Lecture 10 - Congestion

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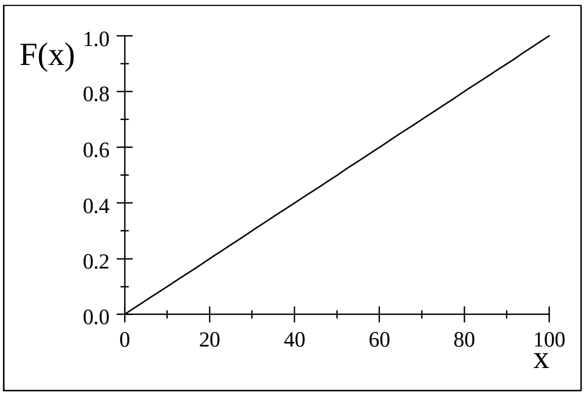
1 Introduction

1. Last lecture sketched a general social welfare maximisation problem and determination of equilibrium in a transportation economy.
2. Today fill in the details of a simple transportation model with congestion.
   (a) Equilibrium.
   (b) Efficiency.
   (c) Optimality.
      i. Efficiency versus distribution of costs and benefits.
   (d) Potential policy interventions.
   (e) Complications.

2 Congestion and pricing

1. Congestion is a problem, externality.
   (a) Externality.
   (b) Policies: Taxes, quotas, regulations.
   (c) Long run issues, investment.
   (d) Realistic pricing: tolls, gas tax, parking tax, subsidize public transit
2. Congestion model.
   (a) In a simple economy, there are N consumers. All the consumers commute to work either by train or by car on the public highway. If they commute by car the personal or private cost of travel is $C(n_d)$ where n_d is the total number of drivers on the highway. For example, it might be the case that $C(n_d) = F + n_d$. The cost of travel on the highway rises with the number of drivers on the highway. The cost of highway travel is the same for every consumer on the highway. For

each person on the train, however, the cost of travel is c_i. Each person i has a different cost of train travel perhaps because each person lives at a different distance from the train station. We assume that $c_i \in [0, 100]$ and that c_i has a distribution in the population that is uniform between 0 and 100. That is, $c_i \in [0, 100]$ and if one picks an arbitrary number x between 0 and 100, then the fraction of the population with c_i less than x is $\frac{x}{100}$. This can also be viewed graphically. Let $F(x)$ equal the fraction of the population with c_i less than x. Then $F(x) = \frac{x}{100}$



Fraction of population with c_i less than x

The graph shows for instance that, $\frac{1}{10}$ the population has cost c_i less than 10. $\frac{1}{2}$ the population has cost less than 50. $\frac{6}{10}$ has cost less than 60. The total number of people in the city is N. The total number with cost less than x is $\frac{Nx}{100}$.

(b) Each consumer uses the mode of transport that is cheapest.

(c) Demand for highway travel

i. Suppose the cost of highway travel is c_h. All the people who have c_i larger than or equal to c_h prefer to drive. All who have $c_i < c_h$ prefer

ii. Let n_h be the number of people who prefer driving to using the train.

iii. How many people have $c_i \geq c_h$?

$$n_h = N\left(1 - \frac{c_h}{100}\right)$$

iv. This is the demand for highway travel.

3. Equilibrium in congestion model.

(a) Demand for road travel satisfies

$$n_h = N\left(1 - \frac{c_h}{100}\right)$$

(b) But, by assumption

$$c_h = C(n_d)$$

where n_d is the number of drivers.

(c) An equilibrium results when $n_h = n_d$. That is when the number of people who want to drive on the highway equals the number of people on the highway.

(d) Compute equilibrium in a special case. Suppose $C(n_h) = F + n_h$. Then

$$n_h = N\left(1 - \frac{F + n_h}{100}\right)$$

$$n_h\left(\frac{100 + N}{100}\right) = N\left(1 - \frac{F}{100}\right)$$

$$n_h = \frac{N(100 - F)}{100 + N}$$

(e) If $N = 100$

$$n_h = \frac{100 - F}{2}.$$

(f) Draw the picture with n_h on the horizontal axis and c_h on the vertical axis.

4. Efficient road use in congestion model.

(a) The above equilibrium level of road use is not efficient. Each person who uses the road only considers his private benefit and private cost of road use. However, each additional road user imposes a cost on other road users. Each additional road user raises the costs of all other road users.

(b) What would an efficient level of road use be in this model? Minimise total cost of transport.

(c) Let n_h be the number of people on the highway.

(d) The total (social) costs of transport in this case equal the total transport costs for those on the highway plus the total transport costs for those on the train.

(e) The total transport costs for those on the highway are $C_r = C(n_h) \cdot n_h$.

(f) The total transport costs of those on the train equals the sum over the costs of different individuals on the train.

(g) To minimise costs, those with low cost should take the train.

