Lecture 4 - Locational Equilibrium Continued

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1 Introduction

1. Review
2. Finish spatial equilibrium with multiple consumer types.
3. Spatial equilibrium with businesses.
4. Discuss other extensions to base model.

2 Review

1. A consumer's "type" is determined by income, preferences, and transport costs. In an equilibrium with multiple consumer types, different types will, in general, live at different locations. The city will segregated into sectors, each sector inhabited by a single type. Everyone of the same type will attain the same utility level but different types will in general attain different utility levels. Comparing the different types, those who value land close to the centre the most will live in a sector close to the centre. Those who value land close to the center the most are those who are willing to "bid" the most or pay the most for land close to the centre. They are the ones who have the highest "bid rent" or the highest "willingness to pay". For consumers, three factors determine who values land close to the centre the most: 1) Income, 2) transport costs, 3) preferences.

2. The bid rent function for a type describes the maximum willingness to pay for land at every location. The maximum willingness to pay of type i at location x is the maximum rent that enables type i to obtain reservation utility V_i at location x.

   (a) This amount satisifies

   $$v_i\left(I_i - t_i x, p, b_i(x)\right) = V_i \tag{1}$$

   where v_i is the indirect utility function for type i (derived form solving the utility maximisation problem for type i), t_i is the transport cost for type i, I_i is the income of type i, and V_i is the reservation utility level of type i. The function $b_i(x)$ that satisfies equation (1) is the bid rent function for type i.

   (b) For example, suppose the indirect utility function is

   $$v_i\left(I_i - t_i x, p, r(x)\right) = \left(I_i - t_i x\right) p^{-0.5} \left[r(x)\right]^{-0.5}. \tag{2}$$

   This is the indirecty utility function for the utility function $u = C^{0.5}L^{0.5}$. In words, this function describes the utility obtained by a consumer with income I_i and transport cost t_i when prices are given by p and $r(x)$ as a function of location choice x. Suppose further, that this household obtains utility V_i in equilibrium. Then, the bid rent function for this household satisfies

   $$V_i = \left(I_i - t_i x\right) p^{-0.5} \left[b_i(x)\right]^{-0.5}$$

   or

   $$b_i(x) = \left(\frac{I_i - t_i x}{V_i}\right)^2 p^{-1}.$$

   A consumer of type i who pays the amount $b_i(x)$ for land at location x will obtain utility level V_i. Therefore, they will be indifferent between paying this amount and living at location x and choosing their next best alternative which by assumption gives utility V_i. This is the maximum willingness to pay because the consumer is not willing to pay anything higher. Any higher rent will yield strictly lower utility. Notice that the bid rent depends on income, transport costs, the reservation utility level, and on the form of preferences. The particular functional form in (2) is determined by the utility function described above. A different utility function would produce a different indirect utility function.

(c) Questions: What is the indirect utility function for a household with utility $u = C^{\alpha} L^{1-\alpha}$? What is the resulting bid rent function?

3. Equilibrium for two consumer types.

   (a) Assume populations of types 1 and 2 equal N_1 and N_2 and assume the boundary rent $r(x_b) = r_A$.

   (b) Guess values for bid rents at centre: $b_1(0)$ and $b_2(0)$.

   (c) Compute the bid rent functions:

$$b_1(x) = b_1(0) - \int_0^x \frac{t}{L^*(I_1 - ts, p, b_1(s))} ds$$

$$b_2(x) = b_2(0) - \int_0^x \frac{t}{L^*(I_2 - ts, p, b_2(s))} ds.$$

   (d) Set $r(x) = \max\{b_1(x), b_2(x)\}$. Set equilibrium rent equal to the highest bid.

