Modelling Movements in Individual Consumption: A Time Series Analysis of Grouped Data♣

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Abstract

In this paper we propose a method to characterize the time series properties of group-level consumption, income and interest rates using micro data. Having estimated the parameters of flexible multivariate MA representations we relate the coefficients of our statistical models to structural parameters of theoretical models of consumption behaviour. Our approach offers a unifying framework that encompasses the Euler equation approach to the study of consumption and the studies that relate income innovations to consumption innovations, such as those that have found the so-called excess smoothness of consumption. Using a long time series of cross sections to construct synthetic panel data for the UK, we estimate our model on different year of birth and education groups, and find that for individuals with post-compulsory education the restrictions implied by the Euler equation are typically not rejected, the elasticity of intertemporal substitution is higher than one and there is evidence of "excess smoothness" of consumption. Households headed by individuals with at most compulsory education, on the other hand, exhibit excess sensitivity of consumption to past income and their estimated elasticity of intertemporal substitution is not statistically different from zero.

JEL: E2, D1, C3
Keywords: Consumption, Euler Equations, Life Cycle Model

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1. Introduction

In the last 25 years, many empirical studies of consumption behaviour have focused on some version of an Euler equation for intertemporal optimization and have estimated structural parameters and tested the model by exploiting the over-identifying restrictions implied by such an equation. These studies, too numerous to be listed here, have used both aggregate and individual level data, reaching different conclusions about the validity of the model and about the magnitude of the structural parameters that can be identified within such a framework. Most of the implications of such theoretical structure are restrictions on the time series properties of household consumption. Moreover, unless one is willing to make strong assumptions about the nature of intertemporal trades available to consumers (like that of complete markets) one can only obtain consistent estimates of the model's parameters from the orthogonality conditions implied by an Euler equation exploiting 'big T' asymptotics. Therefore, the absence of complete markets poses aggregation problems for macro-level studies and consistency problems for any micro-level study where data are not available over a long time horizon.

The results one obtain from an empirical study of consumption behaviour based on Euler equation estimation over a long time period (including parameter estimates and tests of the overidentifying restrictions of the mode), ultimately depend on the time-series properties of consumption, income and interest rates. While much is known about the time-series properties of aggregate consumption in its relation to income, interest rates and other aggregate variables,[1] the same is not true for the time-series properties of individual level consumption or even appropriately aggregated quantities. The latter point is important for the results one gets from Euler equation aggregation, as shown by Attanasio and Weber (1993).

In this paper, we propose a new methodology to analyze the time series properties of consumption expenditure, jointly with those of income and other (possibly aggregate) variables of interest, using a long time-series of household level data. Our approach fills an important gap in the existing literature. Individual longitudinal data have been used to study the time-series properties of hours and earnings by authors such as Lillard and Willis (1979), MaCurdy (1982), Abowd and Card (1989), Moffitt and Gottshalk (1995) and Meghir and Pistaferri (2004), among

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[1] The papers by Sargent (1978), Hall (1978), Flavin (1981), Blinder and Deaton (1985), Campbell (1987), West (1988), Campbell and Deaton 91989), Caballero (1990), Quah (1990) are only some examples of this literature.

others. These studies model only labour market variables. Moreover, they focus on the dynamic properties of purely idiosyncratic components and treat aggregate shocks as nuisance parameters that are eliminated, together with deterministic life-cycle effects, in preliminary regressions. Our contribution, instead, studies mainly consumption. In addition, the availability of a long time-series of cross-section allows us to focus on business cycle fluctuations rather than removing aggregate shocks.

By constructing the theory-consistent possibly non linear aggregates, we can avoid aggregation biases that have been shown to be important in the study of the dynamic properties of consumption. This aspect is crucial as we want to relate findings to the parameters of structural models of consumption behaviour.

A small, but increasing, number of papers emphasizes the presence of heterogeneity in preference parameters, which can explain a number of regularities. Blundell, Browning and Meghir (1994), Attanasio and Browning (1995), Crossley and Low (2009) and Alan and Browning (2010) all find a non-constant elasticity of intertemporal substitution. An advantage of using household level data is that they enable us to stratify our sample according not only to the year of birth of the household head, as is typically done in the studies using pseudo panels, but also to some measure of lifetime wealth, such as the education level, thus allowing for an important source of heterogeneity to have an impact on preference parameters.

We represent the time series behaviour of group-level consumption, income and interest rates as an MA process whose parameters, given some assumptions, we can estimate. These parameters will summarