Lecture 6 - Transportation

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1 Efficiency when there is congestion

1. Efficient road use in congestion model.

   (a) The above equilibrium level of road use is not efficient. Each person who uses the road only considers his private benefit and private cost of road use. However, each additional road user imposes a cost on other road users. Each additional road user raises the costs of all other road users.

   (b) What would an efficient level of road use be in this model? Minimise total cost of transport.

   (c) Let n_h be the number of people on the highway.

   (d) The total (social) costs of transport in this case equal the total transport costs for those on the highway plus the total transport costs for those on the train.

   (e) The total transport costs for those on the highway are $C_r = C(n_h) \cdot n_h$.

   (f) The total transport costs of those on the train equals the sum over the costs of different individuals on the train.

   (g) To minimise costs, those with low cost should take the train.

   (h) If n_h are on the road, then $N - n_h$ are on the train and the fraction on the train is $\frac{N-n_h}{N}$. The lowest cost train traveler has $c = 0$. The highest cost train traveler has $c = c_h$ where

$$c_h = 100 \left(\frac{N - n_h}{N} \right).$$

(i) How many people of type $[c, c+dc]$ are there? $N \cdot \frac{dc}{100}$. The cost per person in this group is c. The total costs of those on the train is

$$C_T = \int_0^{c_h} c \cdot \frac{N}{100} \cdot dc$$

or

$$\begin{aligned} C_T &= \frac{N}{100}\frac{c_h^2}{2} \\ &= \frac{N}{100}\frac{\left(100\left(\frac{N-n_h}{N}\right)\right)^2}{2} \\ &= \frac{100}{N}\frac{\left(N-n_h\right)^2}{2} \end{aligned}$$

(j) The total social transport costs are

$$TC = C\left(n_h\right) n_h + 50\frac{\left(N-n_h\right)^2}{N}.$$

The cost minimising solution solves

$$\frac{\partial TC}{\partial n_h} = C\left(n_h\right) + \frac{\partial C\left(n_h\right)}{\partial n_h} n_h - 100\frac{\left(N-n_h\right)}{N} = 0$$

(k) The marginal social cost of adding one more person to the highway equals the marginal social benefit.

$$\underset{\text{(private cost)}}{C\left(n_h\right)} + \underset{\text{(external cost)}}{n_h \frac{\partial C}{\partial n_h}} - \underset{\text{(social benefit)}}{100\frac{\left(N-n_h\right)}{N}} = 0$$

(l) Recall that in the equilibrium problem, the equilibrium condition was that the number of people on the road increased until the private benefit of adding one person equaled the private cost.

$$\underset{\text{(private cost)}}{C\left(n_h\right)} - \underset{\text{(private benefit)}}{100\frac{\left(N-n_h\right)}{N}} = 0$$

If $\frac{dC}{dN} = 0$, then the equilibrium solution will equal the optimum solution. Otherwise, the equilibrium number of people on the road will exceed the optimum number.

2. Efficient congestion tax.

   (a) One solution that can obtain the optimum is to charge every road user a tax equal to $n_h * \frac{\partial C}{\partial n_h}$.

   (b) With this tax, the private cost will equal the social cost and the equilibrium number of drivers will equal the optimum number.

   (c) Optimum tax depends on:

      i. Details of the congestion cost function. The cost is determined by the speed of travel, the time required for the trip, and the value of time. These variables, especially value of time, vary by time of day, day of the week, origin and destination, and perhaps with characteristics of the travelers. In theory, an optimal tax would vary with these factors. In reality, congestion taxes often do not because more complex congestion taxes are NOT optimal because they are too expensive to implement.

      ii. Transport demand function (benefits of travel). These benefits depend on factors including destination/origin of the trip, time of day, day of the week, type of traveler.

2 Problems with efficient tax.

1. Tax fails to deal with issues related to distribution of benefits

   (a) The tax is efficient but may not be optimal. It need not be optimal because it takes no account of who benefits and who loses from the efficiency improvement. It may be optimal to sacrifice some efficiency to redistribute resources. In general any scheme for addressing congestion should address both efficiency and distributional issues. That is, the distribution of the costs or benefits of the scheme. Most congestion tax proposals do not address distributional issues very well. It is possible to design systems that address both efficiency and distributional issues. For instance, a permit system can address efficiency by restricting the number of users and forcing users to pay the marginal social cost of use. If properly designed, it can address distributional issues by choosing the initial distribution of permits and by allowing people to buy and sell permits.

