Supplementary Material to "Uncertain Demand, Consumer Loss Aversion, and Flat-Rate Tariffs"

Fabian Herweg
University of Munich

Konrad Mierendorff
University of Zurich

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Appendix B: Supplementary Material to Section 6: General Nonlinear Tariffs

The consumer's expected utility from accepting a direct mechanism $\langle q(\theta), P(\theta)\rangle$ for which truth-telling is a personal equilibrium and $q(\theta)$ is increasing, is given by

$$\begin{aligned}\mathbb{E}_\theta[V(\theta,\theta)] &= \int_{\underline{\theta}}^{\bar{\theta}} \left\{ u(q(\theta),\theta) - P(\theta) - \lambda \int_{\underline{\theta}}^{\theta} [P(\theta)-P(z)]f(z)\,dz \right\} f(\theta)d\theta. \\ &= \int_{\underline{\theta}}^{\bar{\theta}} [u(q(\theta),\theta)-P(\theta)]f(\theta)d\theta \\ &\quad - \lambda \int_{\underline{\theta}}^{\bar{\theta}} \int_{\theta}^{\bar{\theta}} [P(z)-P(\theta)]f(z)f(\theta)dzd\theta\end{aligned}$$

Using integration by parts, the last term—the expected loss—can be simplified further.

$$\begin{aligned}& \int_{\underline{\theta}}^{\bar{\theta}} \int_{\theta}^{\bar{\theta}} [P(z)-P(\theta)]f(z)f(\theta)dzd\theta \\ = \quad & \int_{\underline{\theta}}^{\bar{\theta}} \int_{\theta}^{\bar{\theta}} P(z)f(z)dz\, f(\theta)d\theta - \int_{\underline{\theta}}^{\bar{\theta}} P(\theta) \int_{\theta}^{\bar{\theta}} f(z)dz\, f(\theta)d\theta \\ = \quad & \int_{\theta}^{\bar{\theta}} P(z)f(z)dz\, F(\theta)\Big|_{\underline{\theta}}^{\bar{\theta}} - \int_{\underline{\theta}}^{\bar{\theta}} -P(\theta)f(\theta)F(\theta)d\theta - \int_{\underline{\theta}}^{\bar{\theta}} P(\theta)[1-F(\theta)]f(\theta)d\theta \\ = \quad & \int_{\underline{\theta}}^{\bar{\theta}} P(\theta)[2F(\theta)-1]f(\theta)d\theta.\end{aligned}$$

Thus,

$$\mathbb{E}_\theta[V(\theta,\theta)] = \int_{\underline{\theta}}^{\bar{\theta}} \left\{ u(q(\theta),\theta) - P(\theta)\xi(\theta) \right\} f(\theta)d\theta \qquad \text{(B.1)}$$

where $\xi(\theta) = 1 + \lambda(2F(\theta)-1)$.

E-mail: fabian.herweg@lrz.uni-muenchen.de (Herweg); konrad.mierendorff@econ.uzh.ch (Mierendorff)

We will formulate the monopolist's problem as an optimal control problem. In order to incorporate the participation constraint, we define

$$X(\theta)=\int_{\underline{\theta}}^{\theta}\left\{u(q(z),z)-P(z)\xi(z)\right\}f(z)dz,$$

and impose $X(\bar{\theta})\geq 0$. Note that $X(\bar{\theta})=\mathbb{E}_{\theta}[V(\theta,\theta)]$, $X(\underline{\theta})=0$, and

$$X'(\theta)=\{u(q(\theta),\theta)-P(\theta)\xi(\theta)\}f(\theta).$$

In order to solve the monopolist's problem, we want to express the payment rule as a function of the allocation rule. The integral version of the local PE constraint can be derived from the envelope theorem (see Milgrom, 2004):

$$V(\underline{\theta})+\int_{\underline{\theta}}^{\theta}\partial_{\theta}u(q(s),s)ds=u(q(\theta),\theta)-P(\theta)-\lambda\int_{\underline{\theta}}^{\theta}(P(\theta)-P(z))f(z)dz. \quad \text{(B.2)}$$

If P is absolutely continuous, we can equivalently work with the differential version of this constraint

$$P'(\theta)=\frac{\partial_{q}u(q(\theta),\theta)}{\rho(\theta)}q'(\theta). \quad \text{(B.3)}$$

where $\rho(\theta)=1+\lambda F(\theta)$. (This is the revenue equivalence formula (13) from Section 6.)

LEMMA B.1. *Let (q,P) be a mechanism such that q and P are absolutely continuous and satisfy* (B.3). *Then (q,P) satisfies (PE). Moreover,* (B.3) *is necessary for (PE).*

Proof. Differentiating $V(\theta,\varphi)$ with respect to the report θ, we get

$$\begin{aligned}\partial_{\theta}V(\theta,\varphi)&=\partial_{q}u(q(\theta),\varphi)q'(\theta)-P'(\theta)\rho(\theta)\\&=\left[\partial_{q}u(q(\theta),\varphi)-\partial_{q}u(q(\theta),\theta)\right]q'(\theta)\begin{cases}\leq 0, & \text{if } \theta>\varphi,\\ \geq 0, & \text{if } \theta<\varphi.\end{cases}\end{aligned}$$

Hence, truth-telling ($\theta=\varphi$) maximizes $V(\theta,\varphi)$.

(B.3) is clearly necessary for (PE) because it is the first-order condition of the consumer's maximization problem. □

We cannot assume a priori, however, that P is absolutely continuous. Instead, we first solve the monopolist's problem assuming that q is globally Lipschitz continuous (which implies absolute continuity). The following Lemma shows that this also implies Lipschitz continuity of P and hence absolute continuity.

LEMMA B.2. *Let $\langle q(\theta),P(\theta)\rangle_{\theta\in\Theta}$ be a direct mechanism that satisfies (PE). If q is globally Lipschitz continuous and Assumption 1 is fulfilled, then P is also globally Lipschitz continuous.*

Proof. For $\theta_1 < \theta_2$, the personal equilibrium constraint implies

$$\begin{aligned}
P(\theta_2) - P(\theta_1) &\leq u(q(\theta_2),\theta_2) - u(q(\theta_1),\theta_2) \\
&\quad - \lambda \int_{\underline{\theta}}^{\theta_2} [P(\theta_2) - P(z)]\, f(z)dz + \lambda \int_{\underline{\theta}}^{\theta_1} [P(\theta_1) - P(z)]\, f(z)dz \\
&= u(q(\theta_2),\theta_2) - u(q(\theta_1),\theta_2) \\
&\quad - \lambda \int_{\theta_1}^{\theta_2} [P(\theta_2) - P(z)]\, f(z)dz + \lambda\, [P(\theta_1) - P(\theta_2)]\, F(\theta_1) \\
&\leq u(q(\theta_2),\theta_2) - u(q(\theta_1),\theta_2) \\
&\leq K(\theta_2 - \theta_1)
\end{aligned}$$

for some $K < \infty$, where the last line follows from Lipschitz continuity of q and u (Assumption 1). □

Assuming Lipschitz continuity of q, we can restate the problem of the monopolistic seller as the following control problem.

CONTROL PROBLEM:

$$\max_{(q,P,X,v)} \int_{\underline{\theta}}^{\bar{\theta}} [P(\theta) - cq(\theta)] f(\theta)\, d\theta$$

subject to

$$\begin{aligned}
&\text{(i)} && P'(\theta) = \frac{\partial_q u(q,\theta)}{\rho(\theta)} v \\
&\text{(ii)} && q'(\theta) = v \\
&\text{(iii)} && X'(\theta) = (u(q,\theta) - P(\theta)\xi(\theta)) f(\theta) \\
&\text{(iv)} && v(\theta) \in [0,K] \\
&\text{(v)} && q(\underline{\theta}) \geq 0 \\
&\text{(vi)} && X(\underline{\theta}) = X(\bar{\theta}) = 0.
\end{aligned}$$

v is the control variable and P, q, and X are state variables. The state variable X and constraints (iii) and (vi) are introduced to capture the participation constraint. The constant $K > 0$ in constraint (iv) is the Lipschitz-constant.

In the control problem, we impose the additional assumption that q is Lipschitz continuous with constant K. We will show that for K sufficiently large, the constraint $v(\theta) \leq K$ is not binding in the solution to the control problem. The next Lemma shows that an optimal solution for which $v(\theta) < K$, is also an optimal solution to the seller's problem without the Lipschitz constraint.

LEMMA B.3. *Let (P,q,X,v) be an optimal solution to the control problem for some K, that satisfies*

$$v(\theta) < K, \qquad \textit{for all } \theta \in \Theta.$$

Then (q,P) is an optimal solution to the monopolist's problem without the assumption of Lipschitz continuity.