   (e) Compute x_b :

$$r_A = \int_0^{x_b} r(s)\, ds.$$

   (f) Set

$$N_1(x) = \left\{ \begin{array}{cc} \frac{2\pi x}{L(I_1 - tx, p, r(x))} & \text{if } b_1(x) > b_2(x) \\ 0 & \text{if } b_1(x) < b_2(x) \\ \frac{1}{2}\left(\frac{2\pi x}{L(I_1 - tx, p, r(x))}\right) & \text{if } b_1(x) = b_2(x) \end{array} \right\}$$

   and

$$N_2(x) = \left\{ \begin{array}{cc} \frac{2\pi x}{L(I_2 - tx, p, r(x))} & \text{if } b_2(x) > b_1(x) \\ 0 & \text{if } b_2(x) < b_1(x) \\ \frac{1}{2}\left(\frac{2\pi x}{L(I_2 - tx, p, r(x))}\right) & \text{if } b_2(x) = b_1(x) \end{array} \right\}.$$

   This requires supply of land to equal demand for land at every location and requires demand for land of type 1 at location x to be zero if 1 is not the highest bidder at location x. It also requires demand for land of type 2 to be zero at location x if 2 is not the higher bidder at location x.

(g) Check whether

$$N_1 = \int_0^{x_b} N_1(s)\, ds \tag{3}$$

and

$$N_2 = \int_0^{x_b} N_2(s)\, ds. \tag{4}$$

(h) If the right side of (3) is larger than N_1, increase $b_1(0)$. If it is less than N_1, decrease $b_1(0)$.

(i) If the right side of (4) is larger than N_2, increase $b_2(0)$. If it is less than N_2, decrease $b_2(0)$.

(j) In equilibrium, equations (3) and (4) are satisfied.

(k) In equilibrium, in this example type 1, the poor people, will live closer to the centre and type 2, the rich will live farther away. See graph.

(l) How could you change the model to change this conclusion?

4. Equilibrium rules are similar to those in the model with one type of consumer.

   (a) Consumers maximise.

   (b) Identical people who live at different locations in equilibirum, obtain the same utility

   (c) Each plot of land goes to the highest bidder.

   (d) Supply equals demand in every market.

3 Business location choice

1. Now, we will analyse equilibrium in a model with no consumers but with a business sector.

2. As before, suppose the city is a circle.

   (a) The supply of land in every **ring** at distance x is $S_L(x) = 2\pi x$.

3. What is the business demand for land? That is what we want to determine.

4. Assume businesses export output from the transport hub at the city center. (There is increasing returns to scale in export. That is why there is a city in the first place.)

5. Output y is produced with land L and capital K. Assume a constant returns to scale (CRS) production function

$$\begin{aligned} y &= f(K, L) \\ &= K^{\alpha} L^{1-\alpha} \end{aligned} \tag{5}$$

6. Profits of each business are

$$(p - tx) \cdot K^{\alpha} L^{1-\alpha} - r_K K - r(x) L$$

   Businesses hire land and capital to produce y. The price of the output net of transport costs is $p - tx$. The price of capital is r_K. The price of land at location x is $r(x)$.

7. Firm at location x maximises profits. The first order conditions of their maximisation problem are

$$\alpha (p - tx) K^{\alpha-1} L^{1-\alpha} = r_K \tag{6}$$

$$(1-\alpha)(p - tx) K^{\alpha} L^{-\alpha} = r(x) \tag{7}$$

8. These equations can be used to determine the firms' optimal choices of the capital-land ratio $\frac{K^*}{L^*}$. Dividing the left and right sides of equations (11) and (7) we have

$$\frac{\alpha (p - tx) K^{\alpha-1} L^{1-\alpha}}{(1-\alpha)(p - tx) K^{\alpha} L^{-\alpha}} = \frac{r_K}{r(x)}$$

$$\frac{\alpha}{1-\alpha} \cdot \frac{L}{K} = \frac{r_K}{r(x)}$$

$$\frac{K^*}{L^*} = \frac{\alpha}{1-\alpha} \cdot \frac{r(x)}{r_K}$$

9. Since the production function is a CRS function we cannot uniquely define the optimal choice of L^*. If $L^* = L_1$ is an optimal choice then so is $L^* = 2 \cdot L_1$.