2. Political problems.

   (a) Those who remain on train are not affected by the policy.

   (b) Those who switch from highway to train are made worse off. They do not pay the tax, but switch to a mode of transport that is higher cost for them.

   (c) Those who remain on the road must pay the tax but are partially compensated by reduced congestion. Nevertheless, in net they are worse off.

   (d) The government that collects the revenue is better off, they have a large sum of revenue. Unless the revenue is redistributed, spent on other public goods, or used to reduce other taxes, so that those who are harmed by the policy are compensated, there may be no political support for the tax.

3. Quota or voucher or permit system.

   (a) Suppose n_h^* is the efficient number of road users and the government creates n_h^* permits for road use.

   (b) Suppose the government distributes these permits to those who are on the highway giving a fraction of a permit, f_i, to each household.

   (c) The equilibrium price of the permits will adjust until

$$C\left(n_h^*\right) + p = \frac{100}{N}\left(N - n_h^*\right)$$

   where

$$p \ = n_h^* \cdot \frac{\partial C}{\partial n_h}$$

   (d) Households will use the road if

$$c_i - pf_i \geq C\left(n^*\right) + p\left(1 - f_i\right).$$

   Otherwise, they will use the train. The left side is the cost of train travel minus the revenue from selling a fraction of a permit. The people who remain on the road will be net buyers of vouchers. They will support the policy because the cost they have to pay will now be less than the benefit they obtain from lower congestion as long as

$$C(n^*) + p(1 - f_i) \leq C\left(n^E\right)$$

where n^E is the equilibrium number on the road before the permit policy is implemented

(e) The people who switch from road to train will support the policy because they will be net sellers of vouchers. The revenue will compensate them for the higher cost of transport on the train as long as

$$c_i - pf_i \leq C\left(n^E\right).$$

(f) Everyone is better off in the equilibrium with permits.

(g) If optimality requires further redistribution of income, such redistribution could in principle and in part be achieved by the proper allocation of permits.

(h) Alternatively, the government could auction the permits. This would result in the efficient outcome but would not distribute the benefits. The price would be the same, and the government would, as in the tax system, gain all the benefits of the policy change through increased revenues.

4. Both the tax system and the permit system have costs of administration and enforcement. The optimal system depends on which has lower operation costs. Both systems also require the government have a great deal of information. If the optimal number of road users cannot be precisely calculated but can only be estimated, then both systems will fall short of optimality and choice between the two will depend on how close each comes to the optimum. In the extreme case, if operation costs are very high, creating a road use tax system or road permit system would reduce social welfare.

5. Congestion charging and toll collecting practices can worsen congestion because it is costly to collect the tax. If the cost of congestion charging is high, it may be optimal not to charge.

6. Alternative taxes such as a petrol tax or a parking tax have been used to approximate the optimal toll. The petrol tax does not vary by time

of day or location. Also, neither of these taxes is designed to address distributional concerns.

7. Alternatively, instead of taxing road transport, a government can subsidize alternatives like public transit. However, to do this properly one must consider the more general problem of how to price both highways and alternatives like public transit to obtain optimal use of each resource.

8. Interactions with other modes of travel have been ignored. Imposing a tax to reduce congestion on road A may make congestion on road B worse. The analysis above ignores the possibility of multiple roads. If there are multiple roads, the efficient solution may have to consider the interactions between demand for the different roads.

3 Cost structures of alternative modes of transport

The main forms of transport in modern urban economies are train, bus, and automobile. Let us compare the structure of costs on these modes of transport for a simple trip from point A to point B.

Trains have the lowest marginal cost when the number of passengers is low. When a train is empty, the marginal cost of an additional passenger is very nearly the time cost of travel. Marginal costs of travel by train remain low until the train is nearly full. Then the marginal costs rise very rapidly until the train is full. Trains, however, have the highest fixed costs. Major investments in infrastructure are required to run a rail system and each train also has a large fixed cost component.

