Economics C44: Urban Economics
December 2004
There are two sections to the exam.
Part A has five questions. Answer three questions in Part A. Each question in Part A is worth 20 points. The maximum score in Part A is 60 points.
Part B has four questions. Answer two questions in Part B. Each question in Part B is worth 30 points. The maximum score in Part B is 60 points.
The maximum total score is 120 points.
You have two hours to complete the exam.

Part A

Part A has five questions. Answer three questions in Part A. Each question in Part A is worth 20 points. The maximum score in Part A is 60 points.

1) Two types of households live in a city. Households of type 1 are "single" households with one worker who earns w_1 per hour. Households of type 2 are "couples" with two workers, one of whom earns w_2^H per hour and one of whom earns w_2^L per hour. All workers in all households spend t hours per day commuting to the centre of the city to work. Each worker has 9 hours per day and time not spent commuting is spent working. Additionally, each household maximises utility by choosing a location x at which to live, a quantity of housing *h*, and a quantity of food *f* subject to a budget constraint. The price of housing at location *x* is *r(x)*. The price of food is *p*. Household income equals the total labour earnings of the workers in the household. Write down the household budget constraints for each household and the locational equilibrium condition for each household. Explain what factors determine which type of household will live closer to the centre in spatial equilibrium in this city.

2) The local airport owns all the land in a small circle around the airport. It rents the land out to small firms. Each firm located at distance *x* from the airport pays rent $r(x) = r_0 - tx$ where r_0 is the rent at distance zero and *t* is the transport cost per mile. Each firm produces one unit of output which earns a profit of $p - tx - r(x)$ if shipped from the airport. Alternatively, firms can refuse to use the airport in which case they earn V_0. In equilibrium, $r_0 = p - V_0$ and the airport earns profits

$$\Pi = \frac{\pi}{3}\frac{(p - V_0)^3}{t^2} - c$$

where $\pi = 3.1415$ and *c* measures the operating costs of the airport operator. How much profit do the firms earn in equilibrium? What happens to equilibrium rents, profits of the firms, and profits of the airport if *t*, the transport cost per mile, falls? Would the firms benefit from a small tax increase that was used to lower the transport costs?

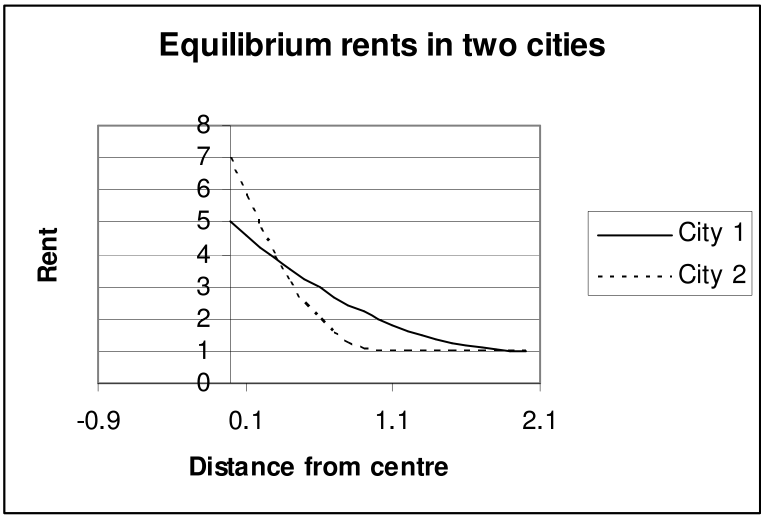
3) Suppose a consumer spends money on housing h and consumption c. The consumer consumes 1 unit of housing. The consumer has income I and must pay taxes $t \cdot I$. Additionally, they receive a housing subsidy equal to $s \cdot p_h$ where p_h is the price of housing. Suppose $p_h = 1$, $t = 0.2$, and the housing subsidy is as follows

$$
\begin{aligned}
s &= 1 && \text{if } I \le 2 \\
s &= 1 - 0.65(I - 2) && \text{if } 2 < I \le 2 + \frac{100}{65} \\
s &= 0 && \text{if } I > 2 + \frac{100}{65}
\end{aligned}
$$

Graph the cost of the subsidy per household versus income and the effective marginal tax rate as a function of income. Comment in a short paragraph or less on the implications for the optimal design of an income tax/benefit system (incentives, efficiency and equity concerns).

4) When purchasing a house worth £100,000, a consumer makes an initial down payment of £10,000 and borrows £90,000 at year 0. The annual interest rate is 5%. The mortgage is a 3 year mortgage with the first two annual payments of £33,000 (at the beginning of years 1 and 2 respectively) followed by a final payment of £35,000 at the beginning of year 3. What is the value of the debt remaining at the beginning of year 1 after having made the first annual payment? What about at the beginning of year 2? What is the present value at year 0 of the cost of the mortgage?

5) The two curves in the figure represent the equilibrium rent functions in two cities. The solid line is the rent function in city 1. The dashed line is the rent function in city 2. In both cities, the rent at the boundary equals $r_F = 1$. In one city, the distance from the centre to the boundary is $x_{b1} = 2$. In the other, the distance to the boundary is $x_{b2} = 1$. Assume that in both cities, rents are determined in a spatial equilibrium in which all residents are identical, all residents consume land (with price dependent on the distance from the centre) and a consumption good (with a fixed price), all residents commute to the centre, populations in both cities are fixed, and the boundary rent is fixed at $r_F = 1$. What factors could explain the differences in the rent functions across the two cities?



Part B

Part B has four questions. Answer two questions in Part B. Each question in Part B is worth 30 points. The maximum score in Part B is 60 points.