Proof. Suppose $\langle q^*(\theta), P^*(\theta)\rangle_{\theta\in\Theta}$ is an optimal mechanism for the monopolist (q^* need not be Lipschitz continuous). If we extend q^* to the real line by setting $q^*(\theta) = q^*(\underline{\theta})$ for $\theta < \underline{\theta}$, and $q^*(\theta) = q^*(\bar{\theta})$ for $\theta > \bar{\theta}$, we can approximate q^* by

$$q_k(\theta) := k\int_{\theta-\frac{1}{2k}}^{\theta+\frac{1}{2k}} q^*(s)ds.$$

This yields a sequence of functions $(q_k)_{k\in\mathbb{N}}$ such that q_k is Lipschitz continuous with constant k, and $q_k(\theta) \to q^*(\theta)$ and $q_k'(\theta) \to q^{*\prime}(\theta)$ as $k \to \infty$ for almost every θ. Using the local PE constraint (i) and the participation constraint (vi), we define the corresponding payment rules $(P_k)_k$.

If we ignore the participation constraint and arbitrarily set the payment of the lowest type to zero, we get a sequence of payments rules $(\tilde{P}_k)_k$. (P_k differs from $\tilde{P}_k$ by the constant $P_k(\underline{\theta})$, which is determined by the participation constraint.) By Helly's theorem, there exists a sub-sequence $(\tilde{P}_{k_n})_n$ and an increasing function $\tilde{P}$, such that $\tilde{P}_{k_n}(\theta) \to \tilde{P}(\theta)$ for almost every θ. Furthermore (possibly after taking sub-sequences again), $P_{k_n}(\underline{\theta})$ converges to some value $\hat{P}(\underline{\theta})$ if $n \to \infty$. For $\theta > \underline{\theta}$ we define $\hat{P}(\theta) = \hat{P}(\underline{\theta}) + \tilde{P}(\theta)$. By definition, each mechanism in the sequence $(\langle q_{k_n}(\theta), P_{k_n}(\theta)\rangle_{\theta\in\Theta})_n$ fulfills the local PE constraint and the participation constraint. Furthermore the sequence converges to the mechanism $\langle q^*(\theta), \hat{P}(\theta)\rangle_{\theta\in\Theta}$. Since the sequence of mechanisms is bounded and converges almost everywhere, the limit also fulfills participation and PE constraints.

Next we show that $\hat{P}(\theta) = P^*(\theta)$. Denote the utility of type θ in the mechanism $\langle q^*(\theta), \hat{P}(\theta)\rangle_{\theta\in\Theta}$ by $\hat{V}(\theta)$ and in the optimal mechanism by $V^*(\theta)$. Define $D(\theta) = P^*(\theta) - \hat{P}(\theta)$. Since both mechanisms fulfill (B.2), we have $D(\underline{\theta}) = \hat{V}(\underline{\theta}) - V^*(\underline{\theta})$. Subtracting (B.2) for the two mechanism yields

$$D(\theta)\rho(\theta) = \hat{V}(\underline{\theta}) - V^*(\underline{\theta}) + \lambda\int_{\underline{\theta}}^{\theta} D(z)f(z)\,dz$$

Hence, D is absolutely continuous. Differentiating and rearranging we get

$$D'(\theta) = 0.$$

Therefore, $D(\underline{\theta}) \neq 0$ is not possible because both mechanisms fulfill the individual rationality constraint with equality. We have shown that $\hat{P} = P^*$.

Now let (q,P,X,v) be an optimal solution to the control problem such that $v(\theta) < K$ for some K. Since the Lipschitz constraint for K is not binding for (q,P,X,v), the expected revenue from (q,P) is at least as high as the revenue from (q_{k_n}, P_{k_n}) for all $k_n > K$. As $q_{k_n}(\theta)$ and $P_{k_n}(\theta)$ converge to $q^*(\theta)$ and $P^*(\theta)$ almost everywhere, the revenue from (q,P) is also weakly greater than the revenue from (q^*,P^*). Hence, (q,P) is an optimal mechanism. □

The Hamiltonian corresponding to the above problem is given by

$$\mathscr{H}(\theta,q,T,X,p_q,p_P,p_X,v) = (T-cq)f(\theta) + p_q v + \frac{p_P}{\rho(\theta)}\partial_q u(q,\theta)v + p_X[u(q,\theta) - P\xi(\theta)]f(\theta).$$

Applying the Pontryagin Maximum Principle (cf. Clarke, 1983) we obtain the following necessary conditions for an optimal control. (p_q, p_P and p_X are absolutely continuous functions.)

(i) adjoint equations: for almost every $\theta \in \Theta$,

$$p_q'(\theta) \begin{cases} = cf(\theta) - \frac{p_P(\theta)}{\rho(\theta)} \partial_{qq} u(q(\theta),\theta)v(\theta) \\ \qquad\qquad - p_X(\theta)\partial_q u(q(\theta),\theta)f(\theta), & \text{if } q(\theta) < q^S(\theta), \\ \in \left[cf(\theta) - \frac{p_P(\theta)}{\rho(\theta)} \partial_{qq}\hat{u}(q^S(\theta),\theta)v(\theta),\, cf(\theta)\right], & \text{if } q(\theta) = q^S(\theta), \\ = cf(\theta), & \text{if } q(\theta) > q^S(\theta). \end{cases} \tag{B.4}$$

$$p_P'(\theta) = -f(\theta) + p_X(\theta)\xi(\theta)f(\theta), \tag{B.5}$$

$$p_X'(\theta) = 0 \quad (\Rightarrow \ p_X'(\theta) = p_X). \tag{B.6}$$

(ii) optimality of control: for almost every $\theta \in \Theta$,

$$v(\theta) \begin{cases} = K, & \text{if } p_q(\theta) + \frac{p_P(\theta)}{\rho(\theta)}\partial_q u(q(\theta),\theta) > 0, \\ \in [0,K], & \text{if } p_q(\theta) + \frac{p_P(\theta)}{\rho(\theta)}\partial_q u(q(\theta),\theta) = 0, \\ = 0, & \text{if } p_q(\theta) + \frac{p_P(\theta)}{\rho(\theta)}\partial_q u(q(\theta),\theta) < 0. \end{cases} \tag{B.7}$$

(iii) transversality conditions:

$$p_q(\bar{\theta}) = p_P(\underline{\theta}) = p_P(\bar{\theta}) = 0, \tag{B.8}$$

$$p_q(\underline{\theta}) \leq 0 \ (\text{“} = \text{” if } q(\underline{\theta}) > 0), \tag{B.9}$$

$$\text{and} \qquad p_X(\underline{\theta}),\ p_X(\bar{\theta}) \text{ free} \tag{B.10}$$

LEMMA B.4. *$p_X = 1$ and $p_P(\theta) = \lambda((F(\theta)^2 - F(\theta))(< 0)$.*

Proof. The adjoint equation for p_P' and the transversality condition imply

$$p_P(\theta) = \int_{\underline{\theta}}^{\theta} -f(s) + p_X f(s)\xi(s)ds. \tag{B.11}$$

Evaluating equation (B.11) at $\theta = \bar{\theta}$ and using the transversality condition for $p_P(\bar{\theta})$ we obtain

$$\begin{aligned} \int_{\underline{\theta}}^{\bar{\theta}} -f(s) + p_X f(s)[1 + \lambda(2F(s) - 1)]ds &= 0 \\ \Leftrightarrow -1 + p_X[1 + \lambda \int_{\underline{\theta}}^{\bar{\theta}} (2F(s) - 1)f(s)ds] &= 0 \\ \Leftrightarrow p_X &= 1. \end{aligned}$$

Inserting $p_X = 1$ into (B.5) and (B.11) we obtain

$$\begin{aligned} p_P'(\theta) &= \lambda(2F(\theta) - 1)f(\theta) \\ p_P(\theta) &= \lambda[(F(\theta))^2 - F(\theta)]. \end{aligned}$$

□

Denote the Lipschitz constants of $\tilde{q}$ (from the main text) and q^S by $\tilde{K}$ and K^S, respectively. Our assumptions on the utility function guarantee that $\max\{\tilde{K}, K^S\} < \infty$.