Bus systems have slightly higher marginal costs and these rise more quickly than the marginal costs of train travel, because additional buses and drivers need to be run. Congestion on buses occurs at lower passenger numbers than for trains. Buses have much lower fixed costs than trains. Their fixed costs, though similar to automobiles, are more similar to those of automobiles. Moreover, some of these fixed costs (i.e. roads) are shared across the two systems.

Finally, car based systems have higher marginal costs at low passenger levels and face congestion at much lower passenger levels than bus systems or train systems. Car systems have the lowest fixed costs of the three modes of travel.

These points have implications for the average costs of travel for each of these modes. For low passenger flows (fewer than about 10,000 – 20,000 per hour), car based modes of travel have the lowest average cost of travel. However, past this point, these costs start to rise rapidly due to congestion. At moderate passenger flows (15,000 – 30,000 per hour), buses have the lowest average costs. At high passenger flows (30,000 – 50,000 per hour) rail based systems dominate in terms of average costs. They have the lowest cost per passenger and can serve the most passengers before congestion becomes a problem.

So, for simple trips, trips along a single link between two destinations, the optimal transport mode in terms of average cost, depends on the number of people who are travelling on that link.

In more complicated systems with multiple links and where people travel along different links at different times of the day or where people must switch between modes of transport, the cost analysis is more complicated. Nevertheless, the basic points above still hold true. When the number of passengers is high, trains dominate. When it is moderate, buses dominate, and when it is low cars dominate. In big cities like London with high population densities, trains and buses tend to dominate because the number of passengers is high on most links in the transport network. Even in central London, there are many trips however, where cars have lower cost. Outside central London, however, trains are only competitive for high passenger flow trips such as trips from the suburbs to the city centre. For other local trips, buses and cars dominate. In rural areas, trains are never the low cost mode of travel for households (unless heavily subsidised by the government).

4 Modal choice

1. Up to now, we have focused on the costs of providing transport services: infrastructure costs, operating and time costs, and pollution and other external costs. We have considered some ways to improve efficiency when there is congestion.

2. The study of congestion brought in some demand considerations. In the congestion model, the consumers attempt to minimise travel costs

by choosing between two modes of transport. The analysis assumed that we know the demand functions for travel by each mode. The spatial equilibrium models we studied in the first part of the course assumed demand for transportation was inelastic. In those models, every consumer commuted and that was that.

3. How do we develop a more complete model of transport mode choice?

4. Modal choice: why is this interesting?

   (a) Study modal choice for two reasons.

      i. Predict transport demand responses to changes in prices or other features of the transport network.

         A. e.g. need to predict demand responses in order to determine optimal congestion charge.

      ii. Measure welfare effects of government policies towards transport.

5. Model of transport mode choice.

   (a) Utility from transport mode depends on:

      i. Observable characteristics of mode (z_j). These typically are things like: time of day, trip duration, travel cost, etc.

      ii. Observable characteristics of people (x_i). These typically are things like: income/wage, location, (possibly) job location, car ownership, etc.

      iii. Unobservable characteristics of both transport modes and people (ε_{ij}). These typically are things like: the degree of comfort or safety of a mode, "tastes" for driving or traveling by train, residential location, or job location.

      iv. All characteristics are observed by consumers. Some are not observed by economists. What is observed may differ depending on the dataset and the application.

6. Utility for person i on transport mode j is

$$U_{ij} = f(x_i, z_j, \varepsilon_{ij}).$$

An example is

$$U_{ij} = (\beta_1 + \beta_2 w_i)\, t_{ij} + \beta_3 p_{ij} + \beta_4 c_j + \varepsilon_{ij}.$$

(a) w_i is the wage or income of person i.

(b) t_{ij} is the time (duration) of travel for person i on mode j.

(c) p_{ij} is the price or fare on monetary cost to person i on mode j.

(d) c_j measures the degree of comfort on transport mode j.

(e) ε_{ij} captures all other features that affect utility of person i on mode j.

