ECON3021: Urban Economics
Exam
May 2008

There are two sections to the exam.

Part A has four questions. Answer three questions in Part A. Each question in Part A is worth 20 points. The maximum score in Part A is 60 points. If you answer more than three questions in Part A, only the first three questions answered will be counted. All remaining answers will be ignored.

Part B has three questions. Answer two questions in Part B. Each question in Part B is worth 30 points. The maximum score in Part B is 60 points. If you answer more than two questions in Part B, only the first two questions answered will be counted. All remaining answers will be ignored.

The maximum total score is 120 points.

You have two hours to complete the exam.

Part A

Part A has four questions. Answer three questions in Part A. Each question in Part A is worth 20 points. The maximum score in Part A is 60 points.

A.1 Suppose there are two types of households living in a city, high wage types who earn w_H per hour and low wage types who earn $w_L < w_H$ per hour. Each household chooses a location x in the city and quantity of consumption c and land L to maximise utility $u(c, L)$ subject to their budget constraint. The price of c is p per unit. The equilibrium price of land at location x is $r(x)$ per unit of land. Each household has 8 hours of time available per day that can be spent working or commuting. All households commute to the centre of the city and must pay t_0 per mile in direct commuting costs plus the cost of time spent commuting. For a household living at location x, total commuting time is $T(x) = t_1 x$.

a. What are the household budget constraints for the two types?
b. What determines which types will live closer to the centre in equilibrium? In what circumstances will high wage types live closer and in what circumstances will low wage types live closer?
c. Describe the equilibrium conditions in this economy and illustrate graphically the equilibrium rent function, the bid rent functions of the two types and where each type will locate in equilibrium. If the problem does not specify all parameters required for solution then make suitable assumptions.

A.2 Suppose a railway can run either 0 trains or 1 train on its rail line and earns profits of £0 and £100 respectively. Farmers can either invest £50 pounds producing £100 of crops or invest £130 producing £200 of crops. However, if a train runs, 25% of the crops are destroyed by fire caused by sparks emitted by the train. Suppose bargaining costs are high so that the railway and the farmers cannot reach private agreements to regulate each other's activities. Two legal rules are under consideration. Under the "no liability" rule, the railway pays no

compensation for crop damage. Under the "full liability" rule, the railway must pay full compensation for all damage. Under these conditions, and assuming the railway chooses the option that maximises profits net of damage payments, what outcomes would be predicted under each of the legal rules and which of the two legal rules maximises social surplus? How would the solution change if bargaining costs were reduced to zero?

A.3 There are two modes of transportation, car and train. Suppose utility for individual *i* if they commute by car is $u_{i1} = -3t_{i1} - 5p_1 + \varepsilon_i$ while utility if they commute by train is $u_{i0} = -2t_0 - 4p_0$. Each individual chooses the mode that provides them with the highest utility. The variable t_{i1} is travel time by car and ε_i is an individual preference shock. If t_{i1} is distributed uniformly between 0 and 1, ε_i is distributed uniformly between -20 and -10, and $t_0 = 0.5$, $p_0 = 10$, and $p_1 = 5$, what fraction of the population travel by car? How does the answer change if t_0 increases to 1? Explain how the change affects utility.

A.4 Suppose owners of housing capital can freely borrow and lend at annual interest rate $i = 0.05$. Let *V* be the property value and suppose owning property entails annual maintenance costs *mV*, and property tax *TV*. Further, suppose that expected annual capital gains are *g* percent per year. What is the user cost of housing as a function of the property value? If annual rent is £36,000, maintenance costs are 1%, and property taxes are 0.5%, what expected capital gains is required to justify an equilibrium house value of £800,000 for a risk neutral investor? If house values grow at this rate, for five years, what new expected capital gains rate is required to justify the new house value assuming the values of other variables remain unchanged? What growth in annual rent would be required to keep the expected capital gains rate constant?

Part B

Part B has three questions. Answer two questions in Part B. Each question in Part B is worth 30 points. The maximum score in Part B is 60 points. If you answer more than two questions in Part B, only the first two questions answered will be counted. All remaining answers will be ignored.

