Economics C44: Urban Economics
May 2005
There are two sections to the exam.
Part A has five questions. Answer three questions in Part A. Each question in Part A is worth 20 points. The maximum score in Part A is 60 points.
Part B has four questions. Answer two questions in Part B. Each question in Part B is worth 30 points. The maximum score in Part B is 60 points.
The maximum total score is 120 points.
You have two hours to complete the exam.

Part A

Part A has five questions. Answer three questions in Part A. Each question in Part A is worth 20 points. The maximum score in Part A is 60 points.

1) Consider a circular city with free migration and fixed boundary rent of r_b per unit of land. Initially, the city is in spatial equilibrium with equilibrium rent function $r^0(x)$. That is, $r^0(x)$ is the rent per unit of land at a distance of *x* miles from the centre. The city has initial equilibrium population N^0 and equilibrium boundary of x_b^0 miles. People are free to move into and out of the city and obtain reservation utility V^0 if they leave. Since the city is in equilibrium all residents initially obtain utility level $V = V^0$. Each of the residents of the city is identical. Each obtains utility from *C* (a consumption good) and *L* (land) and has income *I*. A household living *x* miles from the centre of the city, faces prices for *C* and *L* of *p* and $r^0(x)$ respectively, and must commute to the centre at cost of *t* per mile. Suppose the utility people obtain from leaving the city rises to $V^1 > V^0$. Explain using words and graphs how a new equilibrium will be determined, what it will look like, and how it will compare to the initial equilibrium.

2) Assume two types of firms coexist in a city. All firms maximise profits by choosing a location *x*, capital input *K*, and land input *L*. All firms pay a price r_K per unit of capital and a price $r(x)$ per unit of land at location *x* and ship their output to the city centre where it can be sold for price *p* per unit of output. Firms of type 1 pay a transport cost of t_1 per unit of output per mile. Firms of type 2 pay a transport cost of $t_2 > t_1$ per unit of output per mile. Additionally firms of type 1 have production function $q = K^{0.75}L^{0.25}$ while firms of type 2 have production function $q = K^{0.25}L^{0.75}$. Assume the rent at the boundary of the city is fixed at r_b, the city is in spatial equilibrium with one firm at each location, and that all firms earn zero profits. Is it possible that firms of type 1 are closer to the centre in equilibrium? Why? What about firms of type 2? Draw a graph showing the qualitative features of a possible equilibrium along with the associated bid rent functions of the two types of firms.

3) Suppose a consumer can borrow money at interest rate of 5% and can invest money at interest rate of 3% and must either buy or rent a house. If the consumer had £100,000 in cash, would they prefer to buy the house now for a price of *£100,000* or invest the money and rent the house at an annual rental price of £4,000 per year forever? Assume there are no other costs of either homeownership or of rental. Also, assume the house value will never change and that the first rental payment is payable at the end of the first year. If instead the consumer had no cash but had an income of £4,000 per year (with the first income payment payable at the end of the first year), would they prefer to borrow and buy or rent? How much initial wealth (assuming a £4,000 per year income starting at the end of the first year) would the consumer need to be exactly indifferent between renting and buying in the initial period?

4) A local airport owns all the land in a small circular city surrounding the airport. The boundary of the city is fixed at x_b. It rents the land to firms. Each firm located at distance *x*, pays rent $r(x)$. Each firm produces one unit of output which earns a profit of $p - tx - r(x)$ if shipped from the airport. Alternatively, firms can refuse to use the airport in which case they earn $V(x_b) = V_0 - x_b$ from agricultural production. Profit from agriculture decreases with x_b because of pollution associated with urban expansion. Assume $x_b < \frac{p - V_0}{1 - t}$ and $t > 1$. In equilibrium all firms earn the same profits regardless of location and the airport earns profits

$$\Pi = 2\pi x_b^2 \left(\frac{r_0}{2} - \frac{t x_b}{3} \right) - c$$

If the city is in spatial equilibrium, what is the rent function? What is the rent at the centre? What profits do the firms earn in equilibrium? What happens to equilibrium rents, profits of the firms, and profits of the airport if x_b increases?

5) "The Coase Theorem implies that it doesn't matter whether cigarette smoking is banned in public places. Regardless of the law the same efficient outcome will be realised." Is this claim true, false, or uncertain? Explain.

Part B

Part B has four questions. Answer two questions in Part B. Each question in Part B is worth 30 points. The maximum score in Part B is 60 points.

1) Two types of consumers live in a city. High wage types earn wages w_H pounds per hour. Low wage types earn $w_L < w_H$ pounds per hour. Total income equals the wage times the amount of time spent working. Each consumer must choose a residential location *x* where *x* measures distance from the centre of the city in miles. Each consumer at location *x* must commute to the centre of the city and must pay $r(x)$ pounds per unit of land consumed. Commuting costs t hours per mile and c pounds per mile. All consumers have identical preferences $U(G,T,L) = G^{0.25}T^{0.25}L^{0.50}$ over quantities of a consumption good G, leisure time T, and land L. The consumption good costs *p* pounds per unit. Each consumer has an endowment of 8 hours of time which they can spend working, commuting, or in leisure. The total populations of high and low wage types are N_H and N_L respectively. The rent at the boundary of the city is fixed at r_b. Assume the city is in spatial equilibrium.
   a) What is the budget constraint for a consumer with wages w_L?
   b) Conditional on being at a location *x*, solve the consumer maximization problem for a consumer with wages w_L. What are the first order conditions? What are the conditional demand functions for *G*, *L*, and *T*? Assume that optimal $T < 8 - tx$.
   c) What condition must the slope of the bid rent function for the consumer with wages w_L satisfy? Explain.
   d) What other conditions must equilibrium in this city satisfy? Explain how equilibrium is determined and explain which type of consumer lives closer to the centre in equilibrium. Draw a graph of a potential equilibrium land rent function.

