

1. Assume two types of firms and a single type of consumer coexist in a city. All firms maximise profits by choosing a location x , capital input K , and land input L . Consumers choose location x , land L and consumption C to maximise utility. All firms export from the centre. All consumers commute to the centre to work. Firms of type 1 and 2 have transport cost of t_1 and t_2 respectively per mile per unit of output. Consumers have transport cost per mile of t_3 . In a spatial equilibrium, who locates closest to the centre? Who locates farthest away? Why? Draw a graph showing the qualitative features of a possible equilibrium along with the associated bid rent functions of the three types.
2. Consider equilibrium in an economy with two cities and two types of consumers (Family types and Single types). Family types obtain utility V_F if they live in city B . Single types obtain utility V_S if they live in city B . All consumers in city A choose consumption C , land L , and location x to maximise utility. For those living in city A , the variable x measures the distance from the centre of the city. The Family types who live in city A have utility function

$$u_F(C, L) = C^{0.5} (L - 2)^{0.5}$$

while single types who live in city A have utility function

$$u_S(C, L) = C^{0.5} L^{0.5}.$$

Family types that choose to live in city A must consume at least 2 units of land. Family types have income I_F and single types have income I_S . Both types pay a price p for the consumption good. Each consumer at location x must commute to the centre of the city and must pay $r(x)$ pounds per unit of land consumed and commuting costs of pounds t_F and t_S per mile. The rent at the boundary of city A is fixed at r_A . Assume that the consumers can migrate freely between the two cities and that the cities are in spatial equilibrium.

- (a) What is the budget constraint for each consumer type living in city A ?
- (b) Solve the consumer maximisation problems for the two consumer types assuming at least some consumers of both types live in city A in equilibrium. What are the demand functions for the consumption good and for land conditional on choosing city A and conditional on x ?
- (c) What are the bid rent functions of the two types? Explain what factors determine who will live closer to the centre of the city and who will live in city A and who will live in city B .