Econ 3021 – Urban Economics
Winter 2009
Assignment 4 Solutions: Housing

1.a. For renters, the budget constraints are

$$\begin{aligned} c_1 + p_r + s &= a_1 \\ c_2 &= rs \end{aligned}$$

or

$$c_1 + p_r + \frac{c_2}{r} = a_1.$$

For owners, the budget constraints are

$$\begin{aligned} c_1 + p_1 + s &= a_1 \\ c_{2L} &= rs + p_L \\ c_{2H} &= rs + p_H \end{aligned}$$

1.b. One way to solve this problem for renters is to plug the budget constraints into the objective function and solve

$$\max_s \{\ln(a_1 - s - p_r) + \beta \ln(rs)\}.$$

Note that it must be true

$$a_1 - s - p_r > 0 \text{ and } s > 0.$$

Therefore, we require

$$a_1 - p_r > 0.$$

The first order conditions are

$$\frac{-1}{(a_1 - s - p_r)} + \frac{\beta r}{rs} = 0.$$

Renters equate the marginal utility of consumption in period 1 to the marginal utility of consumption in period 2. By assumption, renters face no uncertainty. Optimal savings is

$$s^* = \frac{\beta(a_1 - p_r)}{1 + \beta}.$$

Therefore,

$$\begin{aligned} c_1^* &= \frac{1}{1+\beta}(a_1 - p_r) \\ c_2^* &= \frac{r\beta(a_1 - p_r)}{1+\beta}. \end{aligned}$$

For owners, we can also plug the budget constraints into the utility function to obtain the maximisation problem

$$\max_s \left\{ \begin{array}{c} \ln(a_1 - s - p_1) + \beta\pi_H \ln\left(rs + p_H^2\right) \\ +\beta(1-\pi_H)\ln\left(rs + p_L^2\right) \end{array} \right\}.$$

Here we require that

$$\begin{array}{rcl} a_1 - s - p_1 & > & 0 \\ rs + p_H^2 & > & 0 \\ rs + p_L^2 & > & 0. \end{array}$$

Since $p_L^2 < p_H^2$, this means that

$$\begin{array}{rcl} s & < & a_1 - p_1 \\ s & > & -\dfrac{p_L^2}{r}. \end{array}$$

Therefore,

$$\frac{-p_L^2}{r} < a_1 - p_1.$$

The first order conditions for owners are

$$\frac{-1}{a_1 - s - p_1} + \frac{\beta\pi_H r}{rs + p_H} + \frac{\beta(1-\pi_H)r}{rs + p_L} = 0$$

Owners equate the marginal utility of period 1 consumption to the expected value of period two marginal utility of consumption.

1.c. The utility of a renter is

$$v_{rent} = \ln\left(\frac{a_1 - p_r}{1+\beta}\right) + \beta\ln\left(\beta r\left(\frac{a_1 - p_r}{1+\beta}\right)\right).$$

The utility of an owner is

$$v_{own} = v(a_1, p_1, r, p_L, p_H, \pi_H).$$

Households choose to rent if $v_{rent} \geq v_{own}$. Rental utility is decreasing in p_r, increasing in a_1, and increasing in r. The utility of owners is increasing in a_1, decreasing in p_1, increasing in the average period 2 housing price, decreasing in the variance of period 2 prices, and increasing in r. One would expect that, given values of the other variables, there is a threshold value of p_r at which households are indifferent between owning and renting. One might also expect, that the riskiness of housing prices has more impact on the choice of households with low levels of assets.

1.d. A reduction in r, will have no impact on first period consumption of renters. It will however reduce second period consumption and lower the utility of renters. In this model renters are savers and so lowering interest rates reduces their utility. The same change in interest rates will reduce savings (or increase borrowing) of owners and so will increase first period consumption. Whether this increases or reduces owner utility will depend on whether, before the change, owners are net savers or net borrowers. If they are savers, it is likely to reduce utility. If they are borrowers, it is likely the increase utility. The net impact on the choice between buying and renting is unclear.