Intertemporal Consumption Choices, Transaction Costs and Limited Participation in Financial Markets: Reconciling Data and Theory

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Abstract

This paper builds a unifying framework based on the theory of intertemporal consumption choices that brings together the limited participation-based explanation of the C-CAPM poor empirical performance and the transaction costs-based explanation of incomplete portfolios. Using the implications of the consumption model and observed household consumption and portfolio choices, we identify the preference parameters of interest and a lower bound for the costs rationalizing non-participation in financial markets. Using the US Consumer Expenditure Survey and assuming isoelastic preferences, we estimate the coefficient of relative risk aversion at 1.7 and a cost bound of 0.4 percent of non-durable consumption. Our estimate of the preference parameter is theoretically plausible and the bound sufficiently small to be likely to be exceeded by the actual total (observable and unobservable) costs of participating in financial markets.

**Keywords:** limited participation in financial markets, fixed participation costs, Euler equation for consumption.
1 Introduction

The dynamics of consumption and saving behavior are obviously related to the demand for assets and, as such, can provide valuable information for equilibrium asset pricing. The pathbreaking contributions of Lucas (1978) and Breeden (1979) made the link between the Euler equation for consumption and equilibrium asset prices explicit and used the first-order conditions of a consumer problem to build what is known as the Consumption Capital Asset Pricing Model (C-CAPM). Unfortunately, despite the formal elegance and the analytical simplicity of the C-CAPM, the empirical performance of the model has been, at best, mixed. Since the early studies by Hansen and Singleton (1982, 1983), it has been clear that observed asset returns were inconsistent with the dynamics of consumption choices, at least as observed in aggregate data. This evidence was reinforced and confirmed in a large number of other studies. Some studies, such as Hansen and Jagannathan (1991), suggested that one of the reasons for the poor empirical performance of the model was the low level of variability of aggregate consumption growth.

Recently, there have been several attempts at rationalizing this discouraging evidence and several studies have explored the possibility that limited participation in financial markets might explain the disparity between theoretical predictions and empirical evidence. More precisely, since the first-order conditions of asset pricing models hold with equality only for those households who own complete portfolios, the models should be tested for this subset of households and not for the whole population. As a consequence, since in practice relatively few households hold shares directly, even abstracting from standard aggregation issues arising from the non-linearity of the marginal rate of substitution, the use of aggregate consumption data in evaluating asset pricing models could be very misleading.

These points have been stressed by Mankiw and Zeldes (1991), Attanasio et al. (2002), Vissing-Jorgensen (2002) and Paiella (2004), among oth-
ers, who propose limited financial market participation as a unified framework for rationalizing the empirical rejection of the C-CAPM. These papers show that accounting for portfolio heterogeneity and in particular for non-participation in financial markets helps to reconcile the predictions of the theory with the empirical evidence. Attanasio et al. (2002), for instance, show that focussing on the consumption of stockholders not only yields estimates of preference parameters that are in line with the theory, but also one does not reject the overidentifying restrictions implied by the model and, relatedly, the moments of the marginal rate of substitution are within the Hansen-Jagannathan bounds.

While these studies have been somewhat successful in reconciling the C-CAPM with the empirical evidence, they take limited participation as given and make no attempt to rationalize it. Limited participation is itself a puzzle for the intertemporal consumption model, just like the observed substantial differences in portfolio composition across agents and over the life cycle. Merton (1969) and Samuelson (1969) have illustrated how such behavior is inconsistent with the maximization of expected lifetime utility, which predicts that rational agents should invest an arbitrarily small amount in all assets with positive expected return, including risky ones, unless there are non-linearities in the budget constraint.

One possible and obvious way to rationalize non participation within the intertemporal consumption model is by invoking non-proportional costs of financial market participation (explicit and non-explicit). As such costs are for the most part unobservable, the plausibility of this explanation depends crucially on the magnitude these costs should have to explain observed data. Should the size of these costs be ‘reasonable’ one might find the explanation attractive. Should instead the size of the participation costs that rationalize observed data be very large, one would probably dismiss it.

One of the first papers to consider this approach was the study by Luttmer (1999) in which, using aggregate data, the author provides a lower bound on
the transaction costs that would rationalize the model in the face of available data. Paiella (2007), using micro data, provides evidence in support of the participation cost hypothesis by bounding from below the costs of participating in some financial markets. Her bounds for the stock market are as small as $130 per year, which implies that it is likely that the true costs of participation may exceed this level in reality.\footnote{Vissing-Jorgensen (2003) provides additional evidence in favor of the participation cost hypothesis, at least for some consumers.}

This paper brings together the limited participation-based explanation of the poor empirical performance of the C-CAPM and the transaction cost-based explanation of limited participation to build a robust test of the theory of the intertemporal allocation of consumption. Using the implications of the consumption model and observed consumption and portfolio choices, we show how to identify jointly the preference parameters of interest and a lower bound for the costs of participating in financial markets rationalizing participation choices. The estimation of the parameters of interest is based on the necessary conditions for the optimality of observed behavior of financial market participants and non-participants.

The methodology relies on the (empirical) distinction between the consumption path of households holding a well-diversified portfolio of assets and the consumption path of financial market non-participants. The former, who pay all costs of financial market participation, exploit all trading opportunities and their consumption dynamics are consistent with the time series properties of asset prices. The latter do not bear the cost of participation. Using the estimated preferences and observed consumption behaviour, we can estimate a lower bound to the gains they forgo by non-investing in some assets with positive excess return. Given this lower bound, we can infer the minimum costs that would rationalize their non-participation.

The costs that we bound embrace all fixed costs that investors may incur in. They include all entry costs of first-time buyers, such as the time/money
spent understanding investment principles and determining and setting up
the optimal portfolio. They include also all per-period participation costs,
such as the time spent determining whether trading is optimal and any fixed
brokerage fee and annual expense.

Recently, several explanations of limited participation have been pro-
posed. These include financial sophistication (Campbell, 2006, and Calvet,
Campbell and Sodini, 2007), ignorance and some aspects of investment mis-
takes (Lusardi and Mitchell, 2005) and trust (Guiso, Sapienza, Zingales, 2004
and 2008). Our cost story could be consistent with most of these stories, as
long as one is willing to give a wide interpretation of participation costs. Our
exercise effectively quantifies the forsaken gains from participating into asset
markets and therefore can provide bounds to the any ‘costs’ of participa-
tion, being those direct monetary costs or psychic costs due to low financial
sophistication.