(f) We would expect $(\beta_1 + \beta_2 w_i) < 0$, $\beta_3 < 0$, and $\beta_4 > 0$. Why?

5 Modal choice

Each person i faces the problem of choosing which transportation mode to use to travel to work. Suppose there are J transportation modes as well as an option to not travel at all. Represent each mode or option by an index j and let $j = 0$ represent the option of not traveling at all. This means that the set of feasible options is $\{0, 1, ..., J\}$ and each $j \in \{0, 1, ..., J\}$. The options might be

Travel options

$j = 0$	Not travel at all
$j = 1$	Travel by car
$j = 2$	Travel by bus
$j = 3$	Travel by train

The utility person i obtains from using transport mode j is given by

$$U_{ij} = f\left(x_i, z_j, \varepsilon_{ij}, \beta\right). \tag{1}$$

Here, U_{ij} is the utility person i obtains if they use mode j, x_i is a set of observable characteristics of the consumer, z_j is a set of observable characteristics of the mode, and ε_{ij} is an unobservable characteristic specific to person i and mode j. The variable β represents a vector of parameters that also affect the choice.

An example of this is

$$U_{ij} = (\beta_1 + \beta_2 w_i)\, t_{ij} + \beta_3 p_{ij} + \beta_4 c_j + \varepsilon_{ij}. \tag{2}$$

In this example w_i is the wage of person i, t_{ij} is the transit time of person i on mode j, p_{ij} is the price charged to person i on mode j, c_j is the degree of comfort of mode j.

Assume that people can only choose one mode of transport and that they choose one of the options that gives them the greatest utility.

That is, person i chooses mode j if

$$U_{ij} > U_{ik} \text{ for all } k \neq j.$$

If $U_{ij} = U_{ik}$ and both are larger than all other options, the consumer is indifferent between j and k. In this case, assume the consumer picks one of the two at random.

In the general model, if one knows the values of β, x_i and the distribution of ε_{ij}, then one can predict demand responses to changes in z_j. How? In general, different people will react differently to changes in z_j. Why? One can also evaluate the welfare effects of change in z_j using this model. How? These welfare impacts will also vary across people. Why?

If one does not know the values of β, then they must be estimated by matching the predictions of the model to some data on transport mode choices. The way this is done is the following.

Define

$$d_{ij} = \left\{ \begin{array}{c} 1 \text{ if person } i \text{ uses mode } j \\ 0 \text{ if person } i \text{ does not use mode } j \end{array} \right\}.$$

If one does not know the value of β, but can observe a data set that contains information on (x_i, z_j, d_{ij}) for $i = 1, 2, ..., N$ people and for $j = 1, 2, ..., J$ modes of transportation, then one can estimate β using this data and this model.

The general procedure to estimate β is the following. The data can be used to measure what fraction of people of each type use each mode. The model predicts that the fraction of each type who use each mode depends on β. Different values of β will lead to different model predictions of how many people of each type use each mode. The value of β that yields model predictions about the fraction of each type who use each mode that are closest to the observed fractions, is the value of β that is most consistent with the data.

To see this in more detail, it is easiest to consider the context of the model in (2) in an example in which $J = 2$ and everyone chooses either 1 or 2. That is, no one chooses $j = 0$, not to travel.

Since each person maximises utility, $d_{i2} = 1$ only if $U_{i2} \geq U_{i1}$. Assume both are larger than U_{i0} for all i. Since U_{i2} is defined in (2), $U_{i2} \geq U_{i1}$ only if

$$\begin{aligned}(\beta_1 + \beta_2 w_i)\, t_{i2} + \beta_3 p_{i2} + \beta_4 c_2 + \varepsilon_{i2} &\geq \\ (\beta_1 + \beta_2 w_i)\, t_{i1} + \beta_3 p_{i1} + \beta_4 c_1 + \varepsilon_{i1}.\end{aligned}$$

This inequality is only true only if

$$\varepsilon_{i1} - \varepsilon_{i2} \leq \begin{pmatrix} (\beta_1 + \beta_2 w_i)(t_{i2} - t_{i1}) + \\ \beta_3 (p_{i2} - p_{i1}) + \\ \beta_4 (c_2 - c_1) \end{pmatrix}. \tag{3}$$