10. However, if the firm earns zero profits, then every value of L is optimal. We can choose to focus on the equilibrium outcome in which all firms earn zero profits, one firm chooses to locate in every location, and the supply of land equals the demand for land. If the supply of land must equal demand, the supply of land is equal to $2\pi x$, and there is 1 firm at every location earning zero profits, then $L^*(x) = 2\pi x$ is an optimal choice for the firm that is consistent with equilibrium.

11. Each firm increases production until all available land is used up. Then combining this fact with the optimal capital-land ratio above implies that $K^*(x) = \frac{\alpha}{1-\alpha} \cdot \frac{r(x)}{r_K} \cdot 2\pi x$ and $y^*(x) = K^*(x)^{\alpha} L^*(x)^{1-\alpha}$.

    (a) Note that $y^*(x) = 2\pi x \left(\frac{\alpha}{1-\alpha} \cdot \frac{r(x)}{r_K} \right)^{\alpha}$

12. $K^*(x), L^*(x)$ demand for land and labor at every location and output $y^*(x)$.

13. Note if $r(x_1) > r(x_2)$, then the optimal capital land ratio will be higher at x_1 than at x_2.

3.1 Business location choice continued

1. Since the production function is a CRS function we cannot uniquely define the optimal choice of L^*. If $L^* = L_1$ is an optimal choice then so is $L^* = 2 \cdot L_1$.

2. However, if the firm earns zero profits, then every value of L is optimal. We can choose to focus on the equilibrium outcome in which all firms earn zero profits, one firm chooses to locate in every location, and the supply of land equals the demand for land. If the supply of land must equal demand, the supply of land is equal to $2\pi x$, and there is 1 firm at every location earning zero profits, then $L^*(x) = 2\pi x$ is an optimal choice for the firm that is consistent with equilibrium.

3. Each firm increases production until all available land is used up. Then combining this fact with the optimal capital-land ratio above implies that
$$K^*(x) = \frac{\alpha}{1-\alpha} \cdot \frac{r(x)}{q} \cdot 2\pi x$$
$$L^*(x) = 2\pi x$$
and
$$\begin{aligned} y^*(x) &= K^*(x)^\alpha L^*(x)^{1-\alpha} \\ &= y^*(x) = 2\pi x \left(\frac{\alpha}{1-\alpha} \cdot \frac{r(x)}{q}\right)^\alpha . \end{aligned}$$

4. $K^*(x), L^*(x)$ demand for land and labor at every location and output $y^*(x)$.

5. Note if $r(x_1) > r(x_2)$, then the optimal capital land ratio will be higher at x_1 than at x_2.

3.2 Locational equilibrium condition for firms

1. Locational equilibrium: all locations earn zero profits
$$(p-tx) \cdot K^*(x)^\alpha L^*(x)^{1-\alpha} - qK^*(x) - r(x) L^*(x) = 0. \tag{8}$$

2. Let $\pi(x)$ be the profit function in (8). Equation (8) states that $\pi(x) = 0$ for all x. In order for this to be true at all locations, it must be the case that $\frac{\partial \pi}{\partial x} = 0$ at all locations. Differentiating $\pi(x)$ with respect to x, we obtain
$$\begin{aligned} &\frac{\partial \pi(x)}{\partial x} = \\ &-t \cdot K^*(x)^\alpha L^*(x)^{1-\alpha} - \frac{\partial r(x)}{\partial x} L^*(x) \\ &+ \left(\alpha(p-tx) K^*(x)^{\alpha-1} L^*(x)^\alpha - q\right)\left(\frac{\partial K^*(x)}{\partial x}\right) \\ &+ \left((1-\alpha)(p-tx) K^*(x)^\alpha L^*(x)^{-\alpha} - r(x)\right)\left(\frac{\partial L^*(x)}{\partial x}\right). \end{aligned}$$

