ECON3021: Urban Economics
Exam
May 2007

There are two sections to the exam.

Part A has four questions. Answer three questions in Part A. Each question in Part A is worth 20 points. The maximum score in Part A is 60 points. If you answer more than three questions in Part A, only the first three questions answered will be counted. All remaining answers will be ignored.

Part B has three questions. Answer two questions in Part B. Each question in Part B is worth 30 points. The maximum score in Part B is 60 points. If you answer more than two questions in Part B, only the first two questions answered will be counted. All remaining answers will be ignored.

The maximum total score is 120 points.

You have two hours to complete the exam.

Part A

Part A has four questions. Answer three questions in Part A. Each question in Part A is worth 20 points. The maximum score in Part A is 60 points.

A.1 A city populated by a large number of identical firms, each with production function $y = K^{\alpha} L^{1-\alpha}$, is initially in spatial equilibrium. All firms earn zero profits and sell output at the centre at price *p*. Firms at each location in the city ship output to the centre at cost *t* per mile per unit of output, rent capital at price r_K per unit, and pay equilibrium rent $r_0(x)$ per unit of land. The rent at the boundary of the city is fixed at r_A, the rent for agricultural land. Suddenly, the city wins a competition to host the Olympic games and a £1 billion investment to reduce transport cost by 20% is announced. The investment is to be paid for through a proportional tax τ on land rents within the city. A new equilibrium results. What is the impact on total output, the price of output, profits, land rents, and size of the city of the combined tax and investment policy?

A.2 An urban economy consists of two cities, *A* and *B*. There are two types of consumers, 1 and 2. Type 1 obtains utility v_1 if they live in city *B* while type 2 obtains utility v_2 if they live in city *B*. If either type lives in city *A*, they choose consumption, housing and location to maximise utility. The bid rent function of type 1 for locations $x \le \frac{I_1}{t_1}$ in city *A* is

$$r_1(x) = \left(\frac{I_1 - t_1 x}{v_1} \right)^2$$

where I_1 is type 1 income, t_1 is type 1 transport cost per mile, and *x* is distance from the centre of city *A*. They are not willing to live at locations $x > \frac{I_1}{t_1}$.

Similarly, the type 2 bid rent function for locations $x \le \frac{I_2}{t_2}$ in city *A* is

$$r_2(x) = \left(\frac{I_2 - t_2 x}{v_2} \right)^2$$

where I_2 is type 2 income and t_2 is type 2 transport cost per mile. They are not willing to live at locations $x > \frac{I_2}{t_2}$. Suppose $\frac{I_1}{v_1} > \frac{I_2}{v_2}$ and $\frac{I_1}{t_1} > \frac{I_2}{t_2}$, what does equilibrium in the two cities look like? Who lives where? Who lives closer to the centre of city *A*, the rich or the poor? What is the equilibrium rent function? Explain. Suppose t_2 falls so that $\frac{I_2}{t_2} > \frac{I_1}{t_1}$, explain how the answers change.

A.3 An airport has profits

$$\pi(f) = pf - cf^2$$

where $p > 0$ is revenue per flight, *f* is the number of flights per day, and $c > 0$ is a cost parameter. These flights produce air and noise pollution that affect nearby households. There are *N* households living near the airport. Each has utility measured in monetary terms of

$$u(f) = -df$$

where *d* is the disutility per flight caused by air and noise pollution.

a. If the airport is not liable for any disutility caused to households and maximises profits and households cannot react, what are the optimal choice of the firm and the resulting equilibrium profits and utilities?
b. Suppose the households could, at cost $b(N)$ per household, reach an agreement among themselves to pay the airport to reduce *f* to the efficient level. What is the efficient level of *f* and how much would the households be willing to pay to reduce flights to this level?
c. How much would the consumers have to pay the airport to reduce to the efficient level?
d. Under what circumstances, would the consumers and the airport both agree to a deal to reduce the number of flights?

A.4 Initially, 60% of households commute to the city by car and 40% by bus. Both road and bus travel suffer from congestion. Then, in response to a new congestion charge of £5 per car for road travel and 25% increased provision of bus service, 20% switch from road to bus. Speed of road travel increases by 10% and speed of bus travel increases by 15%.

a. What are the impacts on utilities of those who continue to travel by car, those who switch to bus, and those who continue to travel by bus?
b. What is the impact on government revenue and spending?

Part B

Part B has three questions. Answer two questions in Part B. Each question in Part B is worth 30 points. The maximum score in Part B is 60 points. If you answer more than two questions in Part B, only the first two questions answered will be counted. All remaining answers will be ignored.

