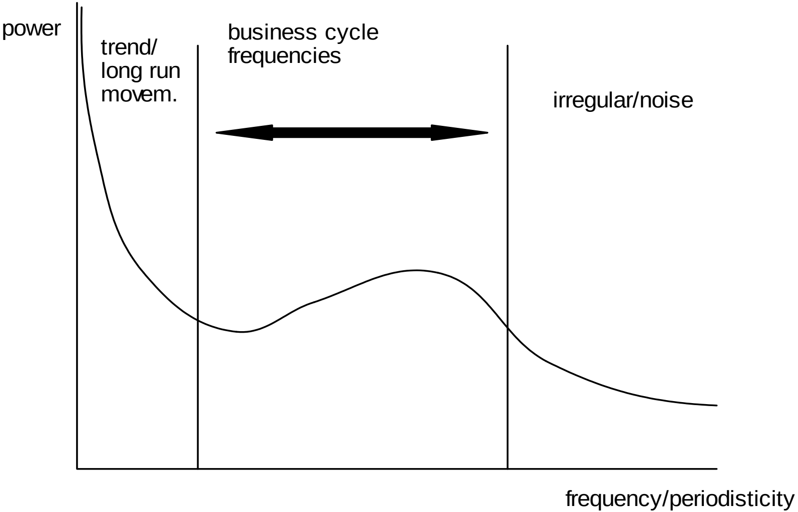
$$y_t = \tau_t + c_t + s_t + e_t$$

$\tau$ is the trend component, $c$ is the cyclical component, $s$ is a seasonal component, $e$ is an irregular component

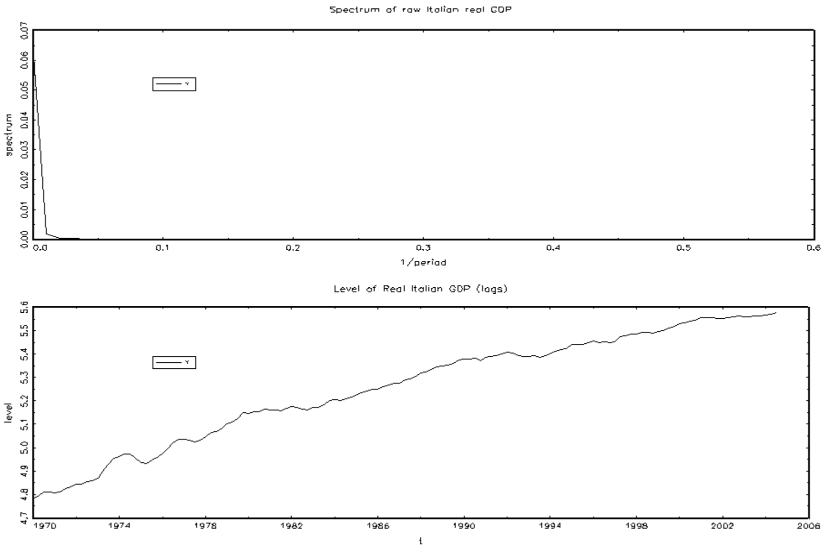
- If we study business cycles, we want to focus attention on the cyclical component - i.e. we want to remove the trend component and possibly seasonals and irregulars
- There are many ways of doing this
- In the business cycle literature it has become common to use the Hodrick-Prescott filter on seasonally adjusted data (i.e. it leaves in irregulars). Use 1600 for quarterly data, 6.25 for annual data.

# Measurement

What we have in mind may be something like the following picture:



# Measurement



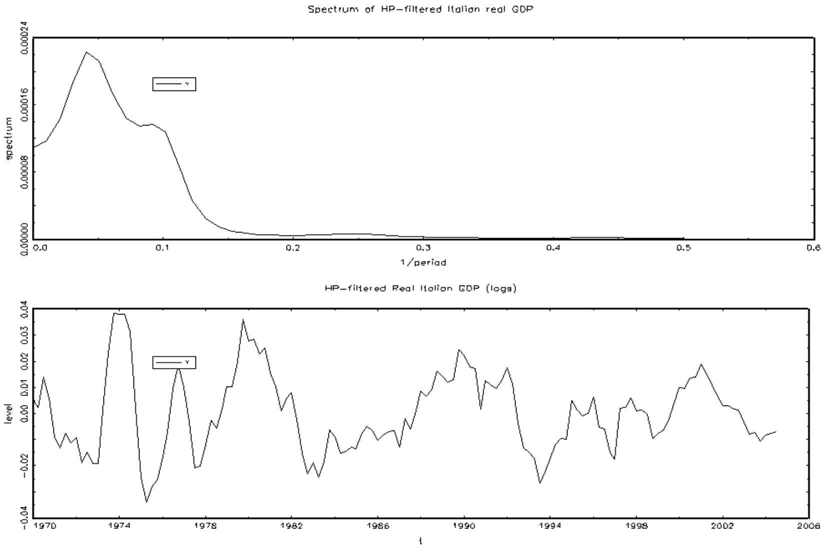
# The Hodrick-Prescott Filter

Specified as:

$$\min_{\{\tau_t\}_{t=0}^{T}} \quad \underbrace{\sum_{t=0}^{T} (y_t - \tau_t)^2}_{\text{"goodness of fit"}} \quad + \quad \lambda \underbrace{\sum_{t=1}^{T-1} \left[(\tau_{t+1} - \tau_t) - (\tau_t - \tau_{t-1})\right]^2}_{\text{squared acceleration of trend}} \tag{1}$$

- $\tau_t$ is the trend component of $y_t$
- $\lambda$: the smoothing parameter
- The smoothing parameter: Determines the trade-off between fit and variability of the trend
- $\lambda \rightarrow 0$ : trend component will become equal to the data series itself
- $\lambda \rightarrow \infty$ : trend component will become linear
- Intermediate values of $\lambda$ : Smooth but non-constant trend

# The Hodrick-Prescott Filter

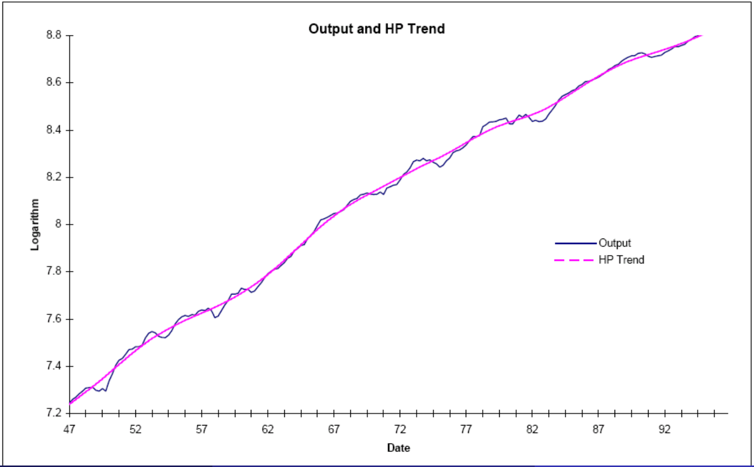


# Measurement

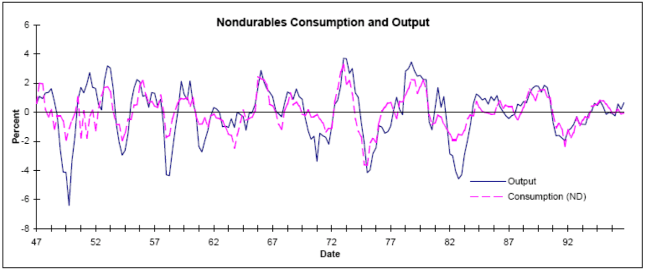
**How to organize and measure the data**
(i) Production inputs: Output - labor input - capital input - inventory stock - plus labor productivity
(ii) Expenditure components: output - consumption - investment - government spending - exports - imports
(iii) Other variables - for example nominal variables such as money supply price level etc.
**Which moments to compute:**
Volatility as measured by standard deviation
Persistence as measured by autocorrelation
Cyclicality as measured by cross correlation with output
If positive - procyclical variable, if negative - countercyclical, if close to zero - acyclical

