ECON3021 Urban Economics
Winter 2008
Assignment 1: Location choice and spatial equilibrium
Due Monday January 21.

Reading assignment: Lecture notes 1, 2, 3, and 4.

1. In a circular city, all consumers commute to the centre. Transportation costs for a consumer living at distance x are $(t_1 + t_2 N)\,x$ where $t_1 + t_2 N$ is the transport cost per mile. Note that if $t_2 > 0$ transport cost per mile increases with population. All consumers have identical incomes I. Consumers also have identical preferences over consumption of b (bread) and h (housing). The price of b is p while $r(x)$ is the price of housing at distance x from the centre. Let the utility function be $u(b,h) = \alpha \ln(b - b_0) + (1-\alpha)\ln(h - h_0)$. The rent for land not used in housing is fixed at r_A. The population is fixed at N.

   (a) What are the first-order conditions characterising the consumers' optimal choices of (b, h, x)?

   (b) For fixed location x, solve for the optimal choices of b and h as functions of x and the other variables.

   (c) What are the equilibrium conditions in this model? Draw a graph showing the approximate shape of the equilibrium price function and explain how the radius and population of the city are determined.

   (d) Assume t_2 is positive and analyse how the equilibrium will change if N increases by 10%.

2. Consider the fictional city of London. It has a population of N consumers each of whom has income I_0. The boundary of the city is fixed at x_b. The supply of housing at every location is $2\pi x$. Initially, everyone commutes to the centre at cost of t_0 per mile. The transport cost can be decreased by investing in Cross-Rail. If i_t is invested in Cross-Rail the transport cost will fall to

$$t = \frac{1}{2} t_0 \left(1 + e^{-i_t}\right).$$

   At the same time, the transport investment must be paid for out of taxes. After tax income is

$$I = I_0 - \frac{i_t}{N}.$$

   Consumers in the city have utility function

$$U(c,h) = c^{0.5} h^{0.5}.$$

   The cost of housing at location x is $r(x)$ and the cost of the consumption good is p. Assume investment in transport is paid for out of income. In an equilibrium of this city, the consumers have demand functions

$$\begin{aligned} c &= \frac{1}{2}\frac{I - tx}{p} \\ h &= \frac{1}{2}\frac{I - tx}{r(x)}. \end{aligned}$$

It turns out that in this example the rent function satisfies

$$r(x) = r_0 (I - tx)^2$$

with

$$r_0 = \frac{N}{\pi x_b^2 \left(2I - \frac{4tx_b}{3}\right)}.$$

(a) Show that this rent function satisfies the locational equilibrium condition and show that all the conditions of locational equilibrium are satisfied. (Hint: In the notes to Lecture 4, we assume the level of the rent at the boundary r_b is fixed. Then the boundary of the city x_b adjusts until the equilibrium conditions are met. In this problem, the only difference is that the boundary x_b is fixed, and the rent at the boundary r_b adjusts to satisfy the conditions.)

(b) Calculate the utility level obtained by the consumers as a function of the parameters (I_0, t_0, x_b, i_t, N).

(c) Calculate the total value of land in the city.

(d) Suppose initially $i_t = 0$. Explain graphically, verbally, or mathematically what will happen to utilities, demand for land, demand for the consumption good, and rents if the government increases i_t to 1. What happens to the total value of land in the city? Should the government increase i_t? Why? Who benefits and who loses? Please be brief.