LEMMA B.5. *If $K > \max\{\tilde{K}, K^S\}$, then $q(\theta) \leq q^S(\theta)$ for all $\theta \in \Theta$.*

Proof. Suppose by contradiction $q(\theta) > q^S(\theta)$ for all θ in a maximal interval (a,b) with $a < b$. Equation (B.4) then implies that $p_q'(\theta) = cf(\theta)$ for $\theta \in (a,b)$. If $p_q(\theta) \geq 0$, for some $\theta < b$ then we must have $v(t) = K$ for all $t \in (\theta, b)$ and $b = \bar{\theta}$. This implies $p_q(\bar{\theta}) > 0$ in contradiction to the transversality condition. Hence, we have $p_q(\theta) < 0$ for all $\theta \in (a,b)$. This implies $v(\theta) = 0$ for all $\theta \in (a,b)$ and since q^S is increasing we must have $a = \underline{\theta}$. But $q(a) > q^S(a) > 0$ implies $p_q(a) = 0$ by the transversality condition. This is a contradiction to $p_q'(\theta) = cf(\theta) > 0$ for $\theta \in (a,b)$. □

LEMMA B.6. *Suppose that $K > \max\{\tilde{K}, K^S\}$. If $q(\theta) = q^S(\theta)$ for all $\theta \in (a,b)$, $a < b$, then for all $\theta \in (a,b)$*

$$cf(\theta) - \frac{p_P(\theta)}{\rho(\theta)} \partial_{qq}\hat{u}(q^S(\theta), \theta) q^{S\prime}(\theta) \leq 0. \tag{B.12}$$

Proof. Since $\partial_q u(q^S(\theta), \theta) = 0$ and $q^{S\prime}(\theta) \in (0, K)$, (B.7) implies that $p_q(\theta) = 0$ for all $\theta \in [a,b]$. Hence, $p_q'(\theta) = 0$ and from (B.4) we obtain (B.12). □

Next we derive properties of the optimal solution if $q(\theta) < q^S(\theta)$. Integrating (B.4), yields

$$\begin{aligned}
p_q(t) &= p_q(s) + \int_s^t cf(r) - \frac{p_P(r)}{\rho(r)} \partial_{qq} u(q(r), r) v(r) - \partial_q u(q(r), r) f(r) dr \\
&= p_q(s) + \int_s^t cf(r) - \frac{p_P(r)}{\rho(r)} \left[\frac{d}{dr} \partial_q u(q(r), r) - \partial_{q\theta} u(q(r), r) \right] \\
&\qquad\qquad - \partial_q u(q(r), r) f(r) dr \\
&= p_q(s) + \frac{p_P(s)}{\rho(s)} \partial_q u(q(s), s) - \frac{p_P(t)}{\rho(t)} \partial_q u(q(t), t) \\
&\quad + \int_s^t cf(r) + \left[\frac{d}{dr} \frac{p_P(r)}{\rho(r)} \right] \partial_q u(q(r), r) - \partial_q u(q(r), r) f(r) \\
&\qquad\qquad + \frac{p_P(r)}{\rho(r)} \partial_{q\theta} u(q(r), r) dr
\end{aligned}$$

Using

$$\left[\frac{d}{d\theta} \frac{p_P(\theta)}{\rho(\theta)} - f(\theta) \right] = -(\lambda + 1) \frac{f(\theta)}{(\rho(\theta))^2}$$

we get

$$\begin{aligned}
p_q(t) = p_q(s) &+ \frac{p_P(s)}{\rho(s)} \partial_q u(q(s), s) - \frac{p_P(t)}{\rho(t)} \partial_q u(q(t), t) \\
&+ \int_s^t cf(r) - (\lambda + 1) \frac{f(r)}{(\rho(r))^2} \partial_q u(q(r), r) + \frac{p_P(r)}{\rho(r)} \partial_{q\theta} u(q(r), r) dr
\end{aligned} \tag{B.13}$$

LEMMA B.7. *If $K > \max\{\tilde{K}, K^S\}$, then $v(\theta) < K$ for all $\theta \in \Theta$.*

Proof. Suppose by contradiction, that for all θ in a maximal interval (a,b), $a < b$, the control variable is $v(\theta) = K$. Then $p_q(\theta) + \frac{p_P(\theta)}{\rho(\theta)} \partial_q u(q(\theta),\theta) \geq 0$ with equality for $\theta \in \{a,b\}$. If the endpoints are $a = \underline{\theta}$ or $b = \bar{\theta}$, respectively, then equality follows from the transversality conditions. Otherwise, it follows because (a,b) is chosen maximally and the left-hand side of the inequality is continuous in θ. A strict inequality at an endpoint would imply that the interval where $v(\theta) = K$, extends beyond the endpoint.

We derive a contradiction by showing that $q(a) \geq q^*(a)$ and $q(b) \leq q^*(b)$. First, suppose by contradiction that $q(a) < q^*(a)$. Using (B.13) for $\theta > a$ close to a we get:

$$
\begin{aligned}
& p_q(\theta) + \frac{p_P(\theta)}{\rho(\theta)} \partial_q u(q(\theta),\theta) \\
&= \int_a^\theta cf(r) - (\lambda+1)\frac{f(r)}{(\rho(r))^2} \partial_q u(q(r),r) + \frac{p_P(r)}{\rho(r)} \partial_{q\theta} u(q(r),r) dr \\
&< \int_a^\theta cf(r) - (\lambda+1)\frac{f(r)}{(\rho(r))^2} \partial_q u(q^*(r),r) + \frac{p_P(r)}{\rho(r)} \partial_{q\theta} u(q^*(r),r) dr \leq 0
\end{aligned}
$$

The strict inequality follows from $q(a) < q^*(a)$ because for r close to a, $q(\theta) < q^*(\theta)$ and hence $\partial_q u(q(r),r) > \partial_q u(q^*(r),r)$ by concavity of u and $\partial_{q\theta} u(q(\theta),\theta) \geq \partial_{q\theta} u(q^*(\theta),\theta)$ by Assumption 3. The weak inequality follows from the definition of q^*. But this contradicts $u(\theta) = K$. Hence $q(a) \geq q^*(a)$. Similarly, it can be shown that $q(b) \leq q^*(b)$. But $q(a) \geq q^*(a)$, $q(b) \leq q^*(b)$ and $v(\theta) = K$ for $\theta \in (a,b)$ cannot be fulfilled simultaneously if $K > \max\{\tilde{K}, K^S\}$. □

LEMMA B.8. *If q^* is strictly increasing, then every optimal solution to the control problem is strictly increasing.*

Proof. Suppose by contradiction, that the control is zero ($v(\theta) = 0$) on a maximal interval (a,b). Then $q(\theta) < q^S(\theta)$ for $\theta \in (a,b)$ and $p_q(\theta) + \frac{p_P(\theta)}{\rho(\theta)} \partial_q u(q(\theta),\theta) \leq 0$, with equality for $\theta = b$ and for $\theta = a$ unless $a = \underline{\theta}$ and $q(a) = 0$. We first show that $q(a) = q^*(a)$. Suppose by contradiction that $q(a) < q^*(a)$. Using (B.13) for $\theta > a$ close to a we get (see the proof of the previous lemma)

$$
\begin{aligned}
& p_q(\theta) + \frac{p_P(\theta)}{\rho(\theta)} \partial_q u(q(\theta),\theta) \\
&< \int_a^\theta cf(r) - (\lambda+1)\frac{f(r)}{(\rho(r))^2} \partial_q u(q^*(r),r) + \frac{p_P(r)}{\rho(r)} \partial_{q\theta} u(q^*(r),r)\, dr \leq 0
\end{aligned}
$$

This implies $p_q(\theta) + \frac{p_P(\theta)}{\rho(\theta)} \partial_q u(q(\theta),\theta) < 0$ and $v(\theta) = 0$ for all $\theta \in (a,\bar{\theta})$ if q^* is strictly increasing. Since $p_P(\bar{\theta}) = 0$, this implies $p_q(\bar{\theta}) < 0$ in contradiction to the transversality condition. Hence $q(a) \geq q^*(a)$. If $q(a) > q^*(a)$, we have $q^*(a) = \tilde{q}(a)$ and $q(a) > \tilde{q}(a)$. Hence, using (B.13) we get $p_q(\theta) + \frac{p_P(\theta)}{\rho(\theta)} \partial_q u(q(\theta),\theta) > 0$ for θ close to a which contradicts $v(\theta) = 0$. Similarly, we can show that $q(b) \geq q^*(b)$ which yields the desired contradiction if $q^*(\theta)$ is strictly increasing. □

Proof of Proposition 4. Consider an optimal solution (q,P,X,v) to the control problem for $K > \max\{\tilde{K}, K^S\}$. By Lemmas B.7 and B.8, we have $v(\theta) \in (0,K)$. If $q(\theta) < q^S(\theta)$ for all $\theta \in (a,b)$ this implies $p_q(\theta) + \frac{p_P(\theta)}{\rho(\theta)}\partial_q u(q(\theta),\theta) = 0$ for all $\theta \in [a,b]$. Inserting into (B.13) we get

$$\int_a^\theta cf(r) - (\lambda+1)\frac{f(r)}{(\rho(r))^2}\partial_q u(q(r),r) + \frac{p_P(r)}{\rho(r)}\partial_{q\theta}u(q(r),r)dr = 0.$$