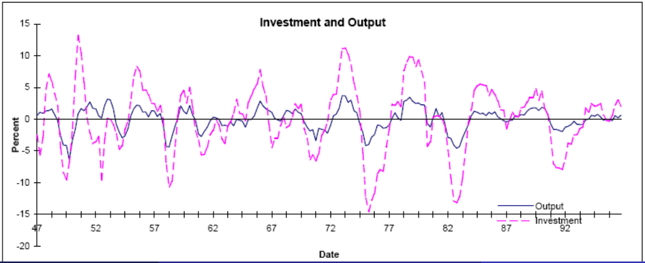
# US Facts and Figures

- US quarterly data for 1947-1996
- All variables are in constant prices and per capita
- Reported in King and Rebelo (Resuscitating Real Business Cycles)



# US Facts and Figures



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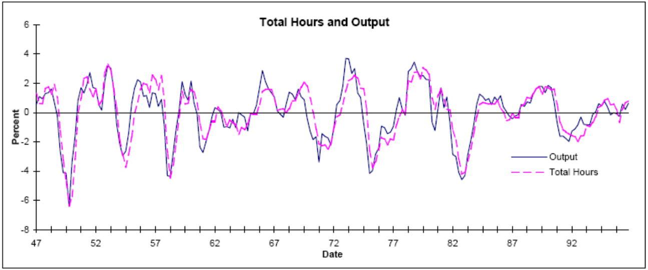


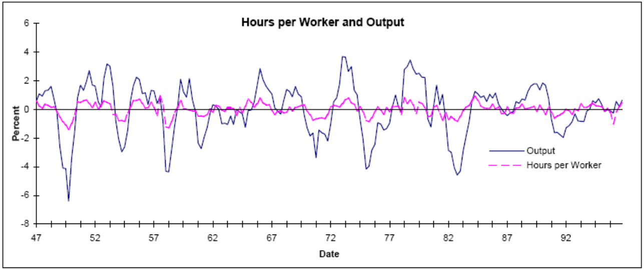

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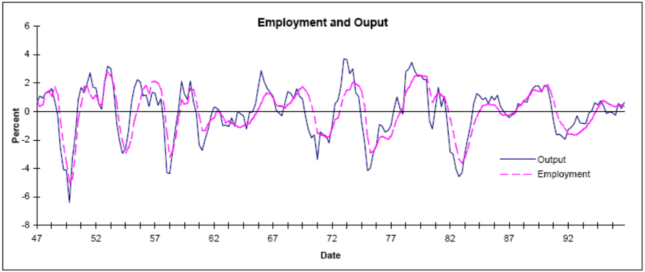


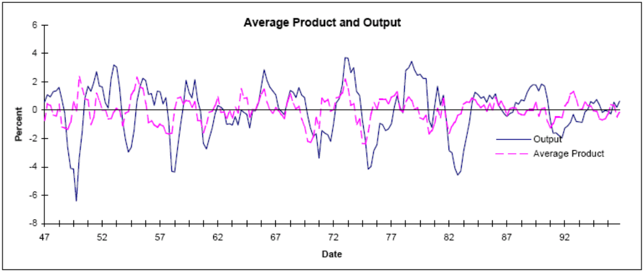

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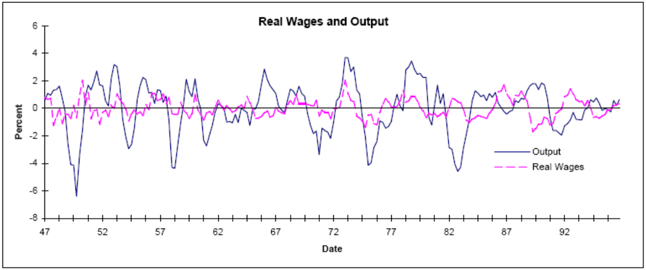


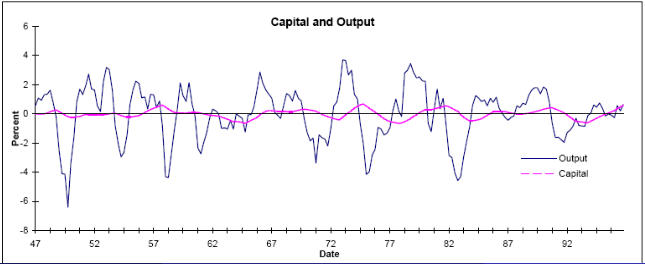



Table 1
Business Cycle Statistics for the U.S. Economy

| | Standard Deviation | Relative Standard Deviation | First Order Auto-correlation | Contemporaneous Correlation with Output |
|---|---|---|---|---|
| Y | 1.81 | 1.00 | 0.84 | 1.00 |
| C | 1.35 | 0.74 | 0.80 | 0.88 |
| I | 5.30 | 2.93 | 0.87 | 0.80 |
| N | 1.79 | 0.99 | 0.88 | 0.88 |
| Y/N | 1.02 | 0.56 | 0.74 | 0.55 |
| w | 0.68 | 0.38 | 0.66 | 0.12 |
| r | 0.30 | 0.16 | 0.60 | -0.35 |
| A | 0.98 | 0.54 | 0.74 | 0.78 |

# US Facts and Figures

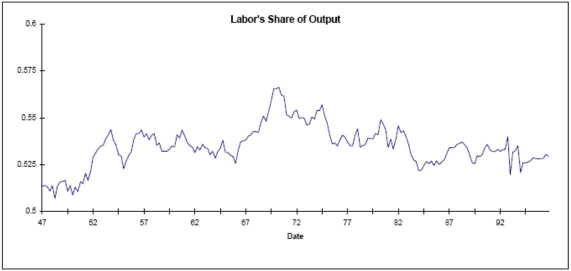
A summary of the facts:

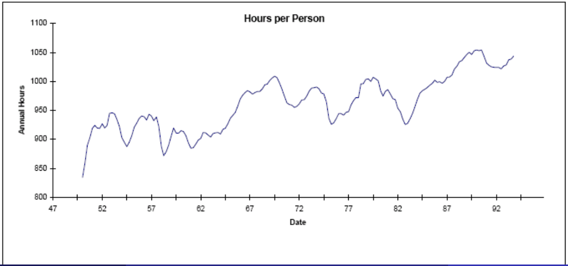
- Output volatility is around 1.8 percent per quarter. Similar numbers for other countries.
- Consumption is smoother than output
- Investment is much more volatile than output
- Total hours worked are about as volatile as output
- Labor productivity is 50-60 percent as volatile as output
- The real wage and the real interest rate are both quite smooth
- All main macroeconomic aggregates are persistent
- Consumption, investment and hours worked are very procyclical
- Productivity is also procyclical, the Solow residual mostly so
- The real wage is almost acyclical although there is a small positive correlation between it and output

