C44 Urban Economics Winter 2008
Assignment 3: Transportation
The due date for this assignment is Monday 25 February. Please place in course mailbox.

1. Utility functions for bus and train travel are

$$u_{bi} = \beta_0 + \beta_1 w_i t_b - (\beta_2 + \beta_3 w_i)\, p_b + \varepsilon_{ib}$$

$$u_{ti} = \beta_1 w_i (t_{t0} + t_{t1} d_i) - (\beta_2 + \beta_3 w_i)\, p_t$$

where w_i is the wage (£ per hour) of person i, t_b is the bus transit time, p_b is the bus fare, $t_{t0} + t_{t1} d_i$ is the train transit time for person i, p_t is the train fare, d_i is the distance from the home of person i to the train station, and ε_{ib} is heterogeneity in other factors that affect the utility of bus travel. Suppose $t_b = 1$, $t_{t0} = 0.5$, $t_{t1} = 1$, $p_b = 1$, $p_t = 3$ and $w_i = 1$. Further suppose $\beta_0 = -12$, $\beta_1 = -4$, $\beta_2 = 5$, and $\beta_3 = -1$. Finally, assume that d_i is uniformly distributed between 0 and 1 and that ε_{ib} is uniformly distributed between 0 and 10.

   (a) What is the marginal value of time in this model? Why?

   (b) If everyone makes 1 trip, what fraction of the population travels by bus and what fraction by train?

   (c) Assume that all bus and train fares are paid to the government. What would be the impact on travel choices, utilities, and government revenue of an increase in the train fare from £3 to £4 pounds? Is this a good policy?

2. All transport in a city is provided by two tube lines, the Northern Line and the Victoria Line. The trains are owned and operated by the government. Both lines suffer from congestion during the morning peak travel period. During this period, the per passenger cost (to the consumer) of travel on the Northern Line is $c_n = f_n + b_n p_n$ where c_n is the per passenger cost of travel (to the consumer) and p_n is the number of passengers on the line during the period. Similarly, the per passenger cost of travel (to the consumer) on the Victoria Line is $c_v = f_v + b_v p_v$ where c_v is the per passenger cost of travel (to the consumer) and p_v is the number of passengers on the line during the period. On both lines, the per passenger cost to the consumer has two components, a fare component (f_n, f_v) and a time cost component $(b_n p_n, b_v p_v)$. The time cost component increases with the number of passengers. The social cost of travel on the Northern line is $s_n = k_n + b_n p_n$ and the social cost of travel on the Victoria line is $s_v = k_v + b_v p_v$. In general, it may be the case that $k_n \neq f_n$ and $k_v \neq f_v$. Assume there are N commuters in the city all of whom make one trip during the morning peak period if the cost of travel is less than or equal to c_0. Each commuter uses the lowest cost tube line.

(a) Assuming c_0 is large so that all consumers travel, what is the equilibrium number of passengers on each line and the equilibrium social cost of travel on each line? What is the equilibrium revenue? Explain.

(b) What is the efficient number of passengers on each line?

(c) Assuming that initially $f_n = f_v = 0$, is the initial equilibrium efficient. What is the impact of increasing the fares to $f_n = f_v = 1$? What level of fares would yield an efficient outcome? Who benefits and who loses if the government increases fares to the efficient level?

(d) Under what circumstances is it efficient to set $f_v = f_n$?