Lecture 12 – Transportation

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1 Introduction

1. Modal choice
2. Last words on transport

2 Modal choice

Each person i faces the problem of choosing which transportation mode to use to travel to work. Suppose there are J transportation modes as well as an option to not travel at all. Represent each mode or option by an index j and let $j = 0$ represent the option of not traveling at all. This means that the set of feasible options is $\{0, 1, ..., J\}$ and each $j \in \{0, 1, ..., J\}$. The options might be

Travel options

$j = 0$	Not travel at all
$j = 1$	Travel by car
$j = 2$	Travel by bus
$j = 3$	Travel by train

The utility person i obtains from using transport mode j is given by

$$U_{ij} = f(x_i, z_j, \varepsilon_{ij}, \beta). \tag{1}$$

Here, U_{ij} is the utility person i obtains if they use mode j, x_i is a set of observable characteristics of the consumer, z_j is a set of observable characteristics of the mode, and ε_{ij} is an unobservable characteristic specific to person i and mode j. The variable β represents a vector of parameters that also affect the choice.

An example of this is

$$U_{ij} = (\beta_1 + \beta_2 w_i)\, t_{ij} + \beta_3 p_{ij} + \beta_4 c_j + \varepsilon_{ij}. \tag{2}$$

In this example w_i is the wage of person i, t_{ij} is the transit time of person i on mode j, p_{ij} is the price charged to person i on mode j, c_j is the degree of comfort of mode j.

Assume that people can only choose one mode of transport and that they choose one of the options that gives them the greatest utility.

That is, person i chooses mode j if

$$U_{ij} > U_{ik} \text{ for all } k \neq j.$$

If $U_{ij} = U_{ik}$ and both are larger than all other options, the consumer is indifferent between j and k. In this case, assume the consumer picks one of the two at random.

In the general model, if one knows the values of β, x_i and the distribution of ε_{ij}, then one can predict demand responses to changes in z_j. How? In general, different people will react differently to changes in z_j. Why? One can also evaluate the welfare effects of change in z_j using this model. How? These welfare impacts will also vary across people. Why?

If one does not know the values of β, then they must be estimated by matching the predictions of the model to some data on transport mode choices. The way this is done is the following.

Define

$$d_{ij} = \left\{ \begin{array}{c} 1 \text{ if person } i \text{ uses mode } j \\ 0 \text{ if person } i \text{ does not use mode } j \end{array} \right\}.$$

If one does not know the value of β, but can observe a data set that contains information on (x_i, z_j, d_{ij}) for $i = 1, 2, ..., N$ people and for $j = 1, 2, ..., J$ modes of transportation, then one can estimate β using this data and this model.

The general procedure to estimate β is the following. The data can be used to measure what fraction of people of each type use each mode. The model predicts that the fraction of each type who use each mode depends on β. Different values of β will lead to different model predictions of how many people of each type use each mode. The value of β that yields model predictions about the fraction of each type who use each mode that are closest to the observed fractions, is the value of β that is most consistent with the data.

To see this in more detail, it is easiest to consider the context of the model in (2) in an example in which $J = 2$ and everyone chooses either 1 or 2. That is, no one chooses $j = 0$, not to travel.

Since each person maximises utility, $d_{i2} = 1$ only if $U_{i2} \geq U_{i1}$. Assume both are larger than U_{i0} for all i. Since U_{i2} is defined in (2), $U_{i2} \geq U_{i1}$ only if

$$\begin{aligned}(\beta_1 + \beta_2 w_i)\, t_{i2} + \beta_3 p_{i2} + \beta_4 c_2 + \varepsilon_{i2} &\geq \\ (\beta_1 + \beta_2 w_i)\, t_{i1} + \beta_3 p_{i1} + \beta_4 c_1 + \varepsilon_{i1}.\end{aligned}$$