Define

$$\Delta U (\beta, w, t, p, c) = \begin{pmatrix} (\beta_1 + \beta_2 w_i)(t_{i2} - t_{i1}) + \\ \beta_3 (p_{i2} - p_{i1}) + \\ \beta_4 (c_2 - c_1) \end{pmatrix}.$$

This variable ΔU is simply shorthand notation for the right side of (3). Summarising the above statements, person i uses mode 2 only if

$$\varepsilon_{i1} - \varepsilon_{i2} \leq \Delta U_{21}.$$

Since, we assume that the $\varepsilon' s$ are not observed in the data, the probability that person i uses mode 2 equals

$$\Pr (\varepsilon_{i1} - \varepsilon_{i2} \leq \Delta U (\beta, w, t, p, c)). \tag{4}$$

This probability clearly depends on the value of β and on the distribution of $\Delta\varepsilon = \varepsilon_{i1} - \varepsilon_{i2}$.

The value of β that makes the probability in (4) close to the population fraction observed using mode 2, is the best estimate of the true value of β.

1. Questions

    (a) Graph the distribution of $\Delta\varepsilon$ and resulting modal choice

    (b) Show two different distributions, uniform on $[0, 1]$ and skewed toward zero, and the resulting demand for modes.

(c) What determines the distribution of $\Delta\varepsilon$?

(d) Show the effect of a change in t_{i1} on demand for modes 1 and 2

$$\begin{aligned}\frac{\partial(\Delta U)}{\partial t_{i1}} &= -(\beta_1 + \beta_2 w_i) \\ &> 0\end{aligned}$$

2. Effect on welfare of change in t_{i1}

(a) $\Delta U(t_{i1}^0)$ increases to $\Delta U(t_{i1}^1)$.

(b) Consumers with $\Delta\varepsilon \leq \Delta U(t_{i1}^0) < \Delta U(t_{i1}^1)$ use mode 2 both before and after change. No change in welfare.

$$\Delta W_i = 0$$

(c) Consumers with $\Delta U(t_{i1}^0) \leq \Delta\varepsilon \leq \Delta U(t_{i1}^1)$ switch from mode 1 to mode 2. Change in welfare of

$$\Delta W_i = \Delta U\left(t_{i1}^0\right) - \Delta\varepsilon_i \leq 0$$

(d) Consumers with $\Delta U(t_{i1}^1) \leq \Delta\varepsilon$ use mode 1 before and after change. Change in welfare is

$$\begin{aligned}\Delta W_i &= \left[(\beta_1 + \beta_2 w_i)\, t_{i1}^1 + \beta_3 p_{i1} + \beta_4 c_1 + \varepsilon_{i1}\right] - \\ &\quad \left[(\beta_1 + \beta_2 w_i)\, t_{i1}^0 + \beta_3 p_{i1} + \beta_4 c_1 + \varepsilon_{i1}\right] \\ &= (\beta_1 + \beta_2 w_i)\left(t_{i1}^1 - t_{i1}^0\right)\end{aligned}$$

(e) Total change found by adding up within and across groups.

With the above model, we have the elements of a positive and normative theory of mode choice. The model is a positive model because we can use it to explain observed mode choices and to predict responses to changing economic circumstances. The model is a normative model because we can use it to measure the impacts on welfare of various policies that impact z_j.

To be useful, the model requires information about U_{ij}. This information can only be obtained by: 1) asking people, 2) observing their choices in different circumstances.

The model as set up here does not have any dynamic dimension. It assumes each person is only making one choice at a single point in time. A more realistic model might model consumers short term choices and long term choices differently. In such a model, the parameters of the utility function might change over time.