If we look closely at the final two lines in this expression, we see that the term multiplying $\frac{\partial K^*(x)}{\partial x}$ is identically equal to zero. This term is equal to zero because at the optimum, the firm sets the marginal product of capital $\left(\alpha (p - tx) K^* (x)^{\alpha-1} L^* (x)^{\alpha}\right)$ equal to the marginal cost (q). We also see that the term multiplying $\frac{\partial L^*(x)}{\partial x}$ is identically equal to zero. This term equals zero because the firm chooses the optimal ratio of land L^* and capital K^* so that the marginal product of land $\left((1-\alpha)(p - tx) K^* (x)^{\alpha} L^* (x)^{-\alpha} - r(x)\right)$ equals the marginal cost $(r(x))$. Hence, the final two lines in the expression equal zero. This is an application of the envelope theorem. As a result the complete expression for $\frac{\partial \pi(x)}{\partial x}$ can be simplified to

$$\frac{\partial \pi (x)}{\partial x} = -t \cdot K^* (x)^{\alpha} L^* (x)^{1-\alpha} - \frac{\partial r (x)}{\partial x} L^* (x) .$$

An incremental increase in distance from the centre reduces profits by an amount equal to the incremental increase in transport costs and increases profits by an amount equal to the incremental reduction in rent. In equilibrium this incremental change must equal zero. Setting $\frac{\partial \pi(x)}{\partial x} = 0$, we have

$$\begin{aligned} \frac{\partial r (x)}{\partial x} &= \frac{-t \cdot K^* (x)^{\alpha} L^* (x)^{1-\alpha}}{L^* (x)} \\ &= -t \cdot \left(\frac{K^* (x)}{L^* (x)} \right)^{\alpha} \\ &= -t \cdot \left(\frac{\alpha}{1-\alpha} \cdot \frac{r (x)}{q} \right)^{\alpha} . \end{aligned} \tag{9}$$

An equilibrium rent function must satisfy this differential equation.

3. This implies that $\frac{\partial r(x)}{\partial x} < 0$ at every location that has at least one firm.

4. It also implies $\frac{\partial^2 r}{\partial x^2} > 0$ if there is input substitution.

   (a) As one moves toward the centre, tranport costs fall, the price of land rises, and firms substitute toward capital. The capital-land ratio rises toward the centre, and the slope of the rent function becomes steeper. That is, if $x_1 < x_2$, $\frac{\partial r(x_1)}{\partial x} < \frac{\partial r(x_2)}{\partial x}$.

3.3 Equilibrium conditions

1. Given, $L^*(x)$ firms choose $K^*(x)$ and $y^*(x)$ to maximize profits.
2. Free entry.
   (a) Profits are zero.
3. Equilibrium in land market and assuming one firm per location.
   (a) $L^*(x) = 2\pi x$.
4. Locational equilibrium conditions ensures that every location earns the same profits.
   (a) Hence, the slope of the rent function must satisfy (9).
   $$\frac{\partial r(x)}{\partial x} = -t \cdot \left(\frac{\alpha}{1-\alpha} \cdot \frac{r(x)}{q} \right)^{\alpha}.$$
   (b) Let rent at centre equal r_0.
   (c) Then the rent function satisfies
   $$r(x) = r_0 + \int_0^x \frac{\partial r(s)}{\partial x} ds$$
   (d) At the urban boundary x_b the rent must equal the agricultural rent r_A. Hence,
   $$r_A = r_0 + \int_0^{x_b} \frac{\partial r(s)}{\partial x} ds \tag{10}$$
   This condition is obtained from the condition that firms earn the same profits at all locations. In this equation r_A is known while x_b and r_0 are unknown.
   (e) When we also impose that firms earn zero profits, we can determine x_b. If firms earn zero profits, the firm choosing $x = x_b$ must earn zero profits. Since $y^*(x_b) = 2\pi x_b \left(\frac{\alpha r_b}{(1-\alpha)q}\right)^{\alpha}$, $K^*(x_b) =$