Our approach for the cost bound identification builds on and generalizes
the losses for leaving unexploited some trading opportunities and proposes
a lower bound on the level of fixed transaction (trading) costs reconciling
per-capita expenditure and asset returns. Instead, we use individual level
data and, by distinguishing between holders and non-holders of risky assets,
we focus on the loss from missing out on the equity premium. Our frictions
include all costs that individual agents must pay in order to invest in risky
assets.\(^2\) Paiella (2007) focuses on the behavior of non-participants. While
her approach delivers lower bounds for the participation cost that are con-
ceptually similar to those we propose here, her estimates are based on specific
assumptions about preference parameters. Instead, we simultaneously esti-

\(^2\)Luttmer’s estimates of the bound to the costs of trading are potentially biased because
they are obtained using aggregate expenditure data, which include both the consumption
of those who hold financial assets and the consumption of those who do not. For the latter
the benefits of trading in financial markets are likely to go beyond those associated with
capturing excess returns.
mate the preference parameters and the bounds on participation costs.

The idea that entry and participation costs may rationalize non-stockholding dates back to a 1995 paper by Haliassos and Bertaut who carry out theoretical simulations of how large entry and/or per-period participation costs would make households stay out of the stock market, based on the basic expected-utility model. More recently, the contributions to the identification of participation costs have taken one of two approaches. The first is that of Luttmer (1999), Vissing-Jorgensen (2003) and Paiella (2007), who estimate directly the threshold level of costs for given preferences for risk. As discussed earlier, our paper builds on these studies. Its contribution is the joint identification of preference parameters and of a lower bound to participation costs, addressing simultaneously the equity premium and the limited participation puzzles.

An alternative approach to the quantification of participation costs which directly develops the original contribution of Haliassos and Bertaut (1995) relies on the calibration and simulation of life cycle models that need to be solved numerically. This is the approach recently taken by Gomes and Michaelides (2006 and 2008) who calibrate a life cycle model with heterogeneity in risk aversion and fixed participation costs. Other interesting studies that take this “simulation-based” approach are the structural studies of Alan (2006) and Kahvecioglu (2005), who estimate participation costs by comparing the value functions in case of participation and non-participation in the stock market.

Both approaches have advantages and weaknesses. The main advantage of the approach we follow is that being based on some specific first order conditions of the dynamic optimization problem avoids the necessity of deriving a solution for consumption and portfolio shares. This, in turn, avoids the necessity to specify all the details of the stochastic environment in which the consumer lives. The main advantage of this approach, that we label as the ‘revealed preferences’ approach, also constitutes its main weakness. Not rely-
ing on the explicit solution of the model, it cannot say much about portfolio shares and their evolution.

The main weakness of the approach based on the solution and simulation of a life cycle model is that one needs to specify all details of the problem, ranging from the stochastic processes that generate earnings and interest rates, to the nature of transaction costs, to pension arrangement, to mortality patterns, to preferences for bequests and so on and so forth. Every single detail will matter for the nature of the solution and the derived solution inherits any shortcomings and remaining puzzles regarding portfolio shares from the model used.

A good example of the type of difficulties one has to face when calibrating and simulating a life cycle model is the exercise in Gomes and Michaelides (2006 and 2008). There, stockholders have to be calibrated to have larger risk aversion than non-stockholders. The reasons is that, in expected utility models, and even with Epstein and Zin preferences, while households with a higher risk aversion invest a lower portfolio share in stocks, they also save more for precautionary reasons and, therefore, accumulate more wealth. The net result is that the more risk averse end up investing larger amounts in stocks. Therefore, Gomes and Michaelides (2006 and 2008) have to make the somewhat counterintuitive assumption that stock holders are more risk averse, while, if anything, the available evidence (Guiso and Paiella, 2007, Barski et al. 2006) points to differences in risk attitudes that go in the opposite direction.

Of course, different portfolio shares could be driven by preference heterogeneity, but also by differences in earning processes, or in pension arrangements, or other institutional features. We can afford to be silent about the specific details and consider the useful and parsimonious case in which we assume homogeneous risk attitudes. We will show that this parsimonious version of the model can fit some important aspects of the data and provides some reasonable bounds on participation costs.
What is bought by the simulation based approach is the ability of deriving portfolio shares (and consumption dynamics) and comparing them with the available data. One possible use of our results would be to calibrate a life cycle model with the preference parameters we have estimated, supplement it with additional assumptions about the relevant stochastic processes (for earnings and rates of return) and institutional details and simulate it. We leave this investigation, which would require searching for an appropriate specification for the stochastic environment faced by consumers and for an appropriate interpretation of costs, to future research.

Using the US Consumer Expenditure Survey and assuming isoelastic preferences with multiplicative preference heterogeneity, we estimate the coefficient of relative risk aversion at 1.7 and a cost bound of 0.4 percent of non-durable consumption. This is one of the first studies that simultaneously estimates preference parameters and a bound on transaction costs using micro data.\(^3\) Moreover, relative to other studies that have estimated the transaction cost bound (such as Paiella (2007)) or preferences (such as Attnasio et al. (2002)) our estimates require much less stringent assumptions on preference heterogeneity and taste shocks. Our estimate of the preference parameter is theoretically plausible and the bound sufficiently small to be likely to be exceeded by the actual total (observable and unobservable) costs of participating in financial markets. Finally, our results are compatible with the evidence coming from the studies that rely on the calibration of fully structural models. Overall, our analysis implies that consumption asset pricing models with standard assumptions regarding preferences provide an

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\(^3\)A referee draw our attention to the recent work of Alan (2006) who estimates jointly stock market entry costs and the coefficient of relative risk aversion matching participation age profiles and using a simulation minimum distance estimator. She estimates the coefficient of relative risk aversion at 1.6, which is very close to our estimate, and the stock market entry cost at around 2 percent of annual permanent income, which is also in line with our results. However and in contrast with our findings, Alan’s estimation resulted in a strong rejection of the overidentifying restrictions, which appears to be due to the sensitivity of the analysis to small changes in the parameters.
accurate description of the data once portfolio heterogeneity and fixed costs of participation are properly accounted for.

The rest of the paper is organized as follows. In Section 2 we present the framework that we use to identify household preference parameters and the bound to the participation costs, within the type of environment specified by the model of intertemporal choice. Section 3 derives our econometric model based on the conditions for the optimality of consumption of stockholders and of non-stockholders. Section 4 presents the data. In Section 5 we discuss the results from the estimation. Section 6 concludes.