2) Suppose the per commuter cost of commuting on a road is $c_d = 5 + 3n_d$ during the day and $c_n = 5 + 3n_n$ at night where n_d and n_n are the numbers of commuters during the day and at night respectively. Cost per commuter increases with the number of commuters due to congestion. Demand for travel on the road during the day is $n_d = 100 - 0.5c_d$ while demand for travel at night is $n_n = 80 - 2c_n$.
   a) What are the equilibrium numbers of commuters and costs per commuter during the day and at night?
   b) For day travel and night travel respectively, when per commuter cost is at level *c*, define consumer surplus to be the area between the demand curve and the vertical axis above the per consumer cost level. Consumer surplus measures the difference between the maximum consumers would be willing to pay and the cost they actually incur. If taxes could be imposed (with zero collection cost), t_d per passenger during the day and t_n per passenger at night, what levels of taxes would maximise consumer surplus plus tax revenue? Why are the optimal taxes positive? Why is the optimal t_d different from the optimal t_n?

c) Suppose instead that when the tax is collected there is a collection cost so that the total per commuter travel cost (commuting cost plus tax cost) becomes

$$c_d = 5 + 3n_d \qquad \text{if } t_d = 0$$
$$c_d = 5 + 6n_d + t_d \quad \text{if } t_d > 0$$

during the day and

$$c_n = 5 + 3n_n \qquad \text{if } t_n = 0$$
$$c_n = 5 + 6n_n + t_n \quad \text{if } t_n > 0$$

at night. What are the optimal day and night taxes (again maximising consumer surplus plus tax revenues)? How does a "collection cost" affect the optimal tax level?

d) Maintaining the assumptions from part c), suppose that as an alternative the government could charge a petrol tax. Suppose that the petrol tax has no collection cost, and works as follows. Each traveller pays the petrol tax regardless of whether they travel during the day or at night and the petrol tax is equivalent to a tax cost per passenger of *p*. Under these conditions, if $t_d = t_n = 0$, what is the efficient petrol tax?

e) Which is more efficient the petrol tax or the congestion taxes? Comment on the efficient design of congestion pricing schemes when different schemes have different collection costs.

3) A government is worried about poverty and housing. Poor people have incomes $I = 2$ and spend money on two services, housing h and medical care *m*. Utility is $U(h,m) = h^{0.75}(m - 0.1)^{0.25}$. The price of housing is $p_h = 1$. The price of medical care is $p_m = 1$.

a) If there are no subsidies on housing or medical care and no income transfers, what is the poor household's optimal consumption of housing and medical care? What is their level of utility?

b) Suppose the government offers a housing subsidy, offering to pay half of the housing expenditures of the poor. What is the new optimal choice of housing, schooling, and level of utility of the poor?

c) What is the cost of the subsidy program?

d) Let *c* equal the cost calculated in part c). Suppose instead of the subsidy program in c), the government gave each poor household *c* in cash. What is the poor household's optimal choice of housing, medical care, and utility level in this case?

e) Under these assumptions, why does the second policy yield higher utility for the poor?

f) Why might a government prefer the first policy even though the poor prefer the second policy under the assumptions stated?

4) Consumers who commute to central London choose from two options, tube and bus. Consumers are heterogeneous. Different consumers have different levels of wages *w* and different levels of a preference parameter ε. For a consumer with wages *w* and preference parameter ε, the utility obtained from a bus trip is

$$U_b = 13.5 - (3 + 0.1w)t_b - (2 - 0.1w)p_b + \varepsilon_1$$

and the utility obtained from a tube trip is

$$U_t = 12.5 - (3 + 0.1w)t_t - (2 - 0.1w)p_t$$

where t_b and t_t are the quantities of time required for bus and tube travel respectively and p_b and p_t are bus and tube fares. Assume ε_1 is distributed uniformly between zero and 5. That is, if *x* is a number between 0 and 5, the fraction of the population with values of $\varepsilon_1 \leq x$ is $F(x) = \frac{x}{5}$. Also assume that initially $t_b = 3$, $p_b = 1$, $t_t = 2$, and $p_t = 2$. Assume $w = 10$.

a) What fraction of the population chooses to use the tube and what fraction chooses to use the bus?

b) If the time required for travel by bus rises to 3.5, what happens to demand for tube and bus?

c) What is the total change in welfare brought about by the change in time of travel?

d) If $t_b = 3$, what change in the bus fare would be required to bring about the same change in demand as in b)?

e) Starting from $t_b = 3$, $p_b = 1$, and the initial level of demand for the bus, what fare increase would be required to cut bus demand in half?