1. Assume there are two types of firms and a single type of consumer in an urban economy. Both firms and consumers can move freely migrate into and out of the city. In equilibrium, firms earn zero profits and consumers obtain utility V_c. The rent for land at the boundary of the city is fixed at r_A. All firms maximise profits by choosing a location x, capital input K, and land input L. Firm i has production function

$$y_i = A_i K_i^{0.5} L_i^{0.5}$$

and transport costs t_i. The net price of firm $i's$ output is $p_i - t_i x_i$ per unit where p_i is the export price, t_i is the transport cost per mile per unit of output, and x_i is the location choice of firm i. The cost of capital is $q = 0.25$. Consumers choose location x, land L and food consumption F to maximise utility

$$u = F^{0.5} L^{0.5}$$

subject to the budget constraint

$$p_F F + r(x) L = I_c - t_c x$$

where p_F is the retail price of food, I_c is consumer income, and t_c is the household transport cost per mile. All firms export from the centre. All consumers commute to the centre to work. It turns out, that in an equilibrium of this economy, the firms' bid rent functions are

$$b_1(x) = A_1 (p_1 - t_1 x)^2$$
$$b_2(x) = A_2 (p_2 - t_2 x)^2$$

and the consumer's bid rent function is

$$b_c(x) = \frac{(I_c - t_c x)^2}{4 p_c V_c^2}$$

(a) Explain under what conditions, only firm type 1 operates in the city. That is, what values of the parameters result in an equilibrium in which only firm type 1 choose to operate in the city? What is the equilibrium rent function in this case? What is the boundary of the city in this case?

(b) Explain under what conditions, firm type 1, firm type 2, and consumers all choose to locate in the city and firm type 1 chooses locations $0 \le x \le x_1$, firm type 2 chooses locations $x_1 \le x \le x_2$, and consumers choose locations $x_2 \le x \le x_B$ where x_B is the equilibrium boundary of the city? What is the equilibrium rent function in this case? What are the boundaries of the 3 sections of the city in this case?

2. Consider equilibrium in an economy with two cities and two types of consumers (Family types and Single types). Family types obtain utility V_F if they live in city B. Single types obtain utility V_S if they live in city B. All consumers in city A choose consumption C, land L, and location x to maximise utility. For those living in city A, the variable x measures the distance from the centre of the city. The Family types who live in city A have utility function

$$u_F(C, L) = C^{0.5}(L-2)^{0.5}$$

while single types who live in city A have utility function

$$u_S(C, L) = C^{0.5}L^{0.5}.$$

Family types that choose to live in city A must consume at least 2 units of land. Family types have income I_F and single types have income I_S. Both types pay a price p for the consumption good. Each consumer at location x must commute to the centre of the city and must pay $r(x)$ pounds per unit of land consumed and commuting costs of pounds t_F and t_S per mile. The rent at the boundary of city A is fixed at r_A. Assume that the consumers can migrate freely between the two cities and that the cities are in spatial equilibrium.

(a) What is the budget constraint for each consumer type living in city A?

(b) Solve the consumer maximisation problems for the two consumer types assuming at least some consumers of both types live in city A in equilibrium. What are the demand functions for the consumption good and for land conditional on choosing city A and conditional on x?

(c) What are the bid rent functions of the two types? Explain what factors determine who will live closer to the centre of the city and who will live in city A and who will live in city B.