2 The model

Investing in financial assets involves information gathering, decision making, brokerage fees and/or other fixed costs that can create a disincentive to portfolio diversification. These frictions may end up offsetting the positive return paid by the asset. The heterogeneity of portfolio choices can then be explained on the basis of differences in socio-demographic and other, observable and unobservable individual specific characteristics, without the need to assume explicitly heterogeneity in the degree of risk aversion. The paper tests this hypothesis by jointly estimating the curvature of households’ utility function and bounding from below the costs that would rationalize non-participation for some consumers, but assuming that consumers are homogeneous in terms of the curvature of the utility function. Heterogeneity is allowed for in discount factors. While preferences and participation costs are jointly estimated using observations on both participants and non-participants, intuitively it is clear that the curvature of the utility function is identified by the consumption dynamics of those holding an optimal portfolio of assets vis-à-vis the dynamics of asset returns. On the other hand, the lower bound on participation cost is identified by the gains that non participants forgo by not diversifying fully.
Consider an environment where households have rational expectations, intertemporally additively separable preferences over consumption, a strictly increasing and concave per-period utility function, \( U(c_{h,t}, \xi_{h,t}) \), and a positive subjective discount factor, \( \beta \). We assume that the instantaneous utility function depends not only on consumption, \( c_{h,t} \), but also on an unobservable taste shock, \( \xi_{h,t} \). In the empirical specification we will assume that this shock enters multiplicatively. It can therefore be interpreted as representing heterogeneity in discount factors.\(^4\)

Households have access to two means to substitute consumption over time: a risky asset yielding \( r_{t+1} \) and a riskfree asset yielding \( r_{t+1}^f \). Let us assume that in order to invest in the risky asset households must pay a fixed cost. This cost is higher than any cost the riskless asset investment may involve. On the basis of portfolio composition, it is possible to distinguish between two types of households: those who hold both risky and riskless financial assets and those who hold only riskless assets. For the risky asset holders, who have paid the fixed cost, the Euler equation for consumption must hold, i.e.:

\[
E_t \left\{ \beta \frac{U'(c_{h,t+1}, \xi_{h,t+1})}{U'(c_{h,t}, \xi_{h,t})} (1 + r_{t+1}) \right\} = 1,  
\]  

(1)

where \( E_t \{} \) denotes the expectation conditional on the information available at time \( t \), \( U() \) is the marginal utility of consumption. Note that consumption at time \( t \) for participants is assumed to be observed after they have borne the fixed cost of participation.

Let us turn to those who have chosen not to invest in the risky asset.

\(^4\)In addition to the unobserved component, the taste shifter can also have an observed component. This specification is consistent with the specifications often used in the empirical literature on Euler equations (see Attanasio, 1999). With CRRA utility, the instantaneous utility function would take the form: \( U(c, z, \xi) = \frac{c^{1-\gamma}}{1-\gamma} \exp(\theta z + \xi) \), where the term \( z \) is a vector of observable variables, and \( \xi \) represents unobserved heterogeneity. In the empirical specification that we use we have not introduced \( z \) variables (such as demographic factors) in an unrestrictive fashion. Our utility is expressed in terms of consumption per adult equivalent.
Let \( \{c^h\}_t, \ t = 1, 2, \ldots T \) be the observable sequence of consumption choices of household \( h \). Since households choose optimally, conditional on the information available, and at time \( t \) they could have chosen any other feasible sequence of consumption bundles, their time \( t \) expected *ex-post* utility gain from deviating from \( \{c^h\}_t \) must be non-positive. More specifically, we assume that at time \( t \) non-shareholders could have paid a fixed cost of \( \delta \) units of consumption invested in the risky asset and adjusted consumption from \( (c^h_{t}, c^h_{t+1}) \) to \( (c^h_{t} + a^h_{t} - \delta c^h_{t}, c^h_{t+1} + b^h_{t+1}) \). \( a^h_{t} \) and \( b^h_{t+1} \) denote a feasible consumption perturbation. Optimality of their observed choices \( (c^h_{t}, c^h_{t+1}) \) implies that:

\[
E_t \left\{ v^{h, t+1} (a^h_{t}, b^h_{t+1}, \delta) \right\} \leq 0. \tag{2}
\]

where \( v^{h, t+1} (a^h_{t}, b^h_{t+1}, \delta) \) is the *ex-post* utility gain that they could have obtained by paying the fixed cost \( \delta c^h_{t} \) and perturbing consumption:

\[
v^{h, t+1} (a^h_{t}, b^h_{t+1}, \delta) = \left\{ U (c^h_{t} + a^h_{t} - \delta c^h_{t}) + \beta U (c^h_{t+1} + b^h_{t+1}) \right\} +\]

\[- \left\{ U (c^h_{t}) + \beta U (c^h_{t+1}) \right\}, \tag{3}\]

where we have suppressed the dependence of the utility function on \( \xi^h_{t} \) for notational convenience. Equation (2) says that, net of the cost \( \delta c^h_{t} \), the expected utility gain from perturbing the observed consumption path is non-positive. Hence, the investment is not worthwhile. Inequalities such as (2) must hold for any \( t \).

The fixed cost \( \delta \) cannot be observed directly and cannot be directly identified. However, following an approach similar to that proposed by Luttmer (1999) and generalized recently by Pakes et al. (2005) (see also Manski, 2003), we can place a lower bound on it. For any given \( (a^h_{t}, b^h_{t+1}) \), the function \( E_t \left\{ v^{h, t+1} (a^h_{t}, b^h_{t+1}, \delta) \right\} \) is continuous and decreasing in \( \delta \), as \( U_h() \) is continuous and increasing. Hence, for any given \( (a^h_{t}, b^h_{t+1}) \), there is a unique value \( d(a^h_{t}, b^h_{t+1}) \) such that (2) is satisfied if \( \delta \geq d(a^h_{t}, b^h_{t+1}) \). The function \( d(a^h_{t}, b^h_{t+1}) \) is defined implicitly as the solution to the equation \( E_t \left\{ v^{h, t+1} (a^h_{t}, b^h_{t+1}, d) \right\} = 0 \). In practice, we are interested in the lower
bound $d$ such that (??) is satisfied for any $\delta \geq d$, i.e. if $d = \max d(a_{h,t}, b_{h,t+1})$. As $E_t \{v_{h,t+1}(a_{h,t}, b_{h,t+1}, \delta)\}$ is continuous and decreasing in $\delta$, this $d$ solves the equation:

$$\max_{a_{h,t}, b_{h,t+1}} E_t \{v_{h,t+1}(a_{h,t}, b_{h,t+1}, d)\} = 0. \quad (4)$$