Differentiating this with respect to θ yields for almost every $\theta \in (a,b)$:

$$cf(\theta) - (\lambda+1)\frac{f(\theta)}{(\rho(\theta))^2}\partial_q u(q(\theta),\theta) + \frac{p_P(\theta)}{\rho(\theta)}\partial_\theta\partial_q u(q(\theta),\theta) = 0$$

Rearranging, we get

$$\begin{aligned}\partial_q u(q(\theta),\theta) &= \frac{(\rho(\theta))^2}{\lambda+1}c + \frac{p_P(\theta)\rho(\theta)}{(\lambda+1)}\frac{\partial_{\theta q}u(q(\theta),\theta)}{f(\theta)} \\ &= \frac{(1+\lambda F(\theta))^2}{\lambda+1}c - F(\theta)(1-F(\theta))\frac{\lambda(1+\lambda F(\theta))}{\lambda+1}\frac{\partial_{q\theta}u(q(\theta),\theta)}{f(\theta)}.\end{aligned} \tag{B.14}$$

This is the first-order condition (14) from the main paper. Inserting $q^{S\prime}(\theta) = -\frac{\partial_{q\theta}\hat{u}(q^S(\theta),\theta)}{\partial_{qq}\hat{u}(q^S(\theta),\theta)}$ into (B.12) yields

$$f(\theta)c + \frac{p_P(\theta)}{\rho(\theta)}\partial_{q\theta}\hat{u}(q^S(\theta),\theta) \leq 0,$$

or equivalently,

$$\frac{(\rho(\theta))^2}{\lambda+1}c + \frac{p_P(\theta)\rho(\theta)}{(\lambda+1)}\frac{\partial_{q\theta}\hat{u}(q^S(\theta),\theta)}{f(\theta)} \leq 0.$$

This is the opposite of condition (15) in the main paper. By Assumption 1, (B.14) has a solution $q(\theta) < q^S(\theta)$ if and only if (B.12) is violated. Assumption 1 also guarantees uniqueness of the solution. Hence, $q(\theta) = \tilde{q}(\theta)$ if (B.12) is violated ((15) is fulfilled). If (B.12) is fulfilled ((15) is violated),

$$0 \in \left[cf(\theta) - \frac{p_P(\theta)}{\rho(\theta)}\partial_{qq}\hat{u}(q^S(\theta),\theta)q^{S\prime}(\theta),\, cf(\theta)\right],$$

and hence $q(\theta) = q^S(\theta)$ fulfills the necessary condition from the maximum principle. We have shown that $q^*(\theta)$ fulfills the necessary conditions for optimality and because $q(\theta) \leq q^S(\theta)$ it is the unique solution. Existence of a solution can be shown by standard techniques. Therefore, we have have constructed an optimal solution to the control problem. By Lemma B.3 it is also a solution to the general problem. □

Appendix C: Proofs of Section 7.1: Heterogeneous Consumers

Proof of Lemma 2. Define $V(\lambda,\theta)$ as the consumer's surplus for a given demand type on the personal equilibrium path. Formally,

$$\begin{aligned} V(\lambda,\theta) &= u(\hat{q}(\theta,p),\theta) - p\hat{q}(\theta,p) - L - \lambda p \int_{\underline{\theta}}^{\theta} [\hat{q}(\theta,p) - \hat{q}(\varphi,p)] f(\varphi) d\varphi \\ &= u(\hat{q}(\theta,p),\theta) - p\hat{q}(\theta,p) - L - \lambda p \int_{\underline{\theta}}^{t(\theta,p)} [\hat{q}(\theta,p) - \hat{q}(\varphi,p)] f(\varphi) d\varphi. \end{aligned}$$

The second line holds because $\hat{q}(\theta,p) = \hat{q}(\varphi,p)$ for $\varphi \in [t(\theta,p),\theta]$. Taking the derivative of $V(\lambda,\theta)$ with respect to λ yields

$$\begin{aligned} V'(\lambda,\theta) &= [\partial_q u(\hat{q}(\theta,p),\theta) - p]\, \partial_\lambda \hat{q}(\theta,p) - p \int_{\underline{\theta}}^{t(\theta,p)} (\hat{q}(\theta,p) - \hat{q}(\varphi,p)) f(\varphi) d\varphi \\ &\qquad - \lambda p \int_{\underline{\theta}}^{t(\theta,p)} (\partial_\lambda \hat{q}(\theta,p) - \partial_\lambda \hat{q}(\varphi,p)) f(\varphi) d\varphi. \\ &= \underbrace{[\partial_q u(\tilde{q}(t(\theta,p),p),\theta) - p(1+\lambda F(t(\theta,p)))]}_{=0} \partial_\lambda \hat{q}(\theta,p) \\ &\qquad - p \int_{\underline{\theta}}^{t(\theta,p)} (\hat{q}(\theta,p) - \hat{q}(\varphi,p)) f(\varphi) d\varphi + \lambda p \int_{\underline{\theta}}^{t(\theta,p)} \partial_\lambda \hat{q}(\varphi,p) f(\varphi) d\varphi. \end{aligned}$$

The first integral in the last line is non-negative since $\hat{q}(\theta,p)$ is non-decreasing in θ. The second integral is negative because $\partial_\lambda \hat{q}(\theta,p) \leq 0$, which follows immediately from $\partial_\lambda \tilde{q}(\theta,p) = pF(\theta)/[\partial_{qq} u(\hat{q}(\theta,p),\theta)] \leq 0$. Hence, we have shown that $V'(\lambda,\theta) \leq 0$.

The consumer's expected utility is given by $\mathbb{E}_\theta[V(\lambda,\theta)] = \int_{\underline{\theta}}^{\bar{\theta}} V(\lambda,\theta) f(\theta) d\theta$. Hence, the change in expected utility due to an increase in the consumer's degree of loss aversion is given by $d\mathbb{E}_\theta[V(\lambda,\theta)]/d\lambda = \int_{\underline{\theta}}^{\bar{\theta}} V'(\lambda,\theta) f(\theta) d\theta \leq 0$. □

Appendix D: Supplementary Material to Section 7.2: Competition

D.1. Market Framework

In this part of the web appendix, a formal model of imperfect competition is considered. Moreover, we allow for heterogeneity among consumers with respect to the degree of loss aversion. Consider a market for one good or service where two firms, A and B, are active. Moreover, there is a continuum of ex ante heterogeneous consumers whose measure is normalized to one.

Players & Timing.—The consumers can be partitioned into two groups that differ in their degrees of loss aversion. Let the two groups be denoted by $j = 1,2$ with $0 \leq \lambda_1 < \lambda_2$. The distribution of demand types is identical for both groups of loss-averse consumers. As before, the demand type is unknown to consumers and firms at the point of contracting.

The two symmetric firms, A and B, produce at constant marginal cost $c > 0$ and without fixed cost. Each firm $i = A, B$ offers a two-part tariff to each group of consumers $j = 1, 2$. The tariff is given by $T_j^i(q) = L_j^i + p_j^i q$, where $q \geq 0$ is the quantity, and L_j^i and p_j^i denote the fixed fee and the unit price, respectively, charged by firm i from consumers of type j. We will analyze the symmetric information case in which firms can observe λ, as well as the asymmetric information case in which λ is private information of the consumers.

The timing is as follows: (1) Firms simultaneously and independently offer a menu of two-part tariffs $\{(L_j^i, p_j^i)\}_{j=1,2}$ to consumers. (2) Each consumer either signs exactly one contract or none. (3) Each consumer privately observes his demand type. Thereafter, each consumer who accepted a contract chooses a quantity. (4) Finally, payments are made according to the demanded quantities and the concluded contracts.

Discrete Choice Framework.—The products of the two firms are symmetrically differentiated. We assume that, next to λ, consumers are ex ante heterogeneous with respect to their brand preferences. Each consumer has idiosyncratic preferences for differing brands of the product (firms), which are parameterized by $\zeta = (\zeta^0, \zeta^A, \zeta^B)$. A consumer with brand preferences ζ has net utility $v^i + \zeta^i$ if he buys from firm i, and net utility ζ^0 if no contract is signed, where $v^i = \mathbb{E}_\theta[U(\cdot)]$. The brand preferences $\zeta = (\zeta^0, \zeta^A, \zeta^B)$ are independently and identically distributed according to a known distribution among the two groups of consumers.

