Lecture 5 - Locational Equilibrium Continued

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23 January 2008

1 Introduction

1. Review.
2. Properties of equilibrium.
3. 4 types of equilibrium.
4. Multiple types of consumer.

2 Review

1. Locational equilibrium.
   (a) Inputs to equilibrium: $t, N, r_A, U(C, L), I, p$. These are the *parameters* of the problem. They are fixed, specified by us, determined outside the model.
   (b) Equilibrium conditions:
      i. Consumers maximise.
      ii. Locational equilibrium.
      iii. Land market equilibrium.
      iv. Consumption good market equilibrium.
   (c) Outputs.
      i. Conditional demand functions: C^*, L^* solve consumer first order conditions holding x fixed. Express demand for consumption and land as functions of prices and income.

ii. Rent function: $r(x, r_0)$, r_0. Expresses rent per unit of land as function of distance from centre.

iii. City size: x_B.

iv. Welfare: $V^* = U(C^*, L^*)$.

v. These are determined in equilibrium.

(d) By choosing different values for the parameters, we can analyse how the outputs of the problem vary.

3 Properties of equilibrium

1. When locational equilibrium condition holds, no change in utility when moving from x_1 to x_2. Change in transport costs exactly compensates for change in price of land. Pure substitution effect. Draw graph.

2. Move from x_1 to $x_2 > x_1$.

   (a) $r(x_1) > r(x_2)$

   (b) $L^*(p, r(x_1), I - tx_1) < L^*(p, r(x_2), I - tx_2)$ because of substitution effect

   (c) The slope of rent function is

$$\frac{dr}{dx} = -\frac{t}{L^*(p_F, r(x), x)}.$$

   (d) Hence, $\frac{-t}{L^*(x_1)} < \frac{-t}{L^*(x_2)}$ and so $\left|\frac{dr(x_1)}{dx}\right| > \left|\frac{dr(x_2)}{dx}\right|$.

   (e) Why do rents increase so dramatically as move to centre?

      i. Higher rents balance lower transport costs. But total rent is $r(x) * L^*$. As rents increase, people near centre consume less land. In order to maintain equilibrium, rent must increase more than proportional to distance.

      ii. Change in rate of increase depends on ability to substitute into C.

      iii. If all consumers need to consume same L^*, impossible to substitute for land, rate of increase is constant.

iv. If eating a little bit more in a much smaller flat maintains same utility than change in slope of rent very fast.

v. Other reasons rate of increase so dramatic.

A. We'll see later, that if different types of people (families with children, people who like a lot of space, different incomes) live in the city who have relatively high demands for land at the centre of the city, then the rate at which rents increase as we move toward the centre will increase.

B. Other demands for land at center, business, transport, retail, government also generate upward pressure on central land rents.

C. Congestion can lead to upward pressure on central land rents.

4 Alternative assumptions about city

1. Model 1: closed city, free boundary.

   (a) Population fixed at N, urban boundary free x_B, boundary rent fixed r_F.

   (b) This is the example we already studied.

   (c) This is the model of a closed city with a free boundary. The city is closed because no migration is allowed. The boundary is free because land can be added to or subtracted from the city by taking land from rural production or giving up land for rural production.

   (d) x_B and V adjust until equilibrium is attained.

2. Model 2: closed city, fixed boundary.

   (a) Population N fixed, urban boundary fixed x_B, boundary rent free.

   (b) This is a model with a closed city and a fixed boundary. The city is closed because migration is not allowed. The boundary is fixed because the total amount of land is fixed perhaps because the city is located on an island. The boundary rent may adjust and is not fixed at a pre-specified level.

(c) In an equilibirum in this city, r_F and V adjust until equilibrium is attained.

(d) How should the equilibrium conditions obtained for model 1 be rewritten to characterize equilibrium in this city?

3. Model 3: open city, free boundary.

(a) Population N is variable, the boundary x_B is free, and the utility level is fixed at V. The rural rent is fixed at r_F.

(b) The is a model of an open city with a free boundary. The city is open because migration is allowed. The boundary is free because land can be freely added to or taken away from the city.

(c) People migrate to the city if the utility obtained in the city is higher than V, the utility obtainable elsewhere.

(d) People leave the city if the utility obtained elswehere, V, is higher than the utility obtainable in the city.

(e) In equilibrium, the utility obtained in the city is $U\left(C^*\left(x\right), L^*\left(x\right)\right)$. In equilibrium, the utility obtained in the city must equal V.

(f) The population, N, and the boundary x_B adjust until equilibrium is attained.

(g) How should the equilibrium conditions obtained in model 1 be adjusted to characterize equilibrium in model 3?

4. Model 4: open city, fixed boundary.

(a) Population N is variable as is the boundary rent r_F. The utility level V is fixed as is the boundary x_B.

(b) This is a model of an open city with a fixed boundary. The city is open because migration is allowed. The boundary is fixed because the total quantity of land available is fixed.

(c) In equilibrium, the utility obtained in the city is $U\left(C^*\left(x\right), L^*\left(x\right)\right)$. In equilibrium, the utility obtained in the city must equal V.

(d) The population, N, and the boundary rent r_F adjust until equilibrium is attained.

(e) How should the equilibrium conditions obtained in model 1 be adjusted to characterize equilibrium in model 4?