The parameter $d$ is the Hicks compensating variation for not investing in an asset yielding $r_{t+1}$. $d$ is a lower bound to the forgone gains for non-investing in assets with positive excess return, which in turn are a lower bound to the cost $\delta$ that would rationalize non-participation. The “true” forgone gains for holding a sub-optimal portfolio are just a lower bound to the participation costs, because the (unobservable) costs $\delta$ may be so large that households are never close to deviating from their actual choices. In this instance, by construction, a level of gains that is much smaller than $\delta$ will suffice to rationalize observed choices. The bound will be closer to the true cost the more profitable the trading rule $(a_{h,t}, b_{h,t+1})$. Further, $d$ is a lower bound to the forgone gains of non-investing in assets with positive excess return: the expected utility gains of deviating from observed portfolio choices may be higher than those captured by equation (??) for at least two reasons. First, the framework behind equation (??) measures the expected gains of using an extra instrument to adjust consumption over two periods. Thus, if the conditioning information set of the agent is larger than that of the econometrician, the agent may actually be able to obtain a higher utility gain than the econometrician can estimate. Second, we are approximating the utility from spreading the gains from the investment over the entire lifetime horizon of the utility maximizing agent with the utility from spreading the gains over the two periods when the investment takes place. This set up leaves households’ consumption plans unchanged at all other dates and allows us to appraise the gains that households forgo for not investing for one period by focussing just on their consumption at two adjacent dates.\(^5\) These

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\(^5\)Another implication of this approximation is that we cannot control for prior participation. Our cost bound is based on the average gains of first-time participants and of past
limitations, which are due to the nature of our data and to the choice of using first-order conditions and actual data rather than calibrations, do not weaken the importance of our findings, which as it will be discussed later, are in line with the conclusions of other studies that use very different approaches.

Overall, $d$ provides the basis for a heuristic test of the cost of participation hypothesis: for the latter to be a plausible explanation of limited participation, any reasonable cost of participation must be higher than our estimated bound. Although this is not the most powerful test, it is indeed the most reliable. A more powerful test would compare the costs with the true forgone gain - not just with a lower bound, as here. However, the estimation of the true forgone gain would require a much larger amount of information and/or assumptions.

3 Empirical specification

The analysis is based on the conventional assumption that utility exhibits constant relative risk aversion. For shareholders we consider a standard Euler equation (which, however, incorporates unobserved heterogeneity). For non-shareholders, instead, we consider the counterfactual gains that would be available in case of participation. In that case, we assume they would follow a certain trading strategy and, after paying the fixed cost for investing in the risky asset, they would adjust their current savings.\footnote{In the appendix, we consider the case where, rather than adjust only their consumption (and savings), households are also allowed to modify their portfolio and shift resources from the risk free asset to the risky one. Given the limited information we have on portfolio composition we preferred to perform the exercise that we report.} Let $x_{h,t}(\alpha^c)$ denote the fraction of time $t$ consumption they would give up and invest in the risky asset. $\alpha^c$ is a vector of parameters to be estimated. We also assume that they would consume all the returns on the investment when they realize it.
The ex-post gain of non-shareholders can then be written as:

\[ v_{h,t+1}(x_{h,t}(\alpha^c), \delta) = U(c_{h,t}(1 - x_{h,t}(\alpha^c) - \delta)) + \beta U \left((c_{h,t+1} + x_{h,t}(\alpha^c)c_{h,t}(1 + r_{t+1})) - \{U(c_{h,t}) + \beta U(c_{h,t+1})\}\right), \]

where we have suppressed again the dependence of the utility function on \( \xi_{h,t} \) for notational convenience.

The estimation of the utility parameter and of the cost bound then relies on two sets of first-order conditions. The first set is the Euler equation in (??), which ensures the optimality of shareholders consumption. The second set consists of the following equations, which follow directly from (??) and must hold for the set of non-shareholders:

\[ E_t \{ D_1 v_{h,t+1}(x_{h,t}(\alpha^c), d) \} = 0, \quad h \in H_{ns}; \quad (5) \]

\[ E_t \{ v_{h,t+1}(x_{h,t}(\alpha^c), d) \} = 0, \quad h \in H_{ns}, \quad (6) \]

where \( D_1 \) denotes the derivative with respect to the first argument of \( v_{h,t+1}(.) \) and \( H_{ns} \) is the set of time \( t \) non-shareholders. Equation (??) determines the optimal trading strategy in case of participation, given the cost. Since, in practice, the actual cost, \( \delta \), is not observed or estimated and only a lower bound to the cost is identified, the optimal portfolio is determined as a function of a cost equal to its estimated bound, \( d \), which is consistent with the rest of the analysis. Equation (??) determines the lower bound \( d \) to the participation cost \( \delta \), given the optimal investment.

Under the assumption of isoelastic preferences, and re-introducing unobserved heterogeneity, the Euler equation for shareholders in (??) becomes:

\[ E_t \left\{ \beta \zeta_{h,t+1} \left(\frac{c_{h,t+1}}{c_{h,t}}\right)^{-\gamma} (1 + r_{t+1}) \right\} = 1, \quad h \in H_s, \quad (7) \]

where \( \gamma \) is the coefficient of relative risk aversion, \( H_s \) the set of shareholders.
and \( \zeta_{h,t+1} = \left( \frac{\zeta_{h,t+1}}{\zeta_{h,t}} \right) \). Equation (7) for non-shareholders can be written as:

\[
E_t \left\{ \beta \zeta_{h,t+1} \left( \frac{1}{c_{h,t+1}} - \left( \frac{c_{h,t+1} + x_c^{e}(\alpha c h,t (1 + r_{t+1}))}{c_{h,t}(1 - x_c^{e}(\alpha c) - d)} \right)^{1-\gamma} - c_{h,t}^{1-\gamma} \right) \right\} = 1, \quad h \in H_{ns},
\]

(8)

which, as we show in Appendix A, can then be approximated as follows:

\[
E_t \left\{ \beta \zeta_{h,t+1} \left( \frac{c_{h,t+1}}{c_{h,t}} \right)^{-\gamma} \left( \frac{1 + r_{t+1}}{1 + d/x_c^{e}(\alpha c)} \right) \right\} \approx 1.
\]

(9)

Under the assumption that consumption and the rate of return on stock are jointly lognormal and homoskedastic, we can loglinearize (8) and obtain:

\[
\log (1 + r_{t+1}) = \alpha_s + \gamma \Delta \log (c_{h,t+1}) + \varepsilon_{h,t+1}, \quad h \in H_s,
\]

(10)

where \( \alpha_s \) is a function of the (conditional) second-order moments of consumption and asset returns and the residual \( \varepsilon_{h,t+1} \) includes the expectation errors as well as the transformation of the unobserved heterogeneity term \( \zeta_{h,t+1} \).\(^7\) Similarly, we can loglinearize equation (9) and obtain:

\[
\log (1 + r_{t+1}) = \alpha_{ns} + \gamma \Delta \log (c_{h,t+1}) + \log \left( \frac{x_c^{e}(\alpha c)}{d + x_c^{e}(\alpha c)} \right) + \varepsilon_{h,t+1}, \quad h \in H_{ns}.
\]

(11)

Equations (10) and (11) together with (8) allow us to identify and estimate the coefficient of relative risk aversion, \( \gamma \), and a lower bound, \( d \), to the costs justifying limited financial market participation.\(^8\) They are conditions for optimality that must be satisfied by consumption choices: (??)