To solve for the tariffs that are offered in the pure-strategy Nash equilibrium by the two firms, we follow the approach of Armstrong and Vickers (2001) and model firms as offering utility directly to consumers. Each two-part tariff can be considered as a deal of a certain expected value that is offered by a firm to its consumers. Thus, firms compete over customers by trying to offer them better deals, i.e., a two-part tariff that yields higher utility (including loss utility). Put differently, we decompose a firm's problem into two parts. First, we solve for the two-part tariff that maximizes profits subject to the constraint that the consumer receives a certain utility level. Thereafter, we solve for the utility levels (v_1^i, v_2^i) a firm i offers to its customers. It is important to note that when λ is unobservable, the two-part tariffs have to be designed such that each group of consumers prefers the offer that is intended for them. Suppose that the utility offered to consumers of group j by firm A and firm B is v_j^A and v_j^B, respectively. Furthermore, assume that the incentive constraints are satisfied. Then, the market share of firm A in the submarket j is $m_j(v_j^A, v_j^B)$ and the market share of firm B is $m_j(v_j^B, v_j^A)$, with $m_j(v_j^A, v_j^B) + m_j(v_j^B, v_j^A) \leq 1$. The market share function $m_j(\cdot)$ is increasing in the first argument and decreasing in the second. Since the brand preferences are identically distributed among the two groups, the market share functions are identical for the two submarkets, i.e., $m_1(\cdot) = m_2(\cdot) = m(\cdot)$. Following Armstrong and Vickers, we impose some regularity conditions in order to guarantee existence of equilibrium. First, we assume that

$$\frac{\partial_{v^A} m(v^A, v^B)}{m(v^A, v^B)} \text{ is non-decreasing in } v^B.$$

Second, we assume that for each submarket the collusive utility level $\tilde{v}_j$ exists which maximizes (symmetric) joint profits.[1]

D.2. Firm's Subproblem: Joint Surplus Maximization

For this part, suppose firms can observe consumers' types $\lambda \in \{\lambda_1, \lambda_2\}$. With consumers' loss-aversion types being observable, the two market segments of types λ_1 and λ_2 can be viewed as distinct markets. Thus, for the analysis we can focus on one market where consumers are homogeneous with respect to their degree of loss aversion, which is denoted by λ.

Suppose firm $i \in \{A,B\}$ offers consumers a "deal" using a two-part tariff (L^i, p^i) that gives them utility v^i. Then, if a consumer with brand preferences $\zeta = (\zeta^0, \zeta^A, \zeta^B)$ purchases from firm i his net utility is $v^i + \zeta^i$. Let $\pi_j(v^i)$ be firm i's maximum profit per customer of type j when offering them a deal that yields utility v^i. The per-consumer profit function is the same for both firms, but in general it depends on the consumer's degree of loss aversion λ. For now we focus on one market segment and therefore the subscript indicating the loss-aversion type can be omitted without confusion. Since $\pi(\cdot)$ is the same for both firms, we will omit firm's superscript in the following. With this notation, $\pi(v)$ is given by the solution to the problem:

$$\begin{aligned} \pi(v) = \max_{L,p \geq 0} L + (p-c) \int_{\underline{\theta}}^{\bar{\theta}} \hat{q}(\theta, p) f(\theta) d\theta \\ \text{s.t.} \quad \mathbb{E}_\theta[U(\hat{q}(\theta,p)|\theta, \langle q(\varphi,p)\rangle)] = v. \end{aligned} \tag{D.1}$$

First, we study the firm's subproblem, that is, we derive the optimal two-part tariff that solves the above problem. Thereafter, we solve for the utility levels and the corresponding tariffs which are offered by the two firms in equilibrium. Put differently, the task is to maximize a firm's profit over the choice variables p and L subject to the constraint that the consumer's expected utility from the offered deal is v. The firm's tariff choice problem can be restated as a problem of choosing only the unit price p. The firm chooses p to maximize $S(p) - v$, i.e., the firm chooses the marginal price p such that the joint surplus of the two contracting parties, the consumer and the firm, is maximized. The optimal marginal price $\hat{p}$ is independent of the utility level v, that the firm offers to the consumer. This immediately implies that $\pi'(v) = -1$. More importantly, the optimal marginal price is characterized by the same conditions as in the case of a monopolistic firm.

In the following we focus on the profit maximization problem of firm A. We assume that Assumption 2 holds for both types of loss-averse consumers, i.e., for $\lambda \in \{\lambda_1, \lambda_2\}$. Moreover, it is assumed that $\Sigma(\lambda_2) \geq c$.

1. For a detailed description of the competition-in-utility-space framework and the needed assumptions see Armstrong and Vickers (2001). A similar approach is used by Rochet and Stole (2002).

D.3. Symmetric Information Case

Consider market segment $j \in \{1,2\}$. For a given utility level v_j^B offered by firm B, the profit maximization problem of firm A is given by

$$\max_{v_j^A} m(v_j^A, v_j^B)\pi_j(v_j^A) . \tag{D.2}$$

The necessary first-order condition for profit maximization amounts to

$$\partial_{v_j^A} m(v_j^A, v_j^B)\, \pi_j(v_j^A) + m(v_j^A, v_j^B)\, \pi_j'(v_j^A) = 0 . \tag{D.3}$$

Remember that $\pi_j'(v^A) = -1$. The optimal marginal price is unaffected by the choice of v_j^A. If firm A offers one unit utility more to consumers, then this is optimally achieved by lowering the fixed fee by one unit. The fixed fee is a one-to-one transfer from the consumer to the firm. Define

$$\Phi(v) \equiv \frac{m(v,v)}{\partial_{v^A} m(v,v)} .$$

Applying Proposition 1 of Armstrong and Vickers (2001), the firm's per customer profit in submarket j in the symmetric equilibrium is given by

$$\pi_j(\hat{v}_j) = \Phi(\hat{v}_j) ,$$

where $\hat{v}_j$ denotes the utility offered to the consumers of type λ_j by both firms in equilibrium. As is shown by Armstrong and Vickers, there are no asymmetric equilibria. Moreover, the equilibrium often is unique.[2] The following proposition summarizes the tariffs offered by the two firms in equilibrium.

PROPOSITION D.1 (Full Information). *Suppose that Assumption 2 holds for consumers of both groups. Then, in equilibrium,*

(i) if $\Sigma(\lambda_1) < c \leq \Sigma(\lambda_2)$ both firms offer the tariff $(\hat{p}, \hat{L})$ with a positive unit price to consumers of type λ_1, and a flat-rate tariff $(0, L^F)$ to consumers of type λ_2.

(ii) if $c \leq \Sigma(\lambda_1) < \Sigma(\lambda_2)$, then both firms offer the flat-rate tariff $(0, L^F)$ to both types of loss-averse consumers.

The tariffs $(\hat{p}, \hat{L})$ and $(0, L^F)$ satisfy: $S_1'(\hat{p}) = 0$,

$$\hat{L} = \Phi(\hat{v}_1) - (\hat{p} - c)\int_{\underline{\theta}}^{\bar{\theta}} \hat{q}_1(\theta, \hat{p}) f(\theta) d\theta$$

$$\text{and} \qquad L^F = \Phi(\hat{v}_2) + c\int_{\underline{\theta}}^{\bar{\theta}} q^S(\theta) f(\theta) d\theta ,$$

respectively, with $\hat{p} \in (0, c]$.

2. See Armstrong and Vickers (2001) for sufficient conditions for a unique equilibrium.

Proof. In order to apply Proposition 1 of Armstrong and Vickers (2001), the following three properties have to be satisfied: (i) $\partial_{v^A} m(v^A, v^B)/m(v^A, v^B)$ is non-decreasing in v^B, (ii) there exists $\tilde{v}_j > -\infty$ that maximizes $m(v,v)\pi_j(v)$ for $j = 1,2$, and (iii) for $j = 1,2$ there exists $\bar{v}_j$ defined by $\pi_j(\bar{v}_j) = 0$, $\pi_j(v) < 0$ if $v > \bar{v}_j$. Since we explicitly assumed (i) and (ii) these properties are satisfied. To see that (iii) is also satisfied note that $\bar{v}_j = \max_p\{S_j(p)\}$. Obviously, $\pi_j(\bar{v}_j) = 0$ and $\pi_j(v) < 0$ if $v > \bar{v}_j$. Hence, we can apply Proposition 1 of Armstrong and Vickers. According to this proposition, there are no asymmetric equilibria and the equilibrium utility level $\hat{v}_j$ satisfies $\hat{v}_j \in (\tilde{v}_j, \bar{v}_j)$. Since $m(v^A, v^B)\pi_j(v^A)$ is continuously differentiable, the equilibrium utility level satisfies the first-order condition of profit maximization. Thus, $\pi_j(\hat{v}_j) = \Phi(\hat{v}_j)$.