$\left(\frac{\alpha}{1-\alpha}\right)\frac{r_A}{r_K}2\pi x_b$, and $L^*(x_b) = 2\pi x_b$, profits at the boundary must satisfy

$$p - tx_b 2\pi x_b \left(\frac{\alpha r_A}{(1-\alpha)q}\right)^{\alpha} - q\left(\frac{\alpha}{1-\alpha}\right)\frac{r_A}{q}2\pi x_b - r_A 2\pi x_b = 0.$$

Dividing both sides by $2\pi x_b$ we obtain

$$(p - tx_b)\left(\frac{\alpha r_A}{(1-\alpha)q}\right)^{\alpha} - \left(\frac{\alpha}{1-\alpha}\right)r_A - r_A = 0$$

or

$$(p - tx_b)\left(\frac{\alpha r_A}{(1-\alpha)q}\right)^{\alpha} = r_A\left(\frac{1}{1-\alpha}\right).$$

This is equivalent to

$$p - tx_b = r_A^{1-\alpha}\left(\frac{1}{1-\alpha}\right)\left(\frac{(1-\alpha)q}{\alpha}\right)^{\alpha}.$$

When solved for x_b this becomes

$$x_b = \frac{p - r_A^{1-\alpha}q^{\alpha}(1-\alpha)^{\alpha-1}\alpha^{-\alpha}}{t} \tag{11}$$

(f) Once we know x_b, we can determine r_0 from (4.4).

5. Supply equals demand for output

$$\begin{aligned}\int_0^{x_b} y^*(x, r(x))\,dx &= D(p)\\ \int_0^{x_b} y^*(x, r(x))\,dx &= D(p)\end{aligned}$$

The function $D(p)$ is the demand for output when price equals p. In the case, of perfectly elastic demand, this means that the price is fixed at p and demand adjusts so that demand equals supply regardless of the quantuty supplied.

(a) Land goes to highest bidder

i. $r(x) \geq r_A$ for all $x \leq x_b$ with equality at $x = x_b$.

ii. Edge of city induces zero profits.

6. Summary

   (a) (Identical) firms get zero profits at every location
   (b) $\frac{\partial r}{\partial x} < 0$
   (c) $\frac{\partial^2 r}{\partial x^2} > 0$
   (d) $r(x)$ solves (9)
   (e) r depends on t, $f(K, L)$, r_A, q, $D(p)$
   (f) Slope depends on how easy it is to substitute capital for land
   (g) $\frac{K}{L}$ decreases with x, $\frac{y}{L}$ decreases with x

7. Comparative statics

   (a) Bigger t smaller city
   (b) Steeper rent
   (c) Offset by more output being produced closer to center. If capital intensive output technology can offset.
   (d) Increase demand for product, increase city size
   (e) Increase cost of capital, makes it harder to substitute capital, increases costs, tends to reduce city size

8. What if $y = K^{\alpha} L^{1-\alpha}$ and α increases (invention of new technology)?

   (a) A bit more complicated to work out
   (b) Technology becomes more capital intensive
   (c) Relatively easy to shift into capital and maintain output
   (d) If capital costs are small fraction of total costs, costs should decline, output increase, x_b could increase or decrease, total land in city should become more valuable
   (e) If capital costs are a large fraction of total costs, costs should increase, total land value should fall

9. In the city with only a business sector, what would be the effect on welfare of a £1 billion investment in transportation that reduced t by 20%?

   (a) There are three effects to consider: the effects on businesses, the effects on landowners, and the effects on final output prices and hence on consumers.

   (b) Since all firms earn zero profits, there is no effect on business profits.