\(^7\)Notice that if we moved away from a setting with expected utility and intertemporally separable preferences the estimated \( \gamma \) would be capturing the inverse of the elasticity of substitution.

\(^8\)One could estimate the parameters of interest by applying the method of moments directly to equations (??) and (??). However, Monte Carlo studies by Vissing-Jorgensen (1999) show that in the presence of measurement error, which is common in micro data, the estimator based on the non-linear version of the conditions for optimal consumption has poorer properties than estimators based on log-linearized versions of the equations. Attanasio and Low (2003) also with a Montecarlo study cast doubts about the small sample properties of non-linear GMM estimates of Euler equation parameters, even in the absence of measurement error.
must hold for shareholders, (??) and (??) for non-shareholders. Note that (??) is non-linear in the parameters of interest.

The two conditions for participants and non-participants can be pooled together to obtain:

\[
\log (1 + r_{t+1}) = \alpha_s p_{h,t} + \alpha_{ns}(1 - p_{h,t}) + \gamma \Delta \log (c_{h,t+1}) + \\
+ (1 - p_{h,t}) \log \left( \frac{x_{h,t}^c(\alpha_c)}{d + x_{h,t}^c(\alpha_c)} \right) + \varepsilon_{h,t+1},
\]

where \( p_{h,t} \) is a dummy variable that takes the value of 1 for participants in the stock market.

Given the parameters of the investment rule, equation (??) can be estimated by standard GMM methods. Any instrument that is uncorrelated with the expectational errors and the unobserved heterogeneity terms will be a valid instrument.\(^9\)

One difficulty with equation (??) is the presence of the term \( \log \left( \frac{x_{h,t}^c(\alpha_c)}{d + x_{h,t}^c(\alpha_c)} \right) \), which implies a non-linearity in parameters. In principle, one could further linearize (??) by applying a first-order Taylor expansion to the only term that is non-linear in parameters. In particular, \( \log \left( \frac{x_{h,t}^c(\alpha_c)}{d + x_{h,t}^c(\alpha_c)} \right) \) could be approximated by \( -\frac{d}{x_{h,t}^c(\alpha_c)} \). However, as we expect this ratio to be in the order of 0.1, the approximation would be a poor one. We therefore prefer to apply non-linear GMM techniques to estimate our parameters.

Equation (??) differs from the standard Euler equation for the term \( \log \left( \frac{x_{h,t}^c(\alpha_c)}{d + x_{h,t}^c(\alpha_c)} \right) \), which captures the difference in consumption growth between the relatively steep consumption path of shareholders and the flatter one of non-shareholders. If returns are high, the optimal investment in case of participation \( (x_{h,t}^c(\alpha_c)) \) would be large,\(^10\) unless non-shareholders’ consumption

\(^9\)The residual terms of these equations will also include the deviation between the conditional second moments in the intercept term and their unconditional value. We will therefore require that these deviations be orthogonal to the instrument used. See Attanasio and Low (2003) for a discussion of these issues.

\(^10\)Notice however that the larger \( x_{h,t}^c \), the smaller time \( t \) consumption, the less smooth the overall consumption path.
is correlated to the return on the risky asset due, for example, to some correlation between individual income and the stock market. In this instance, in order to justify non-shareholding or, equivalently, significant differences between shareholders’ and non-shareholders’ consumption, costs must be high too.

As to the estimation of the investment rules based on equation (??), since the data used for the analysis consist of repeated cross-sections, and not of long individual consumption series, we cannot estimate individual optimal rules. However, we can estimate the trading rules by summing over the set of households who do not invest in the asset considered at \( t \) and taking unconditional expectations, which yields:

\[
E \left\{ \sum_{h \in H_{ns}} D_1 v_{h,t+1} \left( x^c_{h,t}(\alpha^c), d \right) \right\} = 0, \quad h \in H_{ns},
\]

(13)

The trading rule \( x^c_{h,t}(\alpha^c) \) is assumed to be linear in a set of forecasting variables \( z_{h,t} \) that help to select the most profitable level of investment, which is then linear in consumption and wealth. In particular, in what follows we assume \( x^c_{h,t}(\alpha^c) = \alpha^c z_{h,t} \).

\(^{11}\) This set up allows us to capture in the estimate the predictability of the components of asset returns that are correlated with consumption growth and the set of forecasting variables \( z_{h,t} \). As we discuss below, the choice of the variables that determine the trading rule is somewhat arbitrary. The lower bound on the cost structure is then a function of the variables used in the trading rule.

\(^{11}\) In practice, we compute the optimal trading rule, by maximizing average nonstockholders’ forgone gains with respect to \( \alpha^c \), i.e. by solving numerically the following: 

\[
\max_{\alpha^c} \frac{1}{T-1} \sum_{t=1}^{T-1} \left\{ \frac{1}{n_{ns,t}} \sum_{h \in H_{ns,t}} \left[ \left( c_{h,t} - (\alpha^c z_{h,t} + d) c_{h,t} \right)^{1-\gamma} - 1 \right] + \beta \kappa_{h,t+1} \left( (c_{h,t+1} + (1+r_{t+1})(\alpha^c z_{h,t}) c_{h,t} \right)^{1-\gamma} - 1 \right\}. 
\]

We impose that the resulting trading rule is always between zero and one.
The estimation of the preference parameters and of the financial market participation cost bound is based on data from the US Consumer Expenditure Survey, which is run on a continuous basis by the Bureau of Labor Statistics. The CEX is a representative sample of the US population. It is a rotating panel in which interviews occur continuously throughout the year, each consumer unit being interviewed every three months over a twelve-month period, apart from attrition. As households complete their participation, new ones are introduced into the panel on a regular basis and, as a whole, about 4500 households are interviewed each quarter, more or less evenly spread over the three months.

At the time of the last interview, households provide information on their asset holdings at that date and on the dollar difference with respect to the amounts held twelve months earlier. The asset categories in the CEX are: 1. checking, brokerage and other accounts; 2. saving accounts; 3. US savings bonds; 4. stocks, bonds, mutual funds and other securities. As a measure of risky asset holdings, we take the amounts held in stocks, bonds, mutual funds and other securities and US savings bonds. As a measure of riskless asset holdings, we take the amounts held in checking and saving accounts. In order to avoid problems arising from the simultaneity of expenditure growth between $t$ and $t + 1$ and portfolio composition at $t + 1$, the asset holding status must be defined at the beginning of period $t$. For this purpose, for each asset category, we subtract from the stocks held at the time of the last interview the amount of savings (the dollar change) made over the previous twelve months. Hence, for each household we can define only one observation on the expected utility gain, $E_t \left\{ v_{h,t+1} \left( x_{h,t}^c(\alpha^c), d \right) \right\}$.