From Proposition 3 it follows that the optimal marginal price $\hat{p}_j$ is greater than zero if and only if $\Sigma(\lambda_j) < c$. If this is the case, then $\hat{p}_j$ is such that $S_j'(\hat{p}_j) = 0$, as was shown in the proof of Proposition 3. The per-customer profit of a firm is given by

$$\pi_j = L + (p - c)\int_{\underline{\theta}}^{\bar{\theta}} \hat{q}_j(\theta, p) f(\theta) d\theta \ . \tag{D.4}$$

Since, in equilibrium, $\pi_j = \Phi(\hat{v}_j)$ the equilibrium fixed fee is given by

$$L_j = \Phi(\hat{v}_j) - (p_j - c)\int_{\underline{\theta}}^{\bar{\theta}} \hat{q}_j(\theta, p_j) f(\theta) d\theta \ . \tag{D.5}$$

Replacing p_j by $\hat{p}$ and 0, leads to the fixed fees $\hat{L}$ and L^F, respectively. □

If the degree of loss aversion of the less loss-averse consumers is below the threshold given by $\Sigma(\lambda) = c$, then firms offer a measured tariff to these consumers. Next to the measured tariff, firms offer a flat-rate tariff to the more loss-averse consumers. If the degree of loss aversion of both types is above the threshold, then firms offer only a single tariff, which is a flat-rate tariff.

D.4. Asymmetric Information Case

In this subsection, we investigate the tariffs offered by the two firms when facing a screening problem, i.e., when the degree of loss aversion is private information. We show that the firms can screen consumers with respect to the degree of loss aversion without costs, if $\Phi'(v) \geq 0$.[3] The main challenge is to show that consumers self-select into the right tariff if the firms offer a flat rate next to a measured tariff.

To fix ideas, suppose that $\Sigma(\lambda_1) < c \leq \Sigma(\lambda_2)$, so that such a menu would be optimal in the symmetric information case (see Proposition D.1). Furthermore, suppose that the firms offer the same tariffs as in the symmetric information case. $\Phi'(v) \geq 0$ implies that the additional surplus generated for the less loss-averse consumers of group one,

3. For instance, this condition is satisfied for the standard Hotelling model and for the logit demand model, see Section D.5 of this Appendix.

is shared between the two contracting parties. In other words, it implies $\hat{v}_1 > \hat{v}_2$ and that in equilibrium, the profit that a firm earns from a consumer of group one who subscribes to the measured tariff, is greater than the profit from a consumer of group two, who subscribes to the flat rate.

Remember that the expected utility from a flat rate is independent of the degree of loss aversion. Therefore, $\hat{v}_1 > \hat{v}_2$ immediately implies that the less loss-averse consumers of group one do not have an incentive to choose the flat rate. Conversely, we have to show that the more loss-averse consumers of group one do not have an incentive to deviate to the measured tariff. Since $\hat{v}_1 > \hat{v}_2$, we cannot simply use Lemma 2 in order to conclude that such a deviation lowers their utility. By inspecting the profit of a firm from the measured tariff, however, we observe that it decreases with demand because the unit price is below marginal cost. Since demand is decreasing in the degree of loss aversion, the profit from a (deviating) consumer of group two who subscribes to the measured tariff is higher than the profit from a consumer from group one. Furthermore, $\Phi'(v) \geq 0$ implies that the latter profit is greater than the profit from the flat rate. Hence, a firm's profit is increased by a deviation of a consumer of group two. On the other hand, the joint surplus is decreased by the deviation—since $c \leq \Sigma(\lambda_2)$, the flat rate maximizes the joint surplus for consumers of group two. Therefore, the expected utility must decrease if a consumer from group two deviates.

PROPOSITION D.2 (Asymmetric Information). *Suppose that Assumption 2 holds for consumers of both groups and that $\Phi'(v) \geq 0$. Then,*

(i) if $\Sigma(\lambda_1) < c \leq \Sigma(\lambda_2)$ both firms offering tariff $(\hat{p}, \hat{L})$ with a positive unit price to consumers of type λ_1, and flat-rate tariff $(0, L^F)$ to consumers of type λ_2 is an equilibrium.

(ii) If $c \leq \Sigma(\lambda_1) < \Sigma(\lambda_2)$, then in equilibrium both firms offer the flat-rate tariff $(0, L^F)$ to both types of loss-averse consumers.

The tariffs, $(\hat{p}, \hat{L})$ and $(0, L^F)$, are given in Proposition D.1.

Proof. Irrespective of the rival's tariff offer, if the sorting constraint is satisfied it is optimal for a firm to choose p_j such that $S_j(p_j)$ is maximized. Put differently, the firm will choose the method of generating v_j that maximizes its (per-customer) profits. Thus, if no type $\lambda \in \{\lambda_1, \lambda_2\}$ has an incentive to mimic the other type, it is an equilibrium that the firms offer the same tariffs as in the full information case. Obviously, in case (ii) where $c \leq \Sigma(\lambda_1) < \Sigma(\lambda_2)$, both firms offer a flat-rate tariff to consumers. In this case, a flat-rate tariff maximizes $S_1(p)$ as well as $S_2(p)$. Moreover, the generated joint surplus is the same for both types of loss-averse consumers. Since the brand preferences are i.i.d. across the λ_1 and λ_2 types, in any equilibrium each firm offers a single flat-rate tariff to consumers.

In the remaining part of the proof we show that in the case where $\Sigma(\lambda_1) < c \leq \Sigma(\lambda_2)$, neither type λ_1 has an incentive to choose the tariff $(0, L^F)$ nor does type λ_2 have an incentive to choose the tariff $(\hat{p}, \hat{L})$.

CLAIM D.1. $\hat{v}_1 \geq \hat{v}_2$.

Proof. Let $S_j^* \equiv \max_p\{S_j(p)\}$. Note that $S_1(0) = S_2(0) = S_2^*$. The firm's per customer profit from type $j = 1,2$ when offering utility v is

$$\pi_j(v) = S_j^* - v\,. \tag{D.6}$$

Thus, for any v it holds that $\pi_1(v) \geq \pi_2(v)$, since $S_1^* - v \geq S_2^* - v$. The equilibrium utilities are characterized by $\pi_j(\hat{v}_j) = \Phi(\hat{v}_j)$. Hence, we obtain the following relations:

$$\Phi(\hat{v}_1) = \quad \pi_1(\hat{v}_1) \geq \pi_2(\hat{v}_1) \tag{D.7}$$

$$\pi_1(\hat{v}_2) \geq \pi_2(\hat{v}_2) \quad = \Phi(\hat{v}_2)\,. \tag{D.8}$$

Suppose by contradiction, that $\hat{v}_1 < \hat{v}_2$. This immediately implies that $\pi_j(\hat{v}_1) > \pi_j(\hat{v}_2)$. Hence,

$$\Phi(\hat{v}_1) = \pi_1(\hat{v}_1) > \pi_1(\hat{v}_2) \geq \pi_2(\hat{v}_2) = \Phi(\hat{v}_2)\,. \tag{D.9}$$

Since $\Phi'(v) \geq 0$ the above formula holds only if $\hat{v}_1 > \hat{v}_2$, a contradiction. □

With $\hat{v}_1 \geq \hat{v}_2$ and the expected utility from a flat-rate tariff being independent of λ, one can conclude that a consumer of type λ_1 has no incentive to choose the tariff $(0, L^F)$ that is designed for consumers of type λ_2. Finally, we show that type λ_2 has no incentive to mimic type λ_1. Let v_2^{DEV} denote the expected utility of a consumer of type λ_2 who accepts the tariff $(\hat{p}, \hat{L})$ designed for type λ_1.

CLAIM D.2. $v_2^{DEV} < \hat{v}_2$.