B.1 Two types of consumers commute to the centre of a city. Type 1 is low income. Type 2 is high income. All consumers travel either by road or by rail. The road suffers from congestion. For type 1, the road travel cost is

$$c_1 = b_1\left(n_1 + n_2\right)$$

where n_1 is the number of type 1 commuters on the road and n_2 is the number of type 2 commuters on the road. Assume $b_1 > 0$. For type 2, the road travel cost is

$$c_2 = b_2\left(n_1 + n_2\right).$$

Assume $b_2 > b_1$. For both types 1 and 2, the respective train travel costs t_1 and t_2 are distributed uniformly between 0 and 1. That is, for each type, the fraction of households with train travel costs less than x is $F\left(x\right) = x$ for all $x \in \left[0,1\right]$. The total number of type 1 consumers is N_1 and the total number of type 2 consumers is N_2.

a. Calculate the equilibrium numbers of type 1 and type 2 commuters and the equilibrium type 1 and type 2 cost of road travel. Illustrate with graphs and briefly explain your answer. (Assume that the equilibrium values c_1 and c_2 are greater than zero and less than one.)

b. Calculate the efficient outcome and the taxes for type 1 and type 2 that would generate the efficient outcome. (Assume that at the efficient solution, both types have positive demand for road travel.) Briefly explain your answer.

B.2 Consider the fictional city of London. It has a population of N consumers each of whom has income I_0. The boundary of the city is fixed at x_b. The supply of land at every location is $2\pi x$. Initially, everyone commutes to the centre at cost of t_0 per mile. The transport cost can be decreased by investing in diesel trains. If c_t is invested in diesel trains the transport cost is

$$t = 0.5t_0\left(1 + \exp\left(-c_t\right)\right).$$

Consumers in the city have utility function

$$U\left(C, L\right) = C^{0.5}L^{0.5} - a_1 c_t^2.$$

The cost of land is $r\left(x\right)$ and the cost of the consumption good is $p = 1$. Utility declines when money is invested in trains because of air pollution. Assume investment in transport is paid for out of income. If c_t is spent, each consumer's after tax income is

$$I = I_0 - \frac{c_t}{N}$$

In an equilibrium of this city, the consumers have demand functions

$$C = \tfrac{1}{2}(I - tx)$$
$$L = \tfrac{1}{2}(I - tx).$$

It turns out that in this example the rent function satisfies

$$r(x) = r_0\left(1 - \frac{tx}{I}\right)^2$$

with

$$r_0 = \frac{NI}{\pi\left(2 - \frac{4tx_b}{3I}\right)x_b^2}.$$

a. Show that this rent function $r(x)$ satisfies the locational equilibrium condition and show that all the conditions of locational equilibrium are satisfied.
b. Calculate the utility level obtained by the consumers as a function of the parameters (I_0, t_0, x_b, c_t, N).
c. Calculate the total value of land in the city.
d. Suppose initially $c_t = 0$ and the government increases c_t. Explain graphically, verbally, or mathematically what happens to utilities, demand for land, demand for the consumption good, and rents. What happens to the total value of land in the city? Should the government increase c_t? Why? Who benefits and who loses? Please be brief.
e. What role does the value of a_1 play in your answers to d.?

B.3 A consumer lives for two periods. In the first period they have assets a_1 and choose how much to spend on consumption c_1, housing *h*, and savings *s*. First period consumption c_1 has price p_1 per unit. Savings has price 1 per unit. The consumer also chooses whether to buy or rent housing. If they buy housing, they must pay a fixed transaction cost *f* and then a per unit price of p_{h1}. If they rent housing, they must pay a per unit rental price of p_{r1}. If they buy housing, second period assets are $a_2 = rs + p_{h2}h$ where *r* is the gross interest rate and p_{h2} is the price of housing in the second period. Assume $p_{h1} - \frac{p_{h2}}{r} > 0$. If they rent housing, second period assets are $a_2 = rs$. Second period assets are spent on period 2 consumption which has price p_2. Utility depends on c_1, c_2 and housing *h*. The consumer's utility function is

$$u(c_1, c_2, h) = 0.3\ln c_1 + 0.2\ln c_2 + 0.5\ln h$$

a. If the consumer buys a house, what are the first and second period budget constraints? If they rent what are the budget constraints?
b. If the consumer buys a house, what are the demand functions for (c_1, c_2, h)? What are they if they rent?
c. If the consumer buys a house, what is their attained level of utility? If they rent, what is their utility?

d. Explain what factors in this model determine whether the consumer prefers to rent or buy. Under what conditions is the consumer indifferent between buying and renting?