# Long-Run Facts

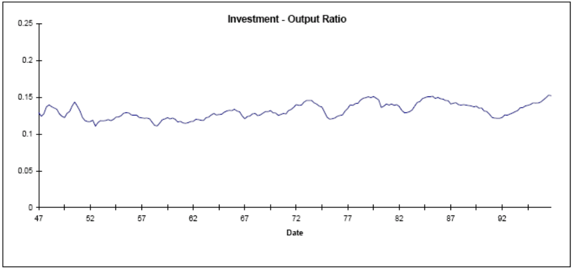
- The "facts" above are useful for evaluating theory
- There's another set of facts that are useful for building theory - these are facts about the long run
  1. Factor income shares are relatively constant over time and are not trending
  2. The consumption and investment shares of output do not trend
  3. Real wages have grown substantially over time. Aggregate hours worked have not.
  4. Output grows over time

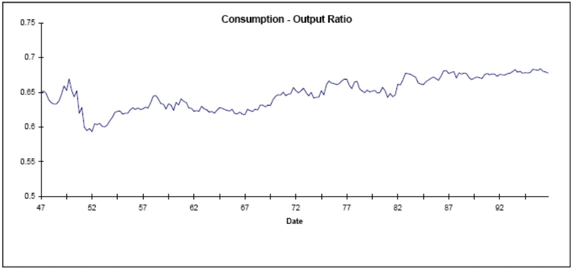
# Long-Run Facts



# Long-Run Facts



# Building a model

- Having seen the facts, we will now build a small model and investigate how it does in accounting for (some of) the empirical facts
- The economy consists of
  1. A large number of identical, infinitely lived households. Households maximize utility which they derive from consumption of goods and consumption of leisure (or disutility of work). They supply labor to firms and rent out capital to firms. They use their income either for consumption or for buying investment goods which they add to their capital stock. They behave competitively taking all prices for given.
  2. A large number of identical firms. Firms rent capital and labor from households. They produce a single good and take all prices for given. We assume that they operate a constant returns to scale technology.
- In order to allow for fluctuations, we will now also incorporate stochastic shocks, and we will specify these as technology shocks

# Specifying the model: Households

- Since we will work in models with stochastic shocks, we must specify the household's preferences taking into account that there is uncertainty. We will assume expected utility meaning that we can formulate preferences as:

$$V_0 = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(c_t, l_t)$$

- Suppose that we denote uncertainty by the state of nature $s_t$ and that $s_t \in S$ can take on $n$ values, $S = (s_1, s_2, .., s_n)$ with probabilities $1 \geq \pi(s_i) \geq 0$, $\sum_i \pi(s_i) = 1$. If the agent computes expectations using the "correct" probabilities, we say that the agent has rational expectations
- Then expected utility in that period is:

$$u_t = \sum_{i=1}^{N} \pi(s_t = s_i) U(c(s_i), l(s_i))$$

which we simply write using the expectations operator as above

# Specifying the model: Households

- We also need to think about how to specify preferences. From last lecture we know that there are restrictions that we must impose on preferences in order to be consistent with the long-run facts
- We will assume that:

$$U(c_t, l_t) = \frac{\left(c_t^{\theta} l_t^{1-\theta}\right)^{1-\kappa} - 1}{1-\kappa}, \ \kappa > 0$$

- Finally, we need to consider the asset market structure
- In general, we will need to distinguish between complete and incomplete markets models
- This is particularly relevant in models with heterogeneous agents
- Here we will have a representative agent and it turns out that a single market for capital is sufficient to have complete markets

# Specifying the model: Households

- Thus, the representative household's problem is:

$$\max \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{\left(c_t^{\theta} l_t^{1-\theta}\right)^{1-\kappa} - 1}{1-\kappa}$$

$$\begin{aligned} c_t + i_t &= w_t h_t + r_t k_t + \pi_t \\ k_{t+1} &= (1-\delta) k_t + i_t \\ l_t + h_t &= T \end{aligned}$$

  where $r_t$ and $w_t$ are exogenous and stochastic (due to productivity shocks)
- A subtle but important difference between the stochastic model and the perfect foresight models that we have seen earlier is that in the present model, agents make contingency plans: They do not necessarily decide the exact levels of consumption, savings etc. from now till eternity - they decide what they want to do subject to the shocks that may hit the economy

# Specifying the model: Households

- The first-order conditions for the household's problem are:

$$\begin{aligned} c_t &: \theta c_t^{\theta(1-\kappa)-1} \left(T - h_t\right)^{(1-\theta)(1-\kappa)} = \lambda_{c,t} \\ h_t &: (1-\theta)\, c_t^{\theta(1-\kappa)} \left(T - h_t\right)^{(1-\theta)(1-\kappa)-1} = \lambda_{c,t} w_t \\ k_{t+1} &: \lambda_{c,t} = \beta \mathbb{E}_t \lambda_{c,t+1} \left(r_{t+1} + (1-\delta)\right) \\ \lambda_{c,t} &: c_t + k_{t+1} = w_t h_t + r_t k_t + (1-\delta)\, k_t + \pi_t \end{aligned}$$

- which can be combined to get:

$$\begin{aligned} \frac{1-\theta}{\theta} \frac{c_t}{T - h_t} &= w_t \\ c_t^{\theta(1-\kappa)-1} \left(T - h_t\right)^{(1-\theta)(1-\kappa)} &= \beta \mathbb{E}_t [c_{t+1}^{\theta(1-\kappa)-1} \left(T - h_{t+1}\right)^{(1-\theta)(1-\kappa)} \\ &\quad \left(r_{t+1} + (1-\delta)\right)] \\ c_t + k_{t+1} &= w_t h_t + r_t k_t + (1-\delta)\, k_t + \pi_t \end{aligned}$$

# Specifying the model: Firms

- Firms are competitive and rent capital and labor from firms after having learned the current period's productivity shock
- They operate Cobb-Douglas technologies:

$$y_t = A_t k_t^{\alpha} h_t^{1-\alpha}$$

and maximize profits:

$$\max_{k_t, h_t} \pi_t = A_t k_t^{\alpha} h_t^{1-\alpha} - r_t k_t - w_t h_t$$

with first-order conditions:

$$\begin{aligned} r_t &= \alpha A_t k_t^{\alpha-1} h_t^{1-\alpha} \\ w_t &= (1-\alpha) A_t k_t^{\alpha} h_t^{-\alpha} \end{aligned}$$

- Notice that productivity shocks move factor prices

# Specifying the model: Productivity Shocks

- We will assume that the only shock to the economy is a productivity shock
- We will assume that $A_t$ is Markovian and its logarithm follows a first-order autoregressive process:

$$\log A_t = \rho \log A_{t-1} + \varepsilon_t$$

- where $\varepsilon_t$ are the stochastic shocks that are the innovations to the TFP process. We assume that these shocks are independently and identically distributed over time with mean 0 and variance $\sigma_\varepsilon^2$, and $\rho$ measures the persistence of the TFP process
- These shocks - productivity shocks - are our candidate for business cycle impulses and they can be measured as Solow residuals:

$$\log A_t = \log y_t - \alpha \log k_t - (1 - \alpha) \log h_t$$

- How persistent is this process? Very - $\rho$ is at least 95 percent per quarter