The consumption measure that we use is seasonally adjusted, real monthly per-adult equivalent expenditure on non-durable goods and services. Each quarterly interview collects household monthly expenditure data on a variety of goods and services for the previous three months. However, since the
information on asset holdings is annual, we use only two observations on consumption and denote as $c_{h,t}$ and as $c_{h,t+1}$ household per-adult equivalent consumption based on the expenditure reported for the first and last month of the year covered by the survey.

The data used for the analysis cover the period from 1982 to 2001, first quarter. Since interviews occur every month, $t$ runs for a total of 208 periods (months). We exclude from the initial sample the households that do not participate in all interviews, those living in rural areas or in university housing and those whose head is under 21 or over 75 years old. We also exclude those with incomplete income responses, those whose financial supplement contains invalid blanks either in the stocks of assets or in the dollar changes occurred with respect to the previous year and those whose stocks of checking and saving accounts and/or of shares and bonds are not positive (15 percent of the sample). Finally, we drop those households whose monthly consumption falls in the 1 percent tails of the distribution or whose consumption growth over the year falls in the 5 percent tails. The results are largely unaffected by reasonable changes in the cutoffs. Overall, the sample used consists of 24,016 households.

The fraction of non-shareholders has fallen from almost 65 percent in the first half of the 1980s to less than 60 percent towards the end of the past decade. Table 1 reports some descriptive statistics for the sample as a whole and for the sub-samples of shareholders and non-shareholders. Stockholders are slightly older than non-stockholders, they are significantly more educated, their consumption is higher and substantially more correlated with stock

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12Around 1985-86 and 1995-1996 the sample design and the household identification numbers were changed and after the first quarter of 1986 and of 1996 no track was kept of those who had entered the survey in 1985 and in 1995, respectively. As a consequence of this and of the fact that the information on financial asset holdings is collected during the last interview, those households that had their first interview in the third and fourth quarter of 1985 or of 1995 had to be excluded from the sample. Thus, the sample used consists of households who had their first interview between January 1982 and June 1985, between January 1986 and June 1995 and between January 1996 and June 2000.
returns. Summary statistics for asset returns and other financial variables used in the analysis are in Table 2. As risky return, we take the total return (capital gain plus dividends) on the Standard & Poor 500 Composite Share Index. As riskless return, we take the return on 3-month US Treasury Bills. Over the period considered stock returns were abnormally high and the mean equity premium that those who do not invest in risky assets forgo is almost 12 percent. The risk premium measure is given by the ratio of the yield of BAA bonds to that of AAA bonds, and the term premium measure is computed as ratio of the yield of 10-year government bonds to 3-month Treasury bills. The price-dividend and earnings ratios are taken from Shiller (2005) and correspond to the ratios of the S&P500 Composite Stock Price Index to the present value of the dividends accruing to the Index and to a 10-year moving average of S&P Composite earnings, respectively.

5 Results

As discussed above, the basis for our estimation consists of equations (??) and (??). These two equations are orthogonality conditions that we exploit to obtain GMM estimates of the structural parameters. As mentioned in the introduction, the use of these equations allows us to be agnostics about many of the details of the optimization problem solved by consumers. It is worth stressing that, although we have several thousands of individual observations, consistency in estimation is achieved by having a large number of time periods. As discussed in Chamberlain (1984), large T asymptotic is necessary in such a situation if one is not willing to assume the presence of complete markets that make aggregate shocks identical for all consumers. Therefore, the fact that we have 208 time periods is crucial. In the estimation, we also recognize the presence of aggregate shocks by allowing for arbitrary correlations among the residuals of individuals observed in the same time period, as described in the appendix. Given the data structure, which
includes annual consumption growth observed at a monthly frequency, residuals for individuals observed in adjacent months are correlated. This correlation declines only for individuals that are further than 11 months apart. As discussed in Appendix B, in the computation of standard errors we take this structure into account.

While in principle it is possible to estimate the parameters of these models considering equations (??) and (??) simultaneously, we use an iterative approach. Given an initial guess for \(d\) and \(\gamma\), we estimate the parameters of the trading rule by maximizing the utility gain in (??). Given the parameters of the trading rule, we then estimate the parameters in (??) by non-linear GMM. This second step gives us new estimates for \(d\) and \(\gamma\). We repeat this procedure until convergence.

5.1 Baseline specification.

The variables that enter the trading rule are somewhat arbitrary. They should provide a hypothetical rule that non participants would use if they were participating in the stock market. The better is the trading rule at exploiting opportunities on the stock market, the higher the utility gains from participation with such a trading rule would be. Therefore, the nature of the trading rule affects the tightness of the bound.

We specify the trading rule as a function of three variables: the risk free rate, the price/earning ratio and the term premium. The last variable is lagged three periods, while the other two are lagged two periods. Given the importance of the details of the trading rule for the bound, we experimented with different specifications. Empirically, it turns out that the specification of the trading rule is also relevant for the precision of the estimates, including that of preference parameters. Our choice of a trading rule was in part driven by the ability to obtain less imprecise estimates of our structural parameters.

For the GMM procedure, we need instruments that are uncorrelated with the unobserved heterogeneity in tastes and that are lagged two or more pe-
riods given the structure of our residuals, arising from the overlapping of the observations on consumption growth.\textsuperscript{13} In addition to the variables that we use for the trading rule, we include a polynomial in the age of the household head, cohort dummies, a time trend and other aggregate (lagged) price variables, such as the return on the risky asset, the risk premium and the price-dividend and the price-earnings ratios. The choice of the instruments is based on a regression aimed at identifying which of the available exogenous variables contributes the most to the prediction of the return on the S&P500.

Table 3 reports the results of the estimation of the parameters of interest in our baseline specification.\textsuperscript{14} The upper panel of the table reports the point estimates of the intercepts and of the slope of equation (\textsuperscript{??}), and of the cost bound, given the investment rule. Consumption is in per-adult equivalent terms to control for observed heterogeneity. Cohort dummies are included to take into account possible differences in tastes (and in particular discount factors) across cohorts. The bottom panel displays the coefficients of non-stockholders’ investment rule in the event of participation, given risk aversion and the cost.