B.1 An economy with two simple circular cities, London and Manchester, is in spatial equilibrium. The combined population of the two cities is *N*. Each consumer maximises utility by choosing in which city to live and within their chosen city by choosing consumption *c*, land quantity *L*, and location *x* (distance from the centre). In equilibrium, the utility obtained by a household living in London at distance *x* from the centre is $V_L = \frac{(I_L - t_L x)}{p^{0.5}(r_L(x))^{0.5}} + V_L^0$ where I_L is household income, t_L is transport cost per mile, *p* is the price of consumption, $r_L(x)$ is the equilibrium rent per unit of land, and V_L^0 is a fixed utility benefit (or detriment) derived from enjoyment of life in London. Similarly, utility obtained by a household living in Manchester at distance *x* from the centre of Manchester is $V_M = \frac{(I_M - t_M x)}{p^{0.5}(r_M(x))^{0.5}} + V_M^0$ where I_M is household income in Manchester, t_M is transport cost per mile in Manchester, *p* is the price of consumption, $r_M(x)$ is the equilibrium rent per unit of land in Manchester, and V_M^0 is a fixed utility benefit (or detriment) derived from enjoyment of life in Manchester. Assume that the boundaries of the two cities are fixed at $\overline{x}_L$ and $\overline{x}_M$ respectively.

a. If V_L^E and V_M^E are the equilibrium utility levels obtained in the two cities, what are the equilibrium rent functions in London and Manchester? Explain what factors in this model determine which city has the highest rent at the centre.
b. In the equilibrium is V_L^E greater than, equal to, or less than V_M^E? Why?
c. Assuming that each city has a fixed supply of land $S(x) = 2\pi x$ at each location *x*, and that the demand for land by each individual is $L_L^* = 0.5\left(\frac{I_L - t_L x}{r_L(x)}\right)$ and $L_M^* = 0.5\left(\frac{I_M - t_M x}{r_M(x)}\right)$ in London and Manchester respectively, what is the equilibrium number of people living at each location? How would you calculate the equilibrium population living in each city in equilibrium?

B.2 Suppose greater London has a population of 10 million each of whom decides to either stay at home or commute to the centre by car or by train. Each person obtains a gross income (before paying transport costs) of £60 per trip if they commute to the centre and receives £10 otherwise. Each maximises income net of transport cost. The commuting cost (in pounds per trip) for someone who drives is $c_d = 5 + 10n_d$ where n_d (measured in millions) is the equilibrium number of people who drive. The train costs vary across individuals and are distributed uniformly in the population ranging from £0 to £100 per trip. That is,

the fraction of the population with train costs less than or equal to x is $\frac{x}{100}$ for all $0 \leq x \leq 100$.

a. If travel costs on the road are c_d, how many people prefer driving to taking the train or staying home?
b. Calculate the equilibrium number who drive, the equilibrium cost per driver on the road, and the equilibrium number on the train.
c. Why is the above outcome not efficient?
d. Write down the total social surplus for this city (total income minus total cost) as a function of the number of people who drive and the number who take the train. Find the values of these numbers that maximise social surplus.

B.3 A consumer lives for two periods. In the first period, they have initial assets a_1 and must either rent or buy one house. Demand for housing is fixed at one unit. If they rent the cost is p_r. If they buy, the cost is p_1. Remaining resources may be spent on first period consumption c_1 or savings *s*. In the second period, savings *s* pays gross return *r*. In addition, if they purchased a house, they may sell it in the second period for an uncertain value. With probability π_H the second period value is p_H and with probability $1 - \pi_H$ it is $p_L < p_H$. Thus, for owners, in the second period in the high house price state, consumption is $c_2^H = rs + p_H$ while in the low house price state it is $c_2^L = rs + p_L$. For renters second period consumption is $c_2^H = c_2^L = rs$ in both states. Utility is $u(c_1, c_2^H, c_2^L) = lnc_1 + \beta[\pi_h lnc_2^H + (1 - \pi_L) lnc_2^L]$.

a. What are the budget constraints for buyers and for renters?
b. What are the demand functions for renters? What are the first order conditions for owners?
c. Explain how values of $(a_1, p_1, p_r, r, p_L, p_H)$ affect the decision to rent or buy.
d. If the government increases *r*, how will that affect first and second period consumption choices of buyers and renters? How will it affect the choice to buy or rent? If the information given is insufficient to determine the direction of either effect then explain why.