1. The costs and benefits of various transport services often depend in important ways on the time of travel and the location of travel.
    (a) Peak versus off peak travel are valued differently by consumers.
    (b) Travel to the centre of the city is valued differently than travel to the countryside and this relative valuation depends on time of day or time of the week.
    (c) Both the costs of operating a transport services and the social costs associated with them, also vary with both time and location. The marginal cost of an additional traveler on the subway is very high during peak hours and very low during off peak hours.
2. Government interventions in the transport industry are large.
    (a) Large components of transport infrastructure are publicly provided.
    (b) Large fractions of transport services are publicly provided in some countries.
    (c) Taxes and subsidies affect use of transport services, use of complementary goods and services, and pollution.
    (d) Governments regulate safety, land use, prices, entry into the industry.
3. Two main benefits from transport.
    (a) Consumer benefit from utility gained from transport.
    (b) Industry benefit from transport of inputs (materials, labor) outputs (goods) and/or customers.
4. Social welfare.
    (a) To maximise social welfare.

i. For example, assume 3 types of transport investments and 6 types of transport services.

A. Investments: k_1 is investment in road 1, k_2 is investment in road 2, and k_3 is investment in rail.

B. Transport services: t_1 is travel by road 1 during peak hours, t_2 is travel by road 1 during off-peak hours, t_3 and t_4 are travel by road 2 during peak and off-peak hours, and t_5 and t_6 are travel by train during peak and off peak hours. Let $T = (t_1, t_2, t_3, t_4, t_5, t_6)$ be the vector of all transport services provided.

ii. Calculate industry benefit as function of transport investments (possibly heterogeneous). Suppose there are I industries. The total benefits to industry i are $B_i(T_i)$ where T_i is the vector of transport services used by industry i.

iii. Calculate consumer utility from transport investments (possibly heterogeneous). Suppose there are J consumers. The total benefits to consumer j are $B_j(T_j)$ where T_j is the vector of transport services consumed by consumer j.

iv. Calculate social costs of each for each level and composition of transport demand : (infrastructure costs + use and operating costs + externalities). Assume the total costs are

$$C(T, k_1, k_2, k_3) = C_1(k_1, k_2, k_3) + C_2(T, k_1, k_2, k_3)$$

A. $\frac{\partial C_1}{\partial k_i} > 0$

B. $\frac{\partial C_2}{\partial k_i} < 0$

C. $\frac{\partial^2 C_2}{\partial t_j \partial k_i} \leq 0$

(b) To maximise social welfare one must decide how much the society values each industry and each agent. Let λ_i be the value of industry i and let γ_j be the value of consumer j. Industries and consumers with high values of λ_i and γ_j are "important" or "valuable" or have high weight.

(c) The social welfare maximisation problem is

$$\max \left\{ \sum_{i=1}^{I} \lambda_i B_i (T_i) + \sum_{j=1}^{J} \gamma_j B_j (T_j) - C (T, k_1, k_2, k_3) \right\}$$

subject to

$$\sum_{i=1}^{I} T_i + \sum_{j=1}^{J} T_j = T$$

(d) This problem can be quite complicated. There are $6 * (I + J) + 3$ choice variables and the functions C and B_i and B_j may be quite complicated.

(e) Efficient investment in infrastructure

i. Marginal cost of investments should all equal zero:

$$\frac{\partial C}{\partial k_1} = \frac{\partial C}{\partial k_2} = \frac{\partial C}{\partial k_3} = 0.$$

(f) Efficient use of transport services

i. Marginal benefits of each use of transport should equal marginal cost of transport

$$\frac{\lambda_i \partial B_i}{\partial T_i} = \frac{\gamma_j \partial B_j}{\partial T_j} = \frac{\partial C}{\partial T}.$$

5. The above problem is the social welfare maximisation problem. Solution is the efficient outcome. However, equilibrium outcome in transport market may be different.

6. Equilibrium in transport market

(a) Government providers maximise utility subject to budgetary and regulatory constraints

(b) Private providers maximise profits subject to regulations, consumer demand, and competition

(c) Consumers choose mode to maximise utility

i. Consumer user cost equals time + out-of-pocket expenses

ii. In general the consumer user cost will not equal the social cost

(d) In general, because of the market imperfections (and when private mechanisms to overcome these imperfections are insufficient) described above, private market equilibrium will not maximise social welfare. Government interventions may improve welfare relative to market equilibrium without government intervention. Government intervention could also reduce welfare if government chooses wrong interventions.