Proof. The expected utility of type λ_2 from the tariff $(\hat{p}, \hat{L})$ equals the generated joint surplus minus the profits of the firm he purchases from. Thus,

$$v_2^{\text{DEV}} = S_2(\hat{p}) - \hat{L} - (\hat{p} - c)\int_{\underline{\theta}}^{\bar{\theta}} \hat{q}_2(\theta, \hat{p}) f(\theta) d\theta, \tag{D.10}$$

where $\hat{q}_2(\theta, p)$ denotes the demand of type λ_2 in the personal equilibrium. Inserting the explicit formula of $\hat{L}$ into (D.10) yields

$$v_2^{\text{DEV}} = S_2(\hat{p}) - \Phi(\hat{v}_1) - (c - \hat{p})\int_{\underline{\theta}}^{\bar{\theta}} [\hat{q}_1(\theta, \hat{p}) - \hat{q}_2(\theta, \hat{p})] f(\theta) d\theta. \tag{D.11}$$

Note that $\hat{q}_1(\theta, \hat{p}) > \hat{q}_2(\theta, \hat{p})$ for all $\theta \in \Theta$, since $\partial_\lambda \hat{q} < 0$ if $p > 0$. By Proposition D.1, $c \geq \hat{p}$, and hence

$$v_2^{\text{DEV}} < S_2(\hat{p}) - \Phi(\hat{v}_1). \tag{D.12}$$

The expected utility of a consumer of type λ_2 when choosing the tariff that is intended for him can be expressed as follows,

$$\hat{v}_2 = S_2^* - \Phi(\hat{v}_2). \tag{D.13}$$

Hence, a deviation is not utility improving if

$$S_2^* - \Phi(\hat{v}_2) \geq S_2(\hat{p}) - \Phi(\hat{v}_1) \tag{D.14}$$
$$\iff [S_2^* - S_2(\hat{p})] + [\Phi(\hat{v}_1) - \Phi(\hat{v}_2)] \geq 0\,. \tag{D.15}$$

The above inequality is satisfied since $\Phi'(\cdot) \geq 0$ and $\hat{v}_1 \geq \hat{v}_2$. □

Thus, if the firms offer the optimal tariffs of the full information case, each type of loss-averse consumer selects the tariff that is designed for him, which completes the proof. □

As in the symmetric information case, if λ_1 is below and λ_2 is above the threshold given by $\Sigma(\lambda) = c$, then firms offer a measured tariff to the less loss-averse types and a flat-rate tariff to the more loss-averse consumers. The fixed fee of the flat-rate tariff is higher than the fixed fee of the measured tariff. In this case, we do not make any claims about the uniqueness of this equilibrium.[4] If the degree of loss aversion of both types exceeds the threshold, then we obtain a pooling equilibrium: each firm offers only a single tariff that is accepted by both types of consumers.

D.5. Examples of Discrete Choice Models

Hotelling Model with Linear Transport Cost.—Suppose consumers' ideal brands are uniformly distributed on the unit interval $[0,1]$. The brands of the two firms, A and B, are located at the two extreme points, brand A at zero and brand B at one. A consumer with ideal brand $x \in [0,1]$ has brand preferences $\zeta = (0, -tx, -t(1-x))$. The parameter $t > 0$ is a consumer's "transport cost" per unit distance between his ideal brand and the brand he purchases from. For the Hotelling specification, the market share function takes the following form,

$$m(v^A, v^B) = \min\left\{\frac{1}{2t}(t + v^A - v^B), \frac{v^A}{t}\right\}. \tag{D.16}$$

The market share function has to be modified if v^A and v^B differ by so much that $m(\cdot) \notin [0,1]$ (this never happens in equilibrium). Moreover, the Hotelling model has the well-known drawback that market shares are kinked. If, however, the transport cost is sufficiently low, then one can focus on the case where the market share function is given by the first term of the above expression and thus well behaved. Formally, for $t \leq (2/3)S_2^*$ it suffices to analyze a firm's profit maximization problem for[5]

$$m(v^A, v^B) = \frac{1}{2t}(t + v^A - v^B). \tag{D.17}$$

4. To analyze all equilibria we cannot apply the competition in utility space framework, since we have to take the sorting constraints explicitly into account.

5. See Lemma 1 of Armstrong and Vickers (2001).

Hence, $\partial_{v^A} m(v^A, v^B) = (2t)^{-1}$ which immediately implies that

$$\Phi(v) \equiv \frac{m(v,v)}{\partial_{v^A} m(v,v)} = t. \tag{D.18}$$

Obviously, $\Phi(\cdot)$ is non-decreasing. Note that

$$\frac{\partial_{v^A} m(v^A, v^B)}{m(v^A, v^B)} = \frac{1}{t + v^A - v^B}. \tag{D.19}$$

It can easily be seen that the above fraction is increasing in v^B. Thus, the Hotelling model satisfies all imposed assumptions if the transport cost is sufficiently low. One can check that the collusive utility level exists. To calculate the collusive utility level one has to use the market share function given in (D.16).

Logit Demand Model.—An obvious drawback of the Hotelling specification is that a firm does not compete with the rival and the outside option at the same time. A model that accounts for this simultaneous competition on two fronts is the logit demand model. Here, a consumer's brand preferences ζ^i for $i = 0, A, B$ are i.i.d. according to the double exponential distribution with mean zero and variance $\mu^2\pi^2/6$, where π (here) denotes the circular constant. Thus, the cumulative distribution function is

$$G(\zeta^i) = \exp\{-\exp[-(\gamma + \zeta^i/\mu)]\}, \tag{D.20}$$

where γ is the Euler-Mascheroni constant and μ is a positive constant. With this specification, the market share of firm A is given by (see Anderson et al., 1992)

$$m(v^A, v^B) = \frac{\exp[v^A/\mu]}{\exp[v^A/\mu] + \exp[v^B/\mu] + 1}. \tag{D.21}$$

The parameter μ captures the degree of heterogeneity among consumers with respect to their brand preferences. Put differently, μ measures the degree of product differentiation. A lower value of μ corresponds to a more competitive market. For $\mu \to \infty$ the firms are local monopolists. Taking the partial derivative of (D.21) with respect to v^A yields

$$\partial_{v^A} m(v^A, v^B) = \frac{\exp[v^A/\mu]\{\exp[v^B/\mu] + 1\}}{\mu\{\exp[v^A/\mu] + \exp[v^B/\mu] + 1\}^2}. \tag{D.22}$$

Thus,

$$\frac{m(v^A, v^B)}{\partial_{v^A} m(v^A, v^B)} = \frac{\mu\{\exp[v^A/\mu] + \exp[v^B/\mu] + 1\}}{\exp[v^B/\mu] + 1}. \tag{D.23}$$

Evaluating the above expression at $v^A = v^B = v$ leads to

$$\Phi(v) = \mu \frac{2\exp[v/\mu] + 1}{\exp[v/\mu] + 1}. \tag{D.24}$$

Taking the derivative of $\Phi(\cdot)$ with respect to v yields

$$\Phi'(v) = \frac{\exp[v/\mu]}{(\exp[v/\mu]+1)^2} > 0. \tag{D.25}$$

Moreover, the derivative of $\partial_{v^A} m(v^A, v^B)/m(v^A, v^B)$ with respect to v^B amounts to

$$\frac{d}{dv^B}\left[\frac{\partial_{v^A} m(v^A, v^B)}{m(v^A, v^B)}\right] = \frac{1}{\mu^2}\,\frac{\exp[v^B/\mu]\{\exp[v^B/\mu]+1\}}{\mu\{\exp[v^B/\mu]+\exp[v^B/\mu]+1\}^2} > 0. \tag{D.26}$$

The collusive utility level $\tilde{v}$ maximizes $m(v,v)\pi(v)$. Note that $m(v,v) \to 0$ for $v \to -\infty$ and $\pi(v) \leq 0$ if $v \geq \max_p\{S(p)\}$. Thus, the collusive utility exists, since $m(v,v)\pi(v)$ is continuously differentiable.

Appendix E: Supplementary Material to Section 7.3: Loss Aversion in Both Dimensions

Proof of Proposition 6. The monopolist maximizes the expected joint surplus by choosing the unit price p. Given that the consumer plays the personal equilibrium $\langle \hat{q}(\theta, p)\rangle$ characterized by (18), the joint surplus is given by

$$S(p) = \int_{\underline{\theta}}^{\bar{\theta}} \Big\{ u(\hat{q}(\theta,p),\theta) - c\hat{q}(\theta,p) - \lambda \int_{\underline{\theta}}^{\theta} \big[u(\hat{q}(z,p),z) - u(\hat{q}(\theta,p),\theta)\big] f(z)\, dz \\ - \lambda \int_{\underline{\theta}}^{\theta} p\big[\hat{q}(\theta,p) - \hat{q}(z,p)\big] f(z)\, dz \Big\} f(\theta)\, d\theta. \tag{E.1}$$

Taking the derivative of $S(p)$ with respect to p and using the personal equilibrium condition (18), we obtain

$$S'(p) = (p-c)\int_{\underline{\theta}}^{\bar{\theta}} \partial_p \hat{q}(p,\theta) f(\theta) d\theta - \lambda \int_{\underline{\theta}}^{\bar{\theta}} \int_{\underline{\theta}}^{\theta} [\hat{q}(p,\theta) - \hat{q}(p,z)] f(z) f(\theta) dz d\theta \tag{E.2}$$