# Equilibrium

We can now put everything together. Clearing the markets we get:

$$
\begin{aligned}
\frac{1-\theta}{\theta}\frac{c_t}{T-h_t} &= (1-\alpha) A_t k_t^{\alpha} h_t^{-\alpha} \\
c_t^{\theta(1-\kappa)-1}(T-h_t)^{(1-\theta)(1-\kappa)} &= \beta \mathbb{E}_t [c_{t+1}^{\theta(1-\kappa)-1}(T-h_{t+1})^{(1-\theta)(1-\kappa)} \\
& \quad (\alpha A_{t+1} k_{t+1}^{\alpha-1} h_{t+1}^{1-\alpha} + (1-\delta))] \\
c_t + k_{t+1} &= A_t k_t^{\alpha} h_t^{1-\alpha} + (1-\delta) k_t
\end{aligned}
$$

- which looks very complicated
- But let us log-linearize it around the deterministic steady-state

1. Find the deterministic steady-state as the equilibrium of the model if $A = 1$ forever
2. Log-Linearize the first-order necessary conditions around the steady-state

# The steady-state

We can find the deterministic steady-state from the following equations:

$$
\begin{aligned}
\frac{1-\theta}{\theta}\frac{\overline{c}}{T-\overline{h}} &= (1-\alpha)\frac{\overline{y}}{\overline{h}} \\
1 &= \beta\left(\alpha\frac{\overline{y}}{\overline{k}}+(1-\delta)\right) \\
\overline{y} &= \overline{k}^{\alpha}\overline{h}^{1-\alpha} \\
\overline{y} &= \overline{c}+\overline{i} \\
\overline{i} &= \delta\overline{k}
\end{aligned}
$$

# The steady-state

The equations imply that the steady-state is determined by the equations:

$$
\begin{aligned}
\overline{y} &= \left(\frac{\overline{k}}{\overline{y}}\right)^{\alpha/(1-\alpha)} \overline{h} \\
\frac{\overline{k}}{\overline{y}} &= \frac{\alpha}{1/\beta - (1-\delta)} \\
\frac{\overline{i}}{\overline{y}} &= \delta \frac{\overline{k}}{\overline{y}} \\
\frac{\overline{c}}{\overline{y}} &= 1 - \frac{\overline{i}}{\overline{y}} = 1 - \delta \frac{\overline{k}}{\overline{y}} \\
\frac{\overline{h}}{T - \overline{h}} &= \frac{\theta}{1-\theta} (1-\alpha) \frac{\overline{y}}{\overline{c}}
\end{aligned}
$$

# Log-linearizing

If we take log-derivatives to the system of equations defining the competitive equilibrium we get that:

$$
\begin{aligned}
\widehat{c}_t + \frac{\overline{h}}{T-\overline{h}}\widehat{h}_t &= \widehat{A}_t + \alpha\widehat{k}_t - \alpha\widehat{h}_t \\
& \left(\theta\left(1-\kappa\right)-1\right)\widehat{c}_t - \left(1-\theta\right)\left(1-\kappa\right)\frac{\overline{h}}{T-\overline{h}}\widehat{h}_t \\
&= \left(\theta\left(1-\kappa\right)-1\right)\mathbb{E}_t\widehat{c}_{t+1} - \left(1-\theta\right)\left(1-\kappa\right)\frac{\overline{h}}{T-\overline{h}}\mathbb{E}_t\widehat{h}_{t+1} \\
& +\beta\alpha\frac{\overline{y}}{\overline{k}}\left(\mathbb{E}_t\widehat{A}_{t+1} + \left(\alpha-1\right)\widehat{k}_{t+1} + \left(1-\alpha\right)\mathbb{E}_t\widehat{h}_{t+1}\right) \\
\frac{\overline{c}}{\overline{y}}\widehat{c}_t + \frac{\overline{k}}{\overline{y}}\widehat{k}_{t+1} &= \widehat{A}_t + \alpha\widehat{k}_t + \left(1-\alpha\right)\widehat{h}_t + \left(1-\delta\right)\frac{\overline{k}}{\overline{y}}\widehat{k}_t \\
\widehat{A}_{t+1} &= \rho\widehat{A}_t + \varepsilon_{t+1}
\end{aligned}
$$

# Log-linearizing

Now, let us make the following guess on the solution:

$$
\begin{aligned}
\widehat{c}_t &= \gamma_c \widehat{k}_t + \mu_c \widehat{A}_t \\
\widehat{k}_{t+1} &= \gamma_k \widehat{k}_t + \mu_k \widehat{A}_t \\
\widehat{h}_t &= \gamma_h \widehat{k}_t + \mu_h \widehat{A}_t
\end{aligned}
$$

If we insert the guess in the first-order conditions, we can solve for the unknown coefficients in the guess (see appendix). This method is called the method of undetermined coefficients.

- These parameters will depend on all the parameters of preferences, technology etc. in complicated ways
- We need to solve a 2nd order equation for $\gamma_k$ which has a stable and unstable solution. We use the stable one since we know that the model has a saddle path solution.
- Given this parameter, we can find all the other unknown parameters

# The Impact of Technology Shocks

We would now like to answer the following **two questions**:

1. What happens in the economy after a technology shock?
2. Can the model account for the statistics that we looked at earlier?

**What happens after a technology shock?**

- Before looking at the general case, let me just look at the special case we had in last class: Log utility and complete depreciation

$$\begin{aligned} U(c_t, L_t) &= \log c_t + \frac{(1-\theta)}{\theta} \log (T - h_t) \\ \delta &= 1 \end{aligned}$$

- We know from last lecture that the solution becomes:

$$\begin{aligned} c_t &= [1 - \beta\alpha] y_t \\ k_{t+1} &= \beta\alpha y_t \\ y_t &= A_t k_t^{\alpha} \overline{h}^{1-\alpha} \end{aligned}$$

# The Impact of Technology Shocks: Special Case

- We can write output as:

$$\begin{aligned} y_t &= A_t k_t^{\alpha} \overline{h}^{1-\alpha} \\ &= A_t \left(\beta\alpha y_{t-1}\right)^{\alpha} \overline{h}^{1-\alpha} \end{aligned}$$

- Now take logarithms:

$$\begin{aligned} \log y_t &= \log A_t + \alpha \log y_{t-1} + \xi \\ \xi &= \alpha \log\left(\beta\alpha\right) + \left(1-\alpha\right)\log \overline{h} \end{aligned}$$

and recall that:

$$\log A_t = \rho \log A_{t-1} + \varepsilon_t$$

- We have that

$$\begin{aligned} \log y_t &= \log A_t + \alpha \log y_{t-1} + \xi \\ \log y_{t-1} &= \log A_{t-1} + \alpha \log y_{t-2} + \xi \\ &\Rightarrow \\ \rho \log y_{t-1} &= \rho \log A_{t-1} + \rho\alpha \log y_{t-2} + \rho\xi \end{aligned}$$

