Economics C44: Urban Economics
May 2006
There are two sections to the exam.
Part A has five questions. Answer three questions in Part A. Each question in Part A is worth 20 points. The maximum score in Part A is 60 points. If you answer more than three questions in Part A, only the first three questions answered will be counted. All remaining answers will be ignored.
Part B has three questions. Answer two questions in Part B. Each question in Part B is worth 30 points. The maximum score in Part B is 60 points. If you answer more than two questions in Part B, only the first two questions answered will be counted. All remaining answers will be ignored.
The maximum total score is 120 points.
You have two hours to complete the exam.

Part A

Part A has five questions. Answer three questions in Part A. Each question in Part A is worth 20 points. The maximum score in Part A is 60 points.

1) Consider a circular city with free migration and fixed boundary rent of r_b per unit of land. Initially, the city is in spatial equilibrium with equilibrium rent function $r^0(x)$. That is, $r^0(x)$ is the rent per unit of land at a distance of *x* miles from the centre. Initially all land is available for housing and the supply of land at distance *x* from the centre is $2\pi x$. The city has initial equilibrium population N^0 and equilibrium boundary of $x_b^0 > 1$ miles. People are free to move into and out of the city and obtain reservation utility V^R if they leave. Since the city is in equilibrium all residents initially obtain utility level $V^0 = V^R$. Each of the residents of the city is identical. Each obtains utility from *C* (a consumption good) and *L* (land) and has income *I*. A household living *x* miles from the centre of the city, faces prices for *C* and *L* of *p* and $r^0(x)$ respectively, and must commute to the centre at cost of *t* per mile. Suppose a hurricane destroys all the housing within 1 mile of the centre so that after the hurricane the supply of land available for housing at each distance $x \le 1$ is zero. The supply of land at other locations is unaffected. Explain using words and graphs how a new equilibrium will be determined, what it will look like, and how it will compare to the initial equilibrium.

2) Assume two types of firms and a single type of consumer coexist in a city. All firms maximise profits by choosing a location *x*, capital input *K*, and land input *L*. Consumers choose location *x*, land *L* and consumption *C* to maximise utility. All firms export from the centre. All consumers commute to the centre to work. Firms of type 1 and 2 have transport cost of t_1 and t_2 respectively per mile per unit of output. Consumers have transport cost per mile of t_3. In a spatial equilibrium, who locates closest to the centre? Who locates farthest away? Why? Draw a graph showing the qualitative features of a possible equilibrium along with the associated bid rent functions of the three types.

3) Suppose a consumer can borrow money at interest rate of 6% and can invest money at interest rate of 3% and must either buy or rent a house for a period of 2 years. If the consumer starts with £0 in assets and earns £24,000 per year (payable at the beginning of each year) would they prefer to buy the house now for a price of *£425,000* or invest their money and rent the house at an annual rental price of £24,000 per year (payable at the beginning of each year)? Assume there are no other costs of either homeownership or of rental, the house value will never change, and the consumer seeks to maximise wealth available at the end of the two year period. How would the answer change if their initial assets were £10,000? What level of initial assets would make them indifferent between buying and renting in the initial period?

4) Suppose a consumer buys a house of current value V making downpayment E and borrowing $M = V - E$. Suppose the consumer can borrow and lend at nominal annual interest rate i^n and that the inflation rate is π. Suppose further that the annual council tax is T percent of the house value and that all other maintenance costs are c percent of the house value. Assume the consumer pays tax on all labour and interest income at marginal rate t. Assuming the consumer owns the house forever, what further information do you need to calculate the annual user cost of the house. What is the formula for the annual user cost? Does the inflation rate affect the user cost? Why or why not?

5) The government must build a new airport in one of N communities. Assume that if the airport is built in community i, the monetary value of the cost to community i is c_i pounds. Assume that community i is willing to accept the airport if paid at least c_i pounds. Each community knows its own cost. The government does not know the values of c_i for the individual communities and would like to build the airport in the community with the lowest cost. Explain how a second price auction can be used to induce the communities to reveal their costs and can enable the government to build the airport in the lowest cost community. Explain in detail the incentives facing each community in this auction.

Part B

Part B has three questions. Answer two questions in Part B. Each question in Part B is worth 30 points. The maximum score in Part B is 60 points. If you answer more than two questions in Part B, only the first two questions answered will be counted. All remaining answers will be ignored.