For $\lambda > 0$ the optimal unit price fulfills $p^* \in [0, c)$, since $S(p)$ is decreasing for prices above the marginal cost. If the consumer is not loss averse, i.e., $\lambda = 0$, then $p^* = c$. Given that $\partial_{qqq} u(q,\theta) \geq 0$ and $\lambda \leq 1$, the joint surplus is a strictly concave function for $p \leq c$. Formally,

$$S''(p) = -(p-c)\int_{\underline{\theta}}^{\bar{\theta}} \frac{\partial_{qqq} u(\hat{q}(\theta,p),\theta)}{[\partial_{qq} u(\hat{q}(\theta,p),\theta)]^3} f(\theta) d\theta + \int_{\underline{\theta}}^{\bar{\theta}} \frac{1-\lambda[2F(\theta)-1]}{\partial_{qq} u(\hat{q}(\theta,p),\theta)} f(\theta) d\theta < 0, \tag{E.3}$$

for $p \leq c$. With the joint surplus being strictly concave, a flat-rate tariff is optimal when $S'(p)|_{p=0} \leq 0$, which is equivalent to

$$c \leq \lambda \frac{\int_{\underline{\theta}}^{\bar{\theta}} \int_{\underline{\theta}}^{\theta} [q^S(\theta) - q^S(\varphi)] f(\varphi) f(\theta) d\varphi d\theta}{\int_{\underline{\theta}}^{\bar{\theta}} \partial_p \hat{q}(\theta,p)\, f(\theta) d\theta} \equiv \Sigma(\lambda). \tag{E.4}$$

□

Construction of a Personal Equilibrium.— Since a higher demand type is associated with a stronger need for the good, we posit that demand is increasing in the type. For a given quantity, higher types are worse off compared to lower types. We posit that this is still the case in the personal equilibrium. Put differently, the increase in intrinsic utility due to a higher consumption of a higher type does not outweigh the direct negative effect on intrinsic utility of being a higher type. Formally, the second hypothesis with respect to the personal equilibrium requires that the following inequality is satisfied:

$$\partial_q u(\hat{q}(\theta,p),\theta)\,\partial_\theta \hat{q}(\theta,p) + \partial_\theta u(\hat{q}(\theta,p),\theta) \leq 0. \tag{E.5}$$

Given these hypotheses, the consumer's utility can be written as

$$U(q|\theta,\langle\hat{q}(\varphi,p)\rangle) = u(q,\theta) - pq - L - \lambda\int_{\underline{\theta}}^{\beta(q)} \big[u(\hat{q}(z,p),z) - u(q,\theta)\big] f(z)\,dz \\ - \lambda\int_{\underline{\theta}}^{\alpha(q)} p\big[q - \hat{q}(z,p)\big] f(z)\,dz, \tag{E.6}$$

where $\alpha(q)$ and $\beta(q)$ are implicitly defined by[6]

$$\hat{q}(\alpha(q),p) \equiv q \quad \text{and} \quad u(\hat{q}(\beta(q)),\beta(q)) \equiv u(q,\theta),$$

respectively. Under the hypotheses, $\alpha'(q) > 0$ and $\beta'(q) < 0$. Differentiating (E.6) with respect to q yields

$$U'(q|\theta,\langle\hat{q}(\varphi,p)\rangle) = \partial_q u(q,\theta)[1+\lambda F(\beta(q))] - p[1+\lambda F(\alpha(q))]. \tag{E.7}$$

The utility function is strictly concave and thus the first-order condition is necessary and sufficient for optimality. Moreover, in equilibrium it has to hold that $\alpha(q) = \beta(q) = \theta$ (cf. Equation (18) in the main text). Hence, the personal equilibrium is characterized by $\partial_q u(\hat{q}(\theta,p),\theta) = p$. Obviously, the demand function characterized by (18) is increasing in the demand type. The second hypothesis (E.5)—higher types achieve lower utility levels in equilibrium—is also satisfied for relatively low marginal prices, i.e., for

$$p \leq \partial_{qq} u(q,\theta)\frac{\partial_\theta u(q,\theta)}{\partial_{q\theta} u(q,\theta)}. \tag{E.8}$$

(Personal) Equilibrium Selection.—A final comment to the personal equilibrium selection is in order. There may exist multiple personal equilibria for the case analyzed above. So far, we constructed only one personal equilibrium. It is reasonable to assume that higher types demand more. Higher types have a higher marginal utility which implies that a higher q increases the intrinsic utility in the good dimension and reduces the loss in the good dimension more for a higher than for a lower type. Moreover, it is reasonable to assume that higher types do not consume so much more such that they achieve a higher intrinsic utility than lower types. Given a personal equilibrium

6. Strictly speaking, $\beta(q) = \beta(q,\theta)$

has to satisfy these features what would be the ex ante optimal plan—the choice acclimating personal equilibrium (CPE)? Using integration by parts, the consumer's ex ante expected utility can be written as

$$\begin{aligned}&\mathbb{E}_\theta[U(q|\theta,\langle\hat{q}(\varphi,p)\rangle)] \qquad \text{(E.9)}\\ &=\int_{\underline{\theta}}^{\bar{\theta}}\left\{u(q(\theta),\theta)[1+\lambda(2F(\theta)-1)]-p[1+\lambda(2F(\theta)-1)]\right\}f(\theta)\,d\theta.\end{aligned}$$

As it is well-known in the literature, with CPE a decision maker is highly risk averse and may prefer stochastically dominated options. This behavior can be ruled out by assuming that loss utility is less important than intrinsic utility, i.e., $\lambda \leq 1$. For $\lambda \leq 1$, the integrand in the expected utility is strictly concave in $q(\theta)$ for all θ. The first-order condition obtained from point wise maximization is

$$\partial_q u(q(\theta),\theta) = p. \qquad \text{(E.10)}$$

Thus, at least for $\lambda \leq 1$ the demand function (18) is the ex ante preferred plan among all plans where higher types consume more but still achieve a lower intrinsic utility level.

Appendix F: Comparison to the loss function of Kőszegi and Rabin (2006, 2007)

For applications, Kőszegi and Rabin (2006, 2007) propose the following gain-loss function:

$$\tilde{\mu}(x) = \begin{cases}\tilde{\eta}x, & \text{if } x \geq 0,\\ \tilde{\eta}\tilde{\lambda}x, & \text{if } x < 0.\end{cases}$$

The parameter $\tilde{\eta} \geq 0$ is the degree of reference dependence and $\tilde{\lambda} \geq 1$ is the degree of loss aversion. ($\tilde{\lambda} = 1$ means that the consumer is not loss averse.)

With loss utility only in the money dimension and reference-dependent preferences ($\tilde{\eta} > 0$), this formulation has the drawback that at the contracting stage, the marginal rate of substitution (MRS) between money and consumption differs from the MRS at the consumption stage. To see this, we rewrite $\tilde{\mu}$ as

$$\tilde{\mu}(x) = \tilde{\eta}x + \begin{cases}0, & \text{if } x \geq 0,\\ \tilde{\eta}(\tilde{\lambda}-1)x, & \text{if } x < 0.\end{cases} \qquad \text{(F.1)}$$

Consider a consumer who is not loss averse ($\tilde{\lambda} = 1$) but has reference-dependent preferences ($\tilde{\eta} > 0$). Ex-post, his marginal utility of money is $1+\tilde{\eta}$. Ex ante, however, gains and losses cancel in expectation because the first part of the gain-loss function in (F.1) is linear. Therefore, ex ante, the marginal utility of money is 1. This time-inconsistency arises because we restrict reference-dependent utility to the money dimension. If preferences were also reference dependent in the good dimension, utility from consumption would also be multiplied by $1+\tilde{\eta}$ ex post, so that the marginal rate of substitution between money and the good would remain unchanged and equal to the ex-ante MRS.

To avoid the shift in the MRS, we use the following modified loss function:[7]

$$\mu(x) = \begin{cases} 0, & \text{if } x \geq 0, \\ \lambda x, & \text{if } x < 0. \end{cases}$$

If we set $\lambda = \tilde{\eta}(\tilde{\lambda} - 1)$, this corresponds to the second part of equation (F.1). This formulation eliminates reference dependence for consumers that are not loss averse because the first part of equation (F.1) was dropped. The ex ante expected utility of a consumer is unchanged because it only depends on $\tilde{\eta}(\tilde{\lambda} - 1)$ (Compare equation (7) in the main paper with equation (5) in an older working paper version, Herweg (2010)). The same argument applies to the ex ante expected joint surplus. Therefore, the condition for the optimality of a flat rate remains qualitatively unchanged between the different formulations.[8]

Since we do not want to model time-inconsistency, the new formulation which holds the MRS constant, is the natural choice. Also, this formulation is closer to the original formulation of Kőszegi and Rabin (2006, 2007) with loss aversion in both dimensions, because this formulation also has a stable MRS.

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7. Since only losses matter, we use $-\mu$ instead of μ in the main text. This is ignored here to facilitate the comparison with the formulation of Kőszegi and Rabin (2006, 2007).

8. Quantitatively, the conditions differ since optimal demand at the consumption stage is depressed in the $\tilde{\eta}$-$\tilde{\lambda}$-formulation compared to the λ-formulation, because of the different MRS.