# The Impact of Technology Shocks: Special Case

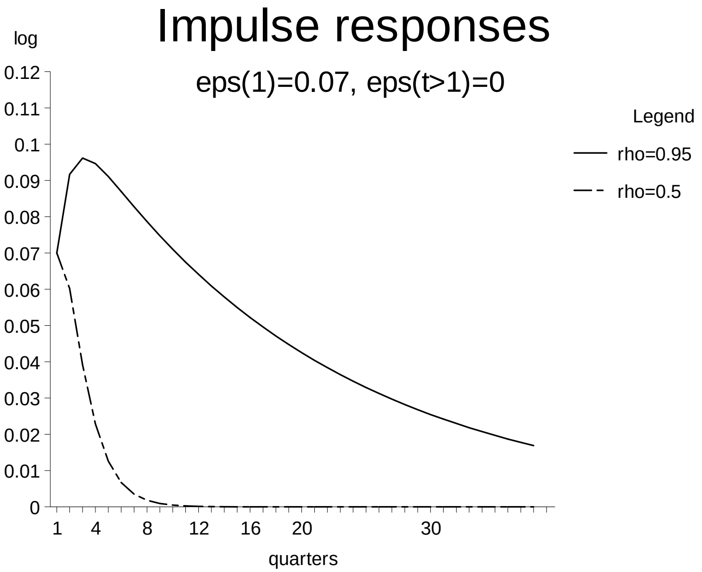
- Now subtract the third equation from the first one:

$$\begin{aligned}\log y_t - \rho \log y_{t-1} &= \log A_t - \rho \log A_{t-1} \\ &\quad + \alpha \log y_{t-1} - \rho\alpha \log y_{t-2} + (1-\rho)\,\xi \\ &\Rightarrow \end{aligned}$$

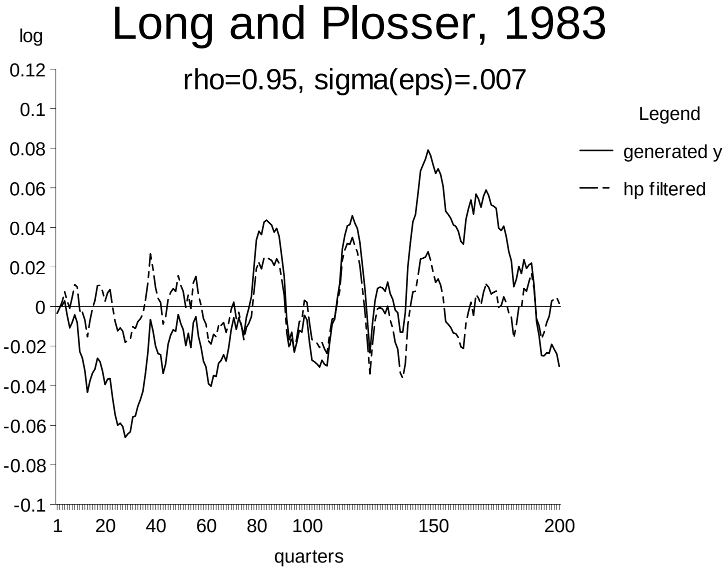
$$\log y_t = (1-\rho)\,\xi + (\alpha + \rho) \log y_{t-1} - \rho\alpha \log y_{t-2} + \varepsilon_t$$

- Thus, output follows an AR(2) process and such a process can have very persistent dynamics
- Here's an example
- Let me set $\alpha = 0.36$ and $\rho$ equal to either 0.25 or 0.95
- The picture below shows the response of output to a technology shock of size 0.07 percent

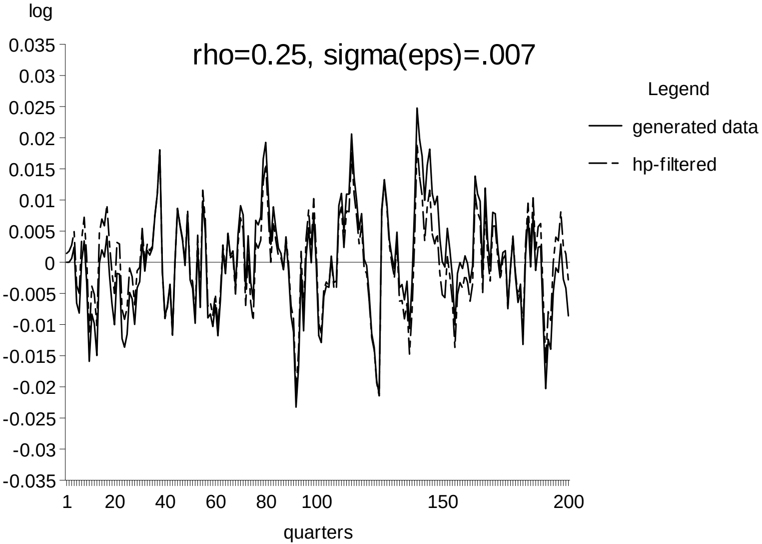
# The Impact of Technology Shocks: Special Case



# The Impact of Technology Shocks: Special Case



# The Impact of Technology Shocks: Special Case



# The Impact of Technology Shocks: The General Case

- In the general case, we will need to ascribe values to the parameters of the model, and then we can use these to compute the parameters of the decision rules that we looked at earlier
- How do we ascribe parameter values? Calibration:
  - select "share parameters" to match model's steady-state implications for "great ratios" with those observed in the data
  - select "curvature parameters" on the basis of econometric estimates
  - select parameters of stochastic driving process by matching these with econometric estimates

# Calibration

Which parameters do we need find?

| parameter | interpretation | type |
|---|---|---|
| $\beta$ | subjective discount factor | share parameter |
| $\theta$ | utility weight | share parameter |
| $1/\kappa$ | Intertemp. elasticity of substitution | curvature parameter |
| $\alpha$ | capital share of income | share parameter |
| $\delta$ | depreciation rate | share parameter |
| $\rho$ | persistence of TFP shock | driving process parameter |
| $\sigma_\varepsilon^2$ | volatility of TFP innovations | driving process parameter |

- We have already derived the steady-state conditions and we now use this to calibrate $\alpha$, $\beta$, $\theta$, and $\delta$

# Calibration

We will set one period equal to 3 months (1 quarter of the year)

1. $\alpha$ is the capital share of income. It is in the region of 30-40 percent. I will use 36 percent
2. From the Euler equation evaluated in the steady-state we know that $\beta = 1/(1+r)$ where $r$ is the real interest rate. The long run real return to capital is 4-6.5 percent per annum or around 1 percent per quarter so $\beta = 1/1.01 = 0.99$
3. The depreciation rate is approximately 10 percent per year, or 2.5 percent per quarter. This implies that

$$\frac{\bar{k}}{\bar{y}} = \frac{\alpha}{1/\beta - (1-\delta)} = \frac{0.36}{1.01 - 0.975} = 10.29$$

$$\frac{\bar{i}}{\bar{y}} = \delta \frac{\bar{k}}{\bar{y}} = 0.257$$

$$\frac{\bar{c}}{\bar{y}} = 1 - \frac{\bar{i}}{\bar{y}} = 1 - \delta \frac{\bar{k}}{\bar{y}} = 0.743$$