The estimates in the upper panel indicate a coefficient of risk aversion of 1.7. While this coefficient is not estimated very precisely, the point estimate indicates a theoretically plausible magnitude. The estimate of the non-linear term implies a point estimate for the cost bound of 0.4 with a 95 percent confidence interval for the cost bound ranging from 0.1 percent to 1 percent of non-durable consumption. The cost bound is sufficiently small to suggest that the actual total (observable and unobservable) costs of participation probably exceed it in reality. In fact, if we take average per-adult equivalent non-stockholders’ monthly non-durable consumption from Table 1 and

\textsuperscript{13}Interviews occur every month of the year.

\textsuperscript{14}For numerical reasons and accelerate the convergence of the algorithm used, for the estimation of the cost bound we rely on $\log \left( x_{c_{t}} / \left( \exp(\delta) + x_{c_{t}} \right) \right)$ rather than $\log \left( x_{c_{t}} / (d + x_{c_{t}}) \right)$. 

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multiply it by 12 and by 2.5, which is the mean of the per-adult equivalent scale, and then multiply this by the estimated bound, we obtain a dollar estimate of the cost bound of approximately $72 per year. Since the model is overidentified, we can test the overidentifying restrictions. The test never rejects the null.

The investment rule coefficients are precisely estimated based on equation (??). They imply that, given a cost of 0.4 percent of non-durable consumption and a risk aversion of 1.7, non-shareholders would maximize their gains from participation by investing in the risky asset 4.2 percent of their current consumption, on average. For costs higher than 0.4 percent they are better off not investing at all.

5.2 Robustness.

While the specification of preferences we report is relatively parsimonious, we did check the robustness of our results in a number of dimensions. In this section, we report some of the results of this analysis. In particular, we focus on how the results change when on changes the specification of preferences (and therefore the Euler equation) and one changes the trading rule.

In Table 4, we show that our main conclusions hold if we remove the cohort dummies from our specification or if we allow additional variables, such as the age and education of the household head, to affect the (marginal) utility of consumption. The essence of our results is not changed by these modifications to the utility function. When we remove cohort dummies, the point estimate of the coefficient of risk aversion increases, but, given the low precisions of the estimates, not significantly so. Our bounds for transaction costs do not change much.

In Table 5, instead, we investigate the robustness of our results to changes in the trading rule. In the first column of the Table we report our baseline results (from Table 3), for comparison purposes. We explore the effect of introducing different lags of the variables we consider for the trading rule as
well as additional variables, such as lagged consumption growth. In the two
panels of the Table we report results with (on the left) and without (on the
right) cohort dummies. In all cases, our results are not affected much.

It is worth stressing that the gains that households forgo can be expected
to be higher the more resources are available for investment. Our investment
rule allows only for the reallocation of current savings into the risky asset
after the payment of the cost and not also for shifting some accumulated
wealth from the risk free asset to the risky one. We chose to perform the
exercise that we report due to limited information on portfolio composition.\textsuperscript{15}
Furthermore, Paiella (2007) shows that, for low levels of risk aversion, the
foregone gains are substantially higher only if households are allowed to shift
into the risky asset almost all their wealth. She finds that for a relative risk
aversion around 2, a value close to our risk aversion estimate, the optimal
investment rule would involve reallocating almost 90 percent of one’s wealth
and the forgone gain would be almost three times as high as when no wealth
reallocation is allowed. If the reallocation involved only 30 percent of wealth,
which is the mean portfolio share in stocks in her CEX sample, the gain would
drop to less than twice.

6 Concluding Remarks

This paper considers the poor empirical performance of the consumption-based
capital asset pricing model and builds a unifying framework that brings
together, within the theory of intertemporal consumption choice, limited par-
ticipation and fixed participation costs in order to appraise their joint role in

\textsuperscript{15}Other data sources, such as the Survey of Consumer Finances (SCF), contain much
more detailed information on portfolio shares than the CEX. However, they do not contain
information on consumption (or consumption growth). We decided against the possibility
of imputing portfolio shares in the CEX, using, for instance, the relationship between
variables observed in both the SCF and the CEX and the portfolio shares in the SCF.
Given our exercise, this would be equivalent to using the covariance between these variables
and consumption.
explaining the disparity between the standard model predictions and the empirical evidence. Allowing explicitly for differences in the consumption paths of shareholders and non-shareholders and for financial market participation frictions, we show how to identify the preference parameters of interest and a bound to the costs rationalizing limited participation.

The costs that we bound from below include all entry and per-period participation fixed costs that financial investments may involve and can be thought of as reflecting the costs of information and transaction that would induce households not to invest in some securities. They “produce” two types of households: one that pays the fixed costs to invest in asset \( j \) and whose consumption, net of the cost, is coherent with the Euler equation for asset \( j \); the other that does not pay the costs because its expected gain from the investment is too low. Differences in the gains from financial market participation and portfolio heterogeneity stem from differences in household observable socio-demographic characteristics and possibly in unobservable attributes, given common investment opportunity and information sets. Alternatively, the participation costs that we bound can be thought of as the costs of following near-rational decision rules. In this instance, households behave according to different decision rules. Those who participate follow rational decision processes that can be modelled as solutions to the maximization of the intertemporal choice model. Those who do not participate follow heuristic decision processes. For the latter the gains of fully optimizing, which we estimate by maximizing their utility under the assumption of full rationality, can be expected to be lower than the costs of solving the model for the optimal intertemporal allocation of consumption, which are primarily costs of information, attention, etc. Differences in socio-demographic characteristics and possibly in unobservable attributes can justify the differences in the costs of behaving according to rational decision processes.

Our approach yields Euler equation-based estimates of relative risk aversion of around 1.7, which is a theoretically plausible value for the curvature
of the utility function. The bound to the costs needed to reconcile the model with observed behavior turns out to be around 0.4 percent of non-durable consumption. Costs higher than this bound would offset the gains of investing optimally in stocks for a large fraction of the population. Our estimate is sufficiently low to make the participation cost-based explanation of limited participation a reasonable explanation, because it is likely that the true total costs of participation exceed this bound. Finally, our results are compatible with the evidence coming from other studies that use different approaches for the identification of participation costs, such as those that rely on the calibration of fully structural models.