1) Suppose you must predict demand and welfare consequences of an improved rail link from London to Paris. Currently, a return trip rail journey costs $p_r = £1.4$ and requires $t_r = 3.5$ hours from city centre to city centre while a return trip flight costs $p_f = £1.0$ and requires $t_f = 4.0$ hours (including all check-in and waiting time). Assume that currently the number of high wage people travelling per day is $n_h = 50{,}000$ while the number of low wage people travelling per day is $n_l = 50{,}000$. High wage people have wages $w_h = 2.0$ while low wage people have wages $w_l = 1.0$. The utility person *i* obtains from a rail journey to Paris is

$$u_{ir} = \beta_0 + (\beta_1 + \beta_2 w_i)t_r + (\beta_3 + \beta_4 w_i)p_r + \varepsilon_{ir}$$

while the utility they obtain from a flight to Paris is

$$u_{if} = (\beta_1 + \beta_2 w_i)t_f + (\beta_3 + \beta_4 w_i)p_f$$

Suppose the low wage people have values of ε_{ir} distributed uniformly between 0 and 3. That is, the fraction of low wage people with values of ε_{ir} less than *x* is

$$F_l(x) = \frac{x}{3}.$$

For example, the fraction of low wage with ε_{ir} less than $x = 0.5$ is $\frac{0.5}{3}$ or $\frac{1}{6}$.

Suppose the high wage people have values of ε_{ir} distributed between 0 and 3 with a distribution function

$$F_h(x) = \frac{x^2}{3}.$$

That is, the fraction of high wage with ε_{ir} less than $x = 0.5$ is $\frac{(0.5)^2}{3}$ or $\frac{1}{12}$.

a) How many low wage and how many high wage people travel by train and by air respectively?

b) If the travel time on the train falls to 3.0, what fraction take each mode?

c) What is the total change in welfare of these two groups brought about by the change in the time of travel?

Part B continued

2) There are two ways to get from Camden to Brixton. By car or by train. Suppose the travel costs for each consumer who drives are $c = 5 + n^2$ where n (measured in thousands) is the number of people who drive. Further suppose that the cost of the train differs across people because different people live at different distances from the train station. Each person in the city travels by car if and only if his or her personal travel cost by train is higher than the travel cost on the road. Travel costs by train are distributed uniformly between 0 and 50. That is, the fraction of the population who have travel costs less than x is x for every x between 0 and 50. The total number of people travelling is $N_0 = 30$.
   a) If travel costs on the road were c, how many people n would choose to drive?
   b) Calculate the equilibrium number of people on the freeway, n_e, the equilibrium cost per person on the freeway, c_e, the total social costs of travelling on the freeway and the total costs of those remaining who travel on the train.
   c) How many people should travel on the freeway to minimise total costs of travel? What are the total travel costs if the cost minimising number of people travel by freeway?
   d) Explain why the equilibrium is not efficient and explain a government policy that could lead to the efficient outcome.

3) Suppose there is a large group of potential landlords each of whom has E in assets to invest and faces the choice between investing in a rental property of value V and investing in a bank. If a landlord invests in rental property of value V, they must invest E and borrow B where $V = B + E$. The annual interest rate charged on the loan is i_b. Additionally, owning the property costs $c \cdot V$ per year in operating costs, earns $g \cdot V$ per year in capital gains income, and R per year in rent. If a landlord invests in a bank, they earn interest at annual rate i_e. Regardless of the investment, a landlord must pay taxes on all taxable income at rate t. Taxable income from the property includes all income minus costs and a tax deductible depreciation cost δV. The amount δV is not a real cost but a landlord may deduct δV from their net income to calculate taxable income. Taxable income from the bank investment equals interest income.
   a) Write down an expression for the total after-tax annual profits from the property investment.
   b) Write down an expression for the total after-tax annual profits from the bank investment.
   c) If equilibrium rent adjusts until all after-tax annual profits from the two investments are equal, what is the equilibrium annual rent.
   d) How do changes in g, i_e, i_b, and t affect equilibrium annual rent?

4) A government can invest in two transport networks, a train network and a road network, and seeks to maximise social welfare (measured in millions of pounds). Let k_r and k_t be the amounts of capital (in millions of pounds) invested in the road and train networks respectively. Let n_t and n_r be the numbers (in millions) of travellers by train and road respectively. Let the social benefits (in millions of pounds) of travel equal

$$B(n_t, n_r) = \ln(n_t) + \ln(n_r) + n_r \cdot n_t$$

The costs of road travel per million travellers (in millions of pounds) are

$$c_r(n_r, k_r) = \frac{k_r}{n_r} + \frac{n_r}{k_r}$$

and the costs of train travel per million travellers (also in millions of pounds) are

$$c_t(k_t, n_t) = \frac{k_t}{n_t} + \frac{1}{3k_t^2}.$$

a) If k_t and k_r are invested and n_t and n_r are the number of travellers, what is the total net social welfare?

b) For fixed values of n_t and n_r, explain how the optimal levels of investment are calculated. If n_t and n_r both equal 10, what is the optimal level of capital investment in the train and road networks?

c) For fixed values of k_r and k_t, what are the optimal numbers of travellers by road and by train?

d) Suppose n_t is fixed and n_r is determined not by the government but by consumer demand and that the demand for road travel is

$$n_r = 10 - 2c_{rr}$$

where $c_{rr} = 2 + \frac{n_r}{k_r}$. What is the equilibrium number of road users, $n_e(k_r)$, as a function of capital investment k_r? What is the optimal level of investment in roads and trains when it is assumed that the number of road users is $n_e(k_r)$? Why is the answer different from the answer in a)?

e) Why is the solution to d) not efficient? How might the government induce an efficient outcome?