# Calibration

5. Finally, from the steady-state conditions for labor supply we know that:

$$\frac{1-\theta}{\theta}\frac{\overline{h}}{T-\overline{h}} = (1-\alpha)\frac{\overline{y}}{\overline{c}} \Rightarrow$$
$$\theta = \frac{\frac{\overline{c}}{\overline{y}}\frac{\overline{h}}{T-\overline{h}}}{(1-\alpha)+\frac{\overline{c}}{\overline{y}}\frac{\overline{h}}{T-\overline{h}}}$$

   Agents work around 25-30 percent of their non-sleeping time (in the US). I will use 25 percent. This gives a value of $\theta = 0.28$

6. We still need to calibrate $\kappa$ and the parameters of the technology shock process. $\kappa$ does not affect the (deterministic) steady-state so it needs to be calibrated without reference to great ratios. Econometric estimates of the coefficient of relative risk aversion are in the range of 0.5 - 5 and many studies use $\kappa = 1$ which implies log-preferences

# Calibration

7. The technology shock process instead can be calibrated from the Solow residuals

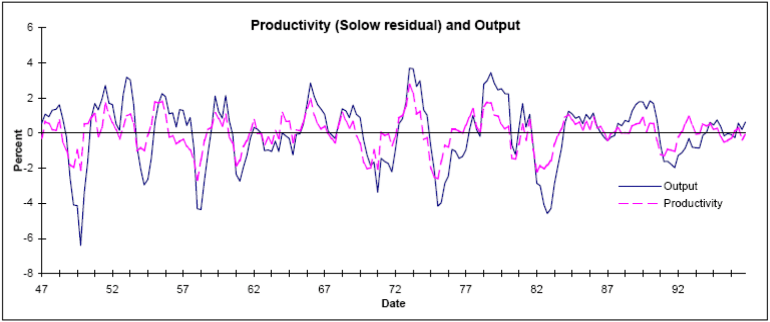
- Take logs to the production function:

$$\begin{aligned} \log y_t &= \log A_t + \alpha \log k_t + (1-\alpha) \log h_t \Rightarrow \\ \log A_t &= \log y_t - \alpha \log k_t - (1-\alpha) \log n_t \end{aligned}$$

- If we remove a trend from this process (to take care of long-run growth) we arrive at our estimate of the log of $A_t$
- From this process we can measure the persistence of the process and the variance of the innovation by fitting an AR(1) process
- For the U.S. this gives the estimates $\rho = 0.972$ and $\sigma_\varepsilon^2 = 0.0072^2$

# Calibration

HP-filtered TFP looks like:



# The Impact of Technology shocks

Given the calibrated parameters, we can solve for the decision rules and perform two experiments:

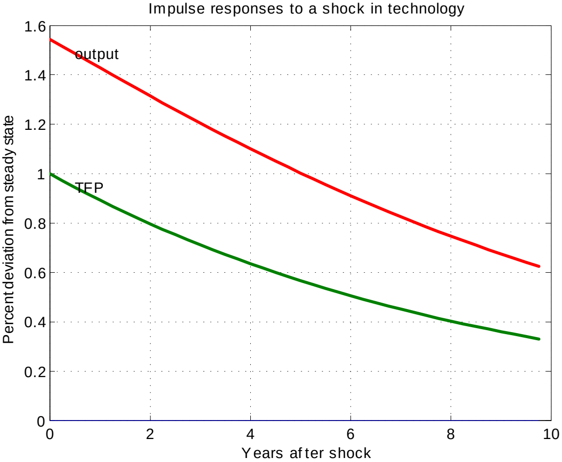
1. How does a one time technology shock affect the economy? This corresponds to computing impulse responses of the variables

$$IR\left(x_{t+i}\right) = E_t z_{t+i} - E_{t-1} z_{t+i}$$

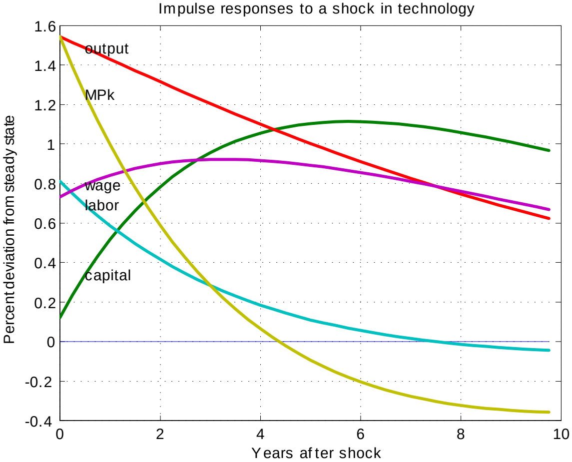
   where $E_{t-1}\varepsilon_t = 0$, $E_t\varepsilon_t = \varepsilon_t$, $E_t\varepsilon_{t+i} = 0$
2. Examine the business cycle moments of the model

- The impulse responses are useful for getting some intuition about the model
- Here I show the responses to a one percent positive technology shock

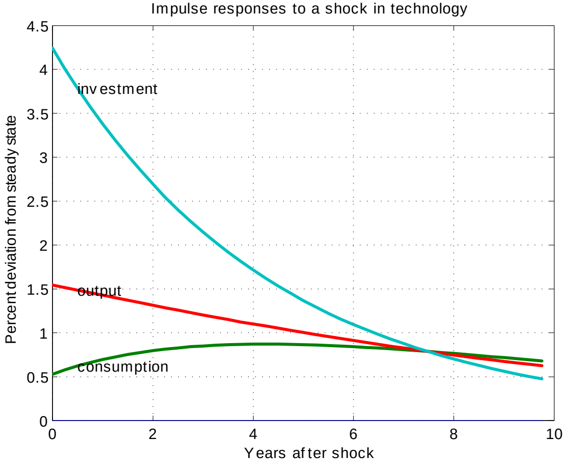
# The Impact of Technology shocks



# The Impact of Technology shocks



# The Impact of Technology shocks



# The Impact of Technology shocks

A technology shock brings about a persistent boom in the economy

- The increase in output is a bit larger than that of TFP but follows much the same shape as the TFP process
- The boom is brought about by:
  - an increase in hours worked: Higher TFP means higher wages which, due to the preference specification leads to higher labor supply.
  - an increase in the capital stock: The increase in TFP is temporary so consumers wish to smooth the consumption response by saving
- It is noticeable that investment is very elastic while consumption is quite smooth: Investment accounts for 25 percent of output and for 2.5 percent of the capital stock in steady-state Hence, it takes large percentage changes in investment to change the capital stock.
- As in the data we see procyclical responses of the output components
- How well does the model account for the moments we observed in the data? To examine this we simulate the model for a sequence of random shocks and we then HP-filter the artificial data and compute average moments over $N$ experiments

# The Impact of Technology shocks

Table 3

Business Cycle Statistics for Basic RBC Model[35]

| | Standard Deviation | Relative Standard Deviation | First Order Auto-correlation | Contemporaneous Correlation with Output |
|---|---|---|---|---|
| Y | 1.39 | 1.00 | 0.72 | 1.00 |
| C | 0.61 | 0.44 | 0.79 | 0.94 |
| I | 4.09 | 2.95 | 0.71 | 0.99 |
| N | 0.67 | 0.48 | 0.71 | 0.97 |
| Y/N | 0.75 | 0.54 | 0.76 | 0.98 |
| w | 0.75 | 0.54 | 0.76 | 0.98 |
| r | 0.05 | 0.04 | 0.71 | 0.95 |
| A | 0.94 | 0.68 | 0.72 | 1.00 |

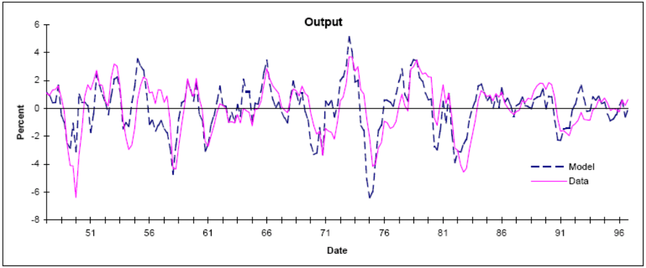
Note: All variables have been logged (with the exception of the real interest rate)and detrended with the HP filter.