1) Consumers who commute to central London choose from two options, tube and bus. Consumers are heterogeneous. Different consumers have different levels of wages *w* and different levels of a preference parameter ε. For a consumer with wages *w* and preference parameter ε, the utility obtained from a bus trip is $U_b = 13.5 - (3 + 0.1w)t_b - (2 - 0.1w)p_b + \varepsilon$ and the utility obtained from a tube trip is $U_t = 12.5 - (3 + 0.1w)t_t - (2 - 0.1w)p_t$ where t_b and t_t are the quantities of time required for bus and tube travel respectively and p_b and p_t are bus and tube fares. Assume ε is distributed uniformly between zero and 5. That is, if *x* is a number between 0 and 5, the fraction of the population with values of $\varepsilon \le x$ is $F(x) = \frac{x}{5}$.
   Also assume that initially $p_b = 1$, $t_t = 2$, and $p_t = 2$. Assume $w = 10$. The variable t_b is a free parameter to be determined. Consumers use the mode of transport that yields the highest utility.
   a. If $t_b = 2.5$, what fraction choose to use the bus?
   b. If t_b is a free variable, calculate the fraction of the population that choose to use the bus as a function of the variable t_b.
   c. Let f_b be a variable measuring the fraction that use the bus. Suppose $t_b = 2.5 + f_b$ so that the time of travel on the bus increases when the fraction using the bus increases. Under these conditions, what is the equilibrium fraction that choose to use the bus?
   d. Is the answer to c) greater or less than the answer to a)? Why?
   e. Under what conditions will the outcome in c) be efficient?

2) Consider equilibrium in an economy with two cities $(A \text{ and } B)$ and two types of consumers (Family types and Single types). Family types obtain utility V_F if they live in city *B*. Single types obtain utility V_S if they live in city *B*. All consumers in city *A* choose consumption *C*, land *L*, and location *x* to maximise utility. For those living in city *A*, the variable *x* measures the distance from the centre of the city. The Family types who live in city *A* have utility function $U_F(C,L) = C^{0.5}(L-2)^{0.5}$ while Single types who live in city *A* have utility function $U_S(C,L) = C^{0.5}L^{0.5}$. Family types that choose to live in city *A* must consume at least 2 units of land. Family types have income I_F and single types have income I_S. Both types pay a price p for the consumption good. Each consumer at location *x* must commute to the centre of the city and must pay $r(x)$ pounds per unit of land consumed and commuting costs of t pounds per mile. The rent at the boundary of city *A* is fixed at r_A. Assume that the consumers can migrate freely between the two cities and that the cities are in spatial equilibrium.

a. What is the budget constraint for each consumer type living in city A?
b. Solve the consumer maximisation problems for the two consumer types assuming at least some consumers of both types live in city A in equilibrium. What are the demand functions for the consumption good and for land conditional on choosing city A and conditional on x?
c. What are the bid rent functions of the two types? Explain what factors determine who will live closer to the centre of the city and who will live in city A and who will live in city B.
d. What other conditions must equilibrium in this economy satisfy? Explain how equilibrium is determined. Draw a graph of a potential equilibrium land rent function.

3) All transport in a city is provided by two tube lines, the Northern Line and the Victoria Line. The trains are owned and operated by the government. Both lines suffer from congestion during the morning peak travel period. During this period, the per passenger cost of travel on the Northern Line is $c_n = f_n + b_n p_n$ where c_n is the per passenger cost of travel and p_n is the number of passengers on the line during the period. Similarly, the per passenger cost of travel on the Victoria Line is $c_v = f_v + b_v p_v$ where c_v is the per passenger cost of travel and p_v is the number of passengers on the line during the period. On both lines, the per passenger cost has two components, a fare component (f_n, f_v) and a time cost component $(b_n p_n, b_v p_v)$. The time cost component increases with the number of passengers. Assume there are N commuters in the city all of whom make one trip during the morning peak period. Each commuter uses the lowest cost tube line.
   a. What is the equilibrium number of passengers on each line and the equilibrium cost of travel on each line? Explain.
   b. What is the socially optimal number of passengers on each line?
   c. Should the government set different tube fares on the two lines? Why or why not?