ECON3021 Urban Economics
Winter 2009
Assignment 1: Location choice and spatial equilibrium
The due date for this assignment is Monday January 26.

Reading assignment: Lecture notes 1, 2.

1. In a circular city, all consumers commute to the centre. Transportation costs for a consumer living at distance x are tx. All consumers have identical incomes y and identical preferences over consumption of b (bread) and h (housing). p the price of b is the same everywhere in the city while $r(x)$ the price of housing varies depending on distance from the centre. Let the utility function be $u(b,h) = (b - b_L)^{0.5} h^{0.5}$.

   (a) What are the first-order conditions characterising the consumers' optimal choices?

   (b) For fixed location x, solve for the optimal choices of b and h as functions of x and the other variables.

   (c) Solve for the differential equation describing the slope of the equilibrium land rent function.

   (d) What are the equilibrium conditions in this model? Draw a graph showing the approximate shape of the equilibrium price function.

   (e) In a full equilibrium of this city, how is the radius of the city and the population determined (Hint: What assumptions were made in the lecture to complete the model?)

2. Consider a circular city with *free migration* and *fixed boundary rent* of r_b per unit of land. Initially, the city is in spatial equilibrium with equilibrium rent function $r^0(x)$. That is, $r^0(x)$ is the rent per unit of land at a distance of x miles from the centre. Initially all land within the city is available for housing and the supply of land at distance x from the centre is $2\pi x$. The city has initial equilibrium population N^0 and equilibrium boundary of $x_b^0 > 1$ miles. People are free to move into and out of the city and obtain reservation utility V_R if they leave. Since the city is in equilibrium all residents initially obtain utility level $V^0 = V_R$. Each of the residents of the city is identical. Each obtains utility from C (a consumption good) and L (land) and has income I. A household living x miles from the centre of the city, faces prices for C and L of p and $r^0(x)$ respectively, and must commute to the centre at cost of t per mile. Suppose a hurricane destroys all the housing within 1 mile of the centre so that after the hurricane the supply of land available for housing at each distance $x \leq 1$ is zero. The supply of land at other locations is unaffected. Explain using words and graphs how a new equilibrium will be determined, what it will look like, and how it will compare to the initial equilibrium.