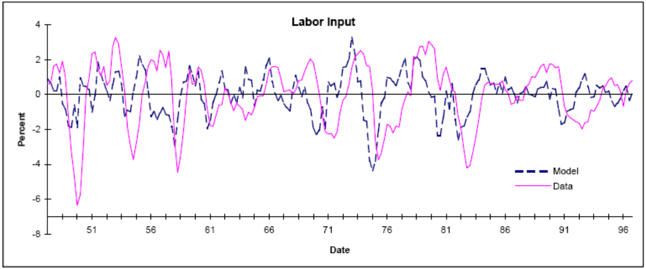
# The Impact of Technology shocks

The results are surprisingly good:

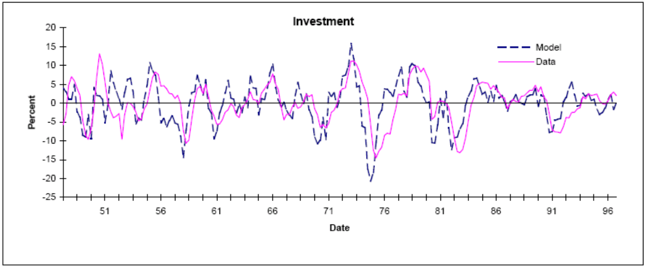
- The standard deviation of output is 1.4 percent per quarter vs. 1.8 per quarter in the US data (the model accounts for 60 percent of the volatility of output at business cycle frequencies
- Like in the US data, consumption is smoother than output while investment is much more volatile than output
- Consumption and investment are both procyclical as in the US data
- Hours worked are procyclical but somewhat smoother than in the data
- We could also do the following experiment: How would the US business cycle have looked like had there been only technology shocks?
- In other words, if we feed in the empirical estimate of the TFP process and the technology shocks, what are the resulting time series for the main macroeconomic aggregates?

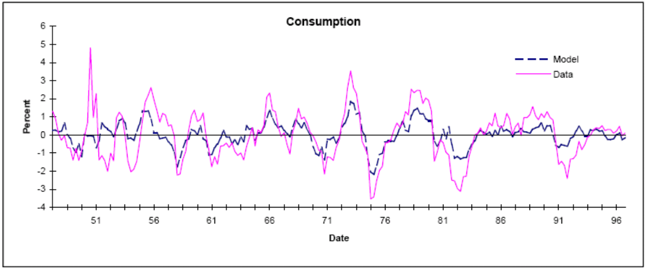
# The Impact of Technology shocks



# The Impact of Technology shocks



So, is that it - is the simple RBC model the "right tool" for considering business cycles?
It's quite successful but there are a number of issues with it:

- Are fluctuations in TFP really exogenous? Probably not all of them
- Other shocks to the economy: Fiscal shocks, monetary policy shocks, labor supply shocks, bursting bubbles etc. In principle, the model can be extended to take such shocks into account
- The model has no role for heterogeneity, frictions, etc.
- Banking - financial frictions - recent event seem to indicate that such features may be important
- What about unemployment? In the model, all the unemployed are voluntarily unemployed

Rather than going through the list above, let me look at a particular failure of the model:

- **The labor supply elasticity is too high in the model**: Micro studies of the labor supply elasticity tend to be low. Labor economists have produced a lot estimates of the "Frisch elasticity of labor supply" (the percentage response of labor supply to changes in the real wage holding wealth constant). These are typically smaller than one.
- In the model that we looked at (remember we are assuming log preferences) the first order condition for labor supply is

$$(1-\theta)\frac{1}{T-h_t} = \lambda_t w_t$$

so the Frisch elasticity is:

$$\zeta = \frac{T-h}{h} \simeq 3-4$$

- which is much higher than the micro estimates

# Indivisible Hours

We will address this issue by introducing indivisible hours - This means a zero elasticity for the individual

- Suppose a worker can work either full time $H < T$ or not at all
- Suppose also that preferences are given as:

$$U(c_t, l_t) = \theta \log c_t + (1-\theta) \log l_t$$

- Workers can either work 0 hours or $H$ hours.
- To analyze this, let us introduce a lottery (this is equivalent to unemployment insurance):
- At the beginning of the period, agents get asked to take part in a lottery with two possible outcomes:
  - with probability $\pi_t$ the agent has to work full time in period $t$
  - with probability $(1-\pi_t)$ the agent does not work in period $t$
  - no matter whether you are drawn to work or not, your consumption is $c_t$ (this can be shown to be optimal due to separable preferences)

# Indivisible Hours

- The expected utility of the representative agent in period $t$ is:

$$\begin{aligned} EU(c_t, l_t) &= \pi_t(\theta \log c_t + (1-\theta)\log(T-H)) \\ &\quad + (1-\pi_t)(\theta \log c_t + (1-\theta)\log T) \\ &= \theta \log c_t + \pi_t(1-\theta)\log\frac{T-H}{T} + (1-\theta)\log T \end{aligned}$$

- The equilibrium employment probability is:

$$\begin{aligned} h_t &= \pi_t H \Rightarrow \\ \pi_t &= h_t / H \end{aligned}$$

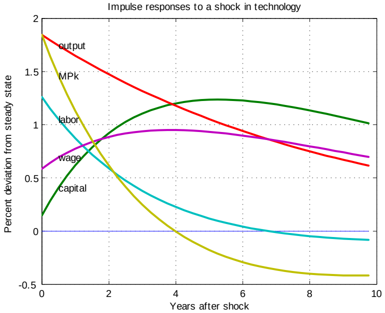
where $h_t$ is per-capita hours. So expected utility of the representative agent is:

$$\begin{aligned} EU(c_t, L_t) &= \theta \log c_t + \pi_t(1-\theta)\log\frac{T-H}{T} + (1-\theta)\log T \\ &= \theta \log c_t + Bh_t + \phi \\ B &= \frac{(1-\theta)}{H}\log\frac{T-H}{T} < 0,\ \phi = (1-\theta)\log T \end{aligned}$$

# Indivisible Hours

Expected utility is linear in per capita hours - i.e. the labor supply elasticity of the representative agent is infinite!!