(h) If n_h are on the road, then $N - n_h$ are on the train and the fraction on the train is $\frac{N-n_h}{N}$. The lowest cost train traveler has $c = 0$. The highest cost train traveler has $c = c_h$ where

$$c_h = 100\left(\frac{N - n_h}{N}\right).$$

(i) How many people of type $[c, c + dc]$ are there? $N \cdot \frac{dc}{100}$. The cost per person in this group is c. The total costs of those on the train is

$$C_T = \int_0^{c_h} c \cdot \frac{N}{100} \cdot dc$$

or

$$\begin{aligned} C_T &= \frac{N}{100}\frac{c_h^2}{2} \\ &= \frac{N}{100}\frac{\left(100\left(\frac{N-n_h}{N}\right)\right)^2}{2} \\ &= \frac{100}{N}\frac{(N - n_h)^2}{2} \end{aligned}$$

(j) The total social transport costs are

$$TC = C(n_h)\, n_h + 50\frac{(N - n_h)^2}{N}.$$

The cost minimising solution solves

$$\frac{\partial TC}{\partial n_h} = C(n_h) + \frac{\partial C(n_h)}{\partial n_h} n_h - 100\frac{(N - n_h)}{N} = 0$$

(k) The marginal social cost of adding one more person to the highway equals the marginal social benefit.

$$\underset{\text{(private cost)}}{C(n_h)} + \underset{\text{(external cost)}}{n_h \frac{\partial C}{\partial n_h}} - \underset{\text{(social benefit)}}{100\frac{(N-n_h)}{N}} = 0$$

(l) Recall that in the equilibrium problem, the equilibrium condition was that the number of people on the road increased until the private benefit of adding one person equaled the private cost.

$$\underset{\text{(private cost)}}{C(n_h)} - \underset{\text{(private benefit)}}{100\frac{(N-n_h)}{N}} = 0$$

If $\frac{dC}{dN} = 0$, then the equilibrium solution will equal the optimum solution. Otherwise, the equilibrium number of people on the road will exceed the optimum number.

5. Efficient congestion tax.

   (a) One solution that can obtain the optimum is to charge every road user a tax equal to $n_h * \frac{\partial C}{\partial n_h}$.

   (b) With this tax, the private cost will equal the social cost and the equilibrium number of drivers will equal the optimum number.

6. This is efficient. Need not be optimal because takes no account of who benefits and who loses from efficiency improvement.

7. Problems with efficient tax.

   (a) Political problems.

      i. Those who remain on train are not affected by the policy.

      ii. Those who switch from highway to train are made worse off. They do not pay the tax, but switch to a mode of transport that is higher cost for them.

      iii. Those who remain on the road must pay the tax but are partially compensated by reduced congestion. Nevertheless, in net they are worse off.

      iv. The government that collects the revenue is better off, they have a large sum of revenue.

      v. Unless the revenue is residistributed to those who are harmed by the policy, there may be no political support for the tax.

   (b) Quota or voucher or permit system.

      i. Suppose n_h^* is the efficient number of road users and the government creates n_h^* permits for road use.

      ii. Suppose the government distributes these permits to those who are on the highway giving more to those with higher costs of train travel.

      iii. The equiibrium price of ther permits will adjust until

$$C(n_h^*) + p = N - n_h^*$$

      where

$$p = n_h^* \cdot \frac{\partial C}{\partial n_h}$$

         A. This will result in the efficient number of road users.

         B. The people who remain on the road will be net buyers of vouchers. They will support the policy because the cost they have to pay will now be less than the benefit they obtain from lower congestion.

         C. The people who switch from road to train will support the policy because they will be net sellers of vouchers. The revenue will compensate them for the higher cost of transport on the train.

D. Everyone is better off than in the equilibrium without permits.

(c) If optimality requires further redistribution of income, such redistribution could in principle and in part be achieved by the proper allocation of permits.

8. In general any scheme for addressing congestion should address both efficiency and distributional issues. That is, the distribution of the costs or benefits of the scheme. Most congestion tax proposals do not address distributional issues very well. It is possible to design systems that address both efficiency and distributional issues. For instance, a permit system can address efficiency by restricting the number of users and forcing users to pay the marginal social cost of use. If properly designed, it can address distributional issues by choosing the initial distribution of permits and by allowing people to buy and sell permits.

3 Problems with realistic congestion pricing

1. Congestion charging and toll collecting practices worsen congestion because it is costly to collect the tax. If the cost of congestion charging is high, it may be optimal not to charge.

2. The problem above assumes that the government can easily calculate the optimal tax or the optimal number of permits. This requires the government to know $C(n_h)$ and the distribution of costs c_i. In reality, both of these are unknown and vary by time of day and by location. They must be estimated and it may not be possible to calculate exactly the optimal congestion tax or the optimal number of road permits.

3. Electronic toll collection or collection by post is technologically possible but involves setup costs, and is not costless. Additionally, privacy concerns have not been addressed in this analysis.

4. Alternative taxes such as a petrol tax or a parking tax have been used to approximate the optimal toll. The petrol tax does not vary by time of day or location. Also, neither of these taxes is designed to address distributional concerns.

5. Alternatively, instead of taxing road transport, a government can subsidize alternatives like public transit. However, to do this properly one must consider the more general problem of how to price both highways and alternatives like public transit to obtain optimal use of each resource.