Overall, our results suggest that the intertemporal consumption model may provides a suitable description of household behavior once fixed costs of participation and unobservable heterogeneity are properly accounted for. The next steps of this research need to investigate the nature of transaction costs. What we have in our model is purposely vague and consistent with many possible justifications of non-participation that have recently proposed in the literature. Moreover, future research will need to establish, possibly using simulation methods, what are the implications of our estimated preferences and cost size, for portfolio share, wealth and consumption over the life cycle under different institutional arrangements.
References


Appendix A: An Alternative Investment Strategy and the Approximation of the First-Order Condition for Non-Stockholders

Let us assume that after paying the fixed cost for investing in the risky asset, non-shareholders may adjust both their savings and their wealth allocation. Let \( x_{h,t}^{c}(\alpha^{c}) \) denote the fraction of time \( t \) consumption they give up and invest in the risky asset and \( x_{w,h,t}(\alpha^{w}) \) the fraction of their wealth \( W_{h,t} \), invested at the riskless rate, that they move into the risky asset. \( \alpha^{c} \) and \( \alpha^{w} \) denote the vectors of parameters. We also assume that they consume the return on the investment when they realize it. The \textit{ex-post} gain of non-shareholders is then given by:

\[
\begin{align*}
\nu_{h,t+1}\left(x_{h,t}^{c}(\alpha^{c}),x_{w,h,t}(\alpha^{w}), \delta\right) &= U\left(c_{h,t}(1-x_{h,t}^{c}(\alpha^{c})-\delta)\right) + \\
&+ \beta U\left((c_{h,t+1} + x_{h,t}^{c}(\alpha^{c})c_{h,t}(1+r_{t+1})\right) + (14) \\
&+ x_{w,h,t}(\alpha^{w})W_{h,t}(r_{t+1} - r_{f,t+1}) \bigg) \\
&- \left\{U\left(c_{h,t}\right) + \beta U\left(c_{h,t+1}\right)\right\}, \\
\end{align*}
\]

where we have suppressed the dependency of the utility function on \( \xi_{h,t} \) for notational convenience. Under the assumption of isoelastic preferences and multiplicative heterogeneity, the condition for the optimality of non-shareholders’ consumption can be written as:

\[
E_{t}\left\{ \beta_{h,t+1} \\
\frac{c_{h,t+1}^{1-\gamma} - \left((c_{h,t+1} + x_{h,t}^{c}(\alpha^{c})c_{h,t}(1+r_{t+1}) + x_{w,h,t}(\alpha^{w})W_{h,t}(r_{t+1} - r_{f,t+1})\right)^{1-\gamma}}{\left(c_{h,t}(1-x_{h,t}^{c}(\alpha^{c}) - \delta)\right)^{1-\gamma} - c_{h,t}^{1-\gamma}} \right\} \\
= 1. \\
\]

Equation (15) can be approximated as follows. If we multiply and divide the numerator of the term on the left-hand side of (15) by \( c_{h,t+1}^{1-\gamma} \) and the
denominator by \( c_{h,t+1}^{1-\gamma} \), we can re-write the ratio as:

\[
\frac{c_{h,t+1}^{1-\gamma}}{c_{h,t}^{1-\gamma}} \left( 1 - \left( 1 + x_{c,t}^c (\alpha_c)^{c_{h,t}} (1 + r_{t+1}) + x_{c,t}^c (\alpha_c)^{W_{h,t}} (r_{t+1} - r_{t+1}^f) \right)^{1-\gamma} \right) \frac{1}{(1 - x_{c,t}^c (\alpha_c) - d)^{1-\gamma} - 1}.
\]

Taking first-order Taylor expansions\(^{16}\) around 1 of the polynomials raised to the \((1 - \gamma)\) at the numerator and denominator, we can approximate (??) by:

\[
\frac{c_{h,t+1}^{1-\gamma}}{c_{h,t}^{1-\gamma}} \left( (1 - \gamma) \left( x_{c,t}^c (\alpha_c)^{c_{h,t+1}} (1 + r_{t+1}) + x_{c,t}^c (\alpha_c)^{W_{h,t}} (r_{t+1} - r_{t+1}^f) \right) \right) \frac{1}{(1 - \gamma) \left( x_{c,t}^c (\alpha_c) + d \right)}.
\]

After simplifying and collecting terms, we can re-write the first-order condition for non-shareholders in (??) as follows:

\[
E_t \left\{ \beta \zeta_{h,t+1} \left( \frac{c_{h,t+1}}{c_{h,t}} \right)^{-\gamma} \frac{(1 + r_{t+1}) + x_{h,t}^w (\alpha_w)^{W_{h,t}} (r_{t+1} - r_{t+1}^f)}{1 + d/x_{c,t}^c (\alpha_c)} \right\} \approx 1. \quad (18)
\]

\( x_{h,t}^w (\alpha_w) = 0 \) corresponds to the case we focus on, where non-stockholders’ consumption must satisfy:

\[
E_t \left\{ \beta \left( \frac{c_{h,t+1}}{c_{h,t}} \right)^{-\gamma} \frac{(1 + r_{t+1})}{1 + d/x_{c,t}^c (\alpha_c)} \right\} \approx 1. \quad (19)
\]

Appendix B: The Variance-Covariance Matrix of the Errors

The error structure of the main equation we estimate is complicated by several factors. First, we deal with annual changes in consumption observed at a monthly frequency. In a time series context this would induce MA(12) residuals. Second, individuals observed over the same time periods or, given the

\(^{16}\)Second- and higher-order terms can be ignored because they are small for reasonable values of the parameters and of the variables.
time frame just mentioned, over adjacent months, will be affected by similar aggregate shocks. This implies correlation in the cross-sectional dimension of the residuals.

While the instrumenting strategy we used takes into account this complex structure and guarantees that we obtain consistent estimates, in computing the standard errors we need to take it into consideration explicitly. The residuals of equation (??) are expectational errors for an Euler equation and can be expressed as the sum of 12 monthly innovations:

\[ \varepsilon_t^h = v_t^h + v_{t-1}^h + \ldots + v_{t-11}^h. \]  

Each of the monthly innovations can be expressed as the sum of two errors, one representing aggregate shocks and one purely idiosyncratic ones. In other words, we express \( v_t^h \) as the sum of its cross-sectional mean and deviations from the same and assume that these deviations are independent across consumers:

\[ v_t^h = \eta_t + u_t^h. \]  

Let:

\[ \text{Var}(\eta_t) = \sigma^2_\eta, \]  
and

\[ \text{Var}(u_t^h) = \sigma^2_{u,t}, \]  
i.e. it is time-varying. Then:

\[ \text{Var}(\varepsilon_t^h) = 12 \sigma^2_\eta + \sum_{j=0}^{11} \sigma^2_{u,t-j}, \]  

\[ \text{Cov}(\varepsilon_t^h \varepsilon_t^k) = 12 \sigma^2_\eta, \]  

and

\[ \text{Cov}(\varepsilon_t^h \varepsilon_{t-j}^k) = \begin{cases} \text{Cov}(u_t^h u_{t-j}^k) \neq 0 & \text{if } 0 < |t - j| \leq 11 \\ 0 & \text{if } |t - j| \geq 11. \end{cases} \]
Our estimate of the elements of the variance-covariance matrix is based on the internal product of the GMM residuals; hence it is heteroskedasticity robust.