- Hence, although individual agents has no choice over hours worked, the representative agent behaves as if the Frisch elasticity is infinite
- This means a much larger hours response to technology shocks




- Thus, the labor market is extremely important for the properties of these models

# Conclusion

- Real Business cycle models provide a nice and coherent lab for studying business cycles
- For sure - these models miss out lots of relevant stuff and many recent papers have extended this type of analysis
- But, the strong point of it is that it works relative well given its simplicity

# Appendix: Undetermined coefficients

Inserting the guess in the consumption-labor condition gives us:

$$\begin{aligned}
\gamma_c \widehat{k}_t + \mu_c \widehat{A}_t + \left(\alpha - \frac{\overline{h}}{T - \overline{h}}\right)\left(\gamma_h \widehat{k}_t + \mu_h \widehat{A}_t\right) &= \widehat{A}_t + \alpha \widehat{k}_t \\
&\Rightarrow \\
\gamma_c + \left(\alpha - \frac{\overline{h}}{T - \overline{h}}\right)\gamma_h &= \alpha \\
\mu_c + \left(\alpha - \frac{\overline{h}}{T - \overline{h}}\right)\mu_h &= 1
\end{aligned}$$

or:

$$\begin{aligned}
\gamma_c + a_1^h \gamma_h &= b_1 \\
\mu_c + c_1^h \mu_h &= d_1 \\
a_1^h &= \left(\alpha - \frac{\overline{h}}{T - \overline{h}}\right),\ b_1 = \alpha \\
c_1^h &= \left(\alpha - \frac{\overline{h}}{T - \overline{h}}\right),\ d_1 = 1
\end{aligned}$$

# Appendix: Undetermined coefficients

Inserting the guess in the resource constraint gives:

$$\frac{\overline{c}}{\overline{y}}\left(\gamma_c\widehat{k}_t+\mu_c\widehat{A}_t\right)+\frac{\overline{k}}{\overline{y}}\left(\gamma_k\widehat{k}_t+\mu_k\widehat{A}_t\right)$$
$$=\ \widehat{A}_t+\alpha\widehat{k}_t+(1-\alpha)\left(\gamma_h\widehat{k}_t+\mu_h\widehat{A}_t\right)+(1-\delta)\frac{\overline{k}}{\overline{y}}\widehat{k}_t\Rightarrow$$

$$a_2^c\gamma_c+a_2^h\gamma_h+a_2^k\gamma_k=b_2$$
$$c_2^c\mu_c+c_2^h\mu_h+c_2^k\mu_k=d_2$$
$$a_2^c=\frac{\overline{c}}{\overline{y}},\ a_2^h=-(1-\alpha),\ a_2^k=\frac{\overline{k}}{\overline{y}},\ b_2=\alpha+(1-\delta)\frac{\overline{k}}{\overline{y}}$$
$$c_2^c=\frac{\overline{c}}{\overline{y}},\ c_2^h=-(1-\alpha),\ c_2^k=\frac{\overline{k}}{\overline{y}},\ d_2=1$$

# Appendix: Undetermined coefficients

Finally, inserting in the intertemporal Euler equation gives:

$$\begin{aligned} a_3^c \gamma_c + a_3^h \gamma_h + a_3^{kc} \gamma_k \gamma_c + a_3^{kh} \gamma_k \gamma_h &= 0 \\ c_3^c \mu_c + c_3^h \mu_h + c_3^{kc} \mu_k \gamma_c + c_3^{kh} \mu_k \gamma_h &= d_3 \end{aligned}$$

where the cofficients are given as:

$$\begin{aligned} a_3^c &= \left( \theta (1-\kappa) - 1 - (1-\theta)(1-\kappa) \frac{\overline{h}}{T-\overline{h}} \right) \\ &\quad -\beta (1-\alpha)(\alpha - 1) \\ a_3^h &= \left( \theta (1-\kappa) - 1 - (1-\theta)(1-\kappa) \frac{\overline{h}}{T-\overline{h}} \right) \\ a_3^{kc} &= -\left( \theta (1-\kappa) - 1 - (1-\theta)(1-\kappa) \frac{\overline{h}}{T-\overline{h}} \right) \\ a_4^{kh} &= -\left( \theta (1-\kappa) - 1 - (1-\theta)(1-\kappa) \frac{\overline{h}}{T-\overline{h}} \right) \\ &\quad - (1-\alpha) \end{aligned}$$

# Appendix: Undetermined coefficients

$$
\begin{aligned}
c_3^c &= \left(\theta(1-\kappa) - 1 - (1-\theta)(1-\kappa)\frac{\overline{h}}{T-\overline{h}}\right)(1-\rho) \\
&\quad + \beta(1-\alpha)^2 \frac{\overline{y}}{\overline{k}} \\
c_3^h &= \left(\theta(1-\kappa) - 1 - (1-\theta)(1-\kappa)\frac{\overline{h}}{T-\overline{h}}\right)(1-\rho) \\
c_3^{kc} &= -\left(\theta(1-\kappa) - 1 - (1-\theta)(1-\kappa)\frac{\overline{h}}{T-\overline{h}}\right) \\
c_3^{kh} &= -\left(\theta(1-\kappa) - 1 - (1-\theta)(1-\kappa)\frac{\overline{h}}{T-\overline{h}}\right) \\
&\quad - \beta(1-\alpha)^2 \frac{\overline{y}}{\overline{k}} \\
d_3 &= \beta(1-\alpha)\frac{\overline{y}}{\overline{k}}\left(\rho + (1-\alpha)\rho\right)
\end{aligned}
$$

# Appendix: Undetermined coefficients

So, altogether:

$$
\begin{aligned}
\gamma_c + a_1^h \gamma_h &= b_1 \\
\mu_c + c_1^h \mu_h &= d_1 \\
a_2^c \gamma_c + a_2^h \gamma_h + a_2^k \gamma_k &= b_2 \\
c_2^c \mu_c + c_2^h \mu_h + c_2^k \mu_k &= d_2 \\
a_3^c \gamma_c + a_3^h \gamma_h + a_3^{kc} \gamma_k \gamma_c + a_3^{kh} \gamma_k \gamma_h &= 0 \\
c_3^c \mu_c + c_3^h \mu_h + c_3^{kc} \mu_k \gamma_c + c_3^{kh} \mu_k \gamma_h &= d_3
\end{aligned}
$$

# Appendix: Undetermined coefficients

The equations for the $\gamma$'s then imply:

$$\gamma_c = b_1 - \frac{a_1^h}{a_2^h - a_2^c a_1^h}\left(b_2 - a_2^c b_1\right) + \frac{a_1^h a_2^k}{a_2^h - a_2^c a_1^h}\gamma_k$$

$$a_2^h \gamma_h = \frac{1}{a_2^h - a_2^c a_1^h}\left(b_2 - a_2^c b_1\right) - \frac{a_2^k}{a_2^h - a_2^c a_1^h}\gamma_k$$

$$0 = \frac{a_2^k\left(a_3^{kc} a_1^h - a_3^{kh}\right)}{a_2^h - a_2^c a_1^h}\gamma_k^2 + \left(\frac{a_2^k\left(a_3^c a_1^h - a_2^k\right)}{a_2^h - a_2^c a_1^h}\right)\gamma_k + a_3^c b_1 + \left(a_3^h - a_3^c a_1^h\right)\frac{\left(b_2 - a_2^c b_1\right)}{a_2^h - a_2^c a_1^h}$$

This last equation is a 2nd order equation in $\gamma_k$ which will have a stable and an unstable solution. We must choose the stable solution for the saddle path solution. Given this, we then get $\gamma_h$ and $\gamma_c$. Given this we can also find the coefficients $\mu_k$, $\mu_c$, and $\mu_h$.