This inequality is only true only if

$$\varepsilon_{i1} - \varepsilon_{i2} \leq \begin{pmatrix} (\beta_1 + \beta_2 w_i)\,(t_{i2} - t_{i1}) + \\ \beta_3\,(p_{i2} - p_{i1}) + \\ \beta_4\,(c_2 - c_1) \end{pmatrix}. \tag{3}$$

Define

$$\Delta U\,(\beta, w, t, p, c) = \begin{pmatrix} (\beta_1 + \beta_2 w_i)\,(t_{i2} - t_{i1}) + \\ \beta_3\,(p_{i2} - p_{i1}) + \\ \beta_4\,(c_2 - c_1) \end{pmatrix}.$$

This variable ΔU is simply shorthand notation for the right side of (3). Summarising the above statements, person i uses mode 2 only if

$$\varepsilon_{i1} - \varepsilon_{i2} \leq \Delta U_{21}.$$

Since, we assume that the $\varepsilon' s$ are not observed in the data, the probability that person i uses mode 2 equals

$$\Pr\left(\varepsilon_{i1} - \varepsilon_{i2} \leq \Delta U\,(\beta, w, t, p, c)\right). \tag{4}$$

This probability clearly depends on the value of β and on the distribution of $\Delta\varepsilon = \varepsilon_{i1} - \varepsilon_{i2}$.

The value of β that makes the probability in (4) close to the population fraction observed using mode 2, is the best estimate of the true value of β.

1. Questions

   (a) Graph the distribution of $\Delta\varepsilon$ and resulting modal choice

(b) Show two different distributions, uniform on $[0, 1]$ and skewed toward zero, and the resulting demand for modes.

(c) What determines the distribution of $\Delta\varepsilon$?

(d) Show the effect of a change in t_{i1} on demand for modes 1 and 2

$$\begin{aligned} \frac{\partial (\Delta U)}{\partial t_{i1}} &= -(\beta_1 + \beta_2 w_i) \\ &> 0 \end{aligned}$$

2. Effect on welfare of change in t_{i1}

(a) $\Delta U(t_{i1}^0)$ increases to $\Delta U(t_{i1}^1)$.

(b) Consumers with $\Delta\varepsilon \leq \Delta U(t_{i1}^0) < \Delta U(t_{i1}^1)$ use mode 2 both before and after change. No change in welfare.

$$\Delta W_i = 0$$

(c) Consumers with $\Delta U(t_{i1}^0) \leq \Delta\varepsilon \leq \Delta U(t_{i1}^1)$ switch from mode 1 to mode 2. Change in welfare of

$$\Delta W_i = \Delta U\left(t_{i1}^0\right) - \Delta\varepsilon_i \leq 0$$

(d) Consumers with $\Delta U(t_{i1}^1) \leq \Delta\varepsilon$ use mode 1 before and after change. Change in welfare is

$$\begin{aligned} \Delta W_i &= \left[(\beta_1 + \beta_2 w_i) t_{i1}^1 + \beta_3 p_{i1} + \beta_4 c_1 + \varepsilon_{i1}\right] - \\ &\quad \left[(\beta_1 + \beta_2 w_i) t_{i1}^0 + \beta_3 p_{i1} + \beta_4 c_1 + \varepsilon_{i1}\right] \\ &= (\beta_1 + \beta_2 w_i)\left(t_{i1}^1 - t_{i1}^0\right) \end{aligned}$$

(e) Total change found by adding up within and across groups.

With the above model, we have the elements of a positive and normative theory of mode choice. The model is a positive model because we can use it to explain observed mode choices and to predict responses to changing economic circumstances. The model is a normative model because we can use it to measure the impacts on welfare of various policies that impact z_j.

To be useful, the model requires information about U_{ij}. This information can only be obtained by: 1) asking people, 2) observing their choices in different circumstances.

The model as set up here does not have any dynamic dimension. It assumes each person is only making one choice at a single point in time. A more realistic model might model consumers short term choices and long term choices differently. In such a model, the parameters of the utility function might change over time.