   (c) The effect on landowners equals the change in rents. Let t_1 be the transport cost prior to the investment, let x_{b1} be the boundary of the city prior to the investment and let $r(x, t_1)$ be the rent function prior to the investment. Similarly, let t_2 be the transport cost after the investment, let x_{b2} be the boundary of the city after the investment, and let $r(x, t_2)$ be the rent function after the investment. The total change in land rents in the city is

   $$\Delta r = \int_0^{x_{b2}} r(x, t_2)\, dx - \int_0^{x_{b1}} r(x, t_1)\, dx.$$

   Draw a graph. This is the total value of land in the city after the investment minus the total value of land in the city before the investment. This measures the total change in the welfare of landowners. In this case, the total change is likely to be positive since total the total costs of production in the city have fallen (because transport costs have fallen). Since the value of land in the city is determined in this example by its value in production, the fall in transport costs make the land more valuable overall. It is possible that land rents decline at some locations.

   (d) Finally, if demand for the output of the city is not perfectly elastic, there will also be an effect on prices of the output. Total supply will increase in response to the investment. This total increase in supply will lower prices of the final output good. This will increase consumer surplus in the economy. If CS_2 is the consumer surplus after the price change and CS_1 is the consumer surplus before the price change. The total change in consumer welfare is $\Delta CS = CS_2 - CS_1$. This will be positive if demand is not perfectly elastic.

(e) The total benefit of the investment is $\Delta r + \Delta CS$

(f) The total cost of the investment is £1 billion plus any deadweight loss associated with raising the revenue required to pay for the investment.

4 Other extensions to basic model

This section lists various ways the simple model can be extended to account for features of the real world economy that we have ignored and briefly discusses some of these extensions.

4.1 Multiple city centres

Suppose not all workers commute to centre or not all businesses export from center. In reality, many people commute from the city centre to the suburbs. In reality, there is not a single city centre that is the destination. What is the center of London? The model can be extended by adding several centres. For example, suppose there were two centres? What would an equilibrium look like? In the simplest case, there is a single type of consumer and both centres are identical except for location. In this case, households living closer to centre 1 will commute to centre 1 and households living closer to centre 2 will commute to centre 2. Since all households are identical, all will obtain the same utility in equilibrium. Those households who live at locations equidistant from the two centres will be indifferent between commuting to centre 1 and centre 2. However, the complexity of the model increases with the number of centres and simple analytical statements about the equilibrium become more difficult to make. Computational models of this sort have been studied to understand cities with more than one centre.

4.2 Varying supply of land at every location or an endogenous supply of land at every location

The simple model assumes that the supply of land at each location is $S(x) = 2\pi x$. In reality, the amount of land at different locations will be more complicated because things like rivers, roads, swamps, and hills make some land unusable for housing or business. In this case, the model can be analysed quite simply using the assumption that $S(x) = f(x)$. The only difference from the baseline model is that the equilbrium relationship between supply and demand for land at location x becomes

$$f(x) = L^*(x) N(x)$$

implying that

$$N(x) = \frac{f(x)}{L^*(x)}.$$

The rest of the analysis is identical to the analysis of the baseline model.

More difficult is the case where the the supply of land at each location can be altered by investment. In this case the supply of land at location x will depend on the rent function. For example, suppose

$$S(x) = 2\pi x + r(x)$$

so that the supply of land increases with rent. Equilibrium in the land market then requires

$$N(x) = \frac{2\pi x + r(x)}{L^*(x)}.$$

The equilbrium equation for the close city model that requires the entire population to be housed becomes

$$N = \int_0^{x_B} \left(\frac{2\pi x + r(x)}{L^*(x)} \right) dx.$$

The equilbrium in this case can be analysed with a computer. The precise predictions of the model are somewhat more complicated to analyse than the baseline case.

4.3 Transport cost could depend on the number of people in the city, the distance to the centre, or on the number of people commuting through a location.

Transport costs could be $t(N)$ where transport cost depends on population. This might be the case if there is congestion. This case is straightforward to analyse using preceisely the same analysis as the base line model. Alternatively, transport costs could be $t(x)$ where transport cost depends on x.

Perhaps transport costs are higher close to the centre. This case can be analysed along the lines of the baseline model. However, precise predictions depend on what is assumed about the transport cost function and require a computer.

Alternatively, transport cost could be $t(n_c(x))$ where $n_c(x)$ is the number of commuters per unit land at distance x. Pick a distance x. Everyone who lives farther from the centre than x must commute through x. Therefore the number of people who must commute through location x is

$$N_c(x) = N - \int_0^x N(x')\,dx'$$

where $N(x)$ is the number of people living at distance x. Then by definition

$$n_c(x) = \frac{N_c(x)}{2\pi x}.$$

This allows transport cost to depend on how congested the transport network is. The higher is $n_c(x)$ at location x, the more people are crammed onto the network at location x. This model would assume that the marginal transport cost increases with $n_c(x)$.

In each of these cases, equilibrium can be computed and studied. How would you expect the equilibrium in an urban economy to be affected by these alternative assumptions?

4.4 Moving costs, timing of sales, dynamics

The basic model assumed that there is only one period and that each consumer or firm can costlessly move to their desired location. There is no cost of moving and there is no future. In reality, moving costs are important and when making location decisions people worry not only about the current equilibrium but also about future changes in the economy. As one example, with the current credit crisis, there is a great deal of uncertainty about not only overall housing prices but, more importantly from a spatial economic perspective, about how prices might change in different parts of the city. How much will prices fall in the City vs in Hackney? How much will the fall in East London vs. West London.

As another example, the transport infrastructure in East London is currently under development for the Olympics. Living near the construction is not very desirable. However, everyone knows that living near the new developments will be very valuable. Thus, many people were buying properties in East London in expectation of these future changes. If there were no moving costs, then this wouldn't matter. At any point in time, equilibrium in the spatial market would be instaneously established just as the simple model specifies. At every point in time, people would move costlessly from East London to West London or from North to South to establish equilibrium.

However, if there are moving costs, then this is not the case. Suppose a household is at location x_0 paying rent $r_0(x_0)$ and currently obtains utility $V_0 = v(I - tx_0, p, r_0(x_0))$. Suppose rents change to $r_1(x)$ so that utility changes to $V_1 = v(I - tx_0, p, r_1(x_0))$. Suppose with the new rent $r_1(x)$, it is the case that there is an alternative location x_1 such that $v(I - tx_1, p, r_1(x_1)) > V_1$. That is, the household would prefer to move to location x_1 if there were no moving costs. Suppose moving costs are c. The household will not move if

$$v(I - c - tx_1, p, r_1(x_1)) < V_1 = v(I - tx_0, p, r_1(x_0)).$$

They will only move if the utility gain outweighs the cost.

If people plan ahead, then this moving cost will also have dynamic effects. A household wants to choose a location today, so that they are unlikely to have to move in the future. They would prefer to avoid paying moving costs if possible. To study the economics of spatial equilibrium with dynamics and moving costs requires an explicit model of dynamics. Mills page 148 has a brief discussion of speculation in urban economies. In general, each consumer would take into account both present and future payoffs when making a location choice and would only move when the benefits to moving outweigh the costs. Key parameters that will affect the dynamic equilibrium include 1) the same parameters that determine equilibrium in the static model, 2) expectations about future changes in the values of those parameters, 3) movings costs.

Some of these issues we will talk about later in the course, some will be addressed at least in part in the homework, others are beyond the scope of this course. In the next few lectures, we will focus on intracity transport of people, goods and services. To date, our model of transportation cost has been very simple. Every household must pay a fixed cost per mile. We have not discussed at all what determines the transport cost, not what are some of the major issues surrounding the transportation industry. The next few lectures will examine some of the economic issues involved in the transportation inuistry and develop some economic models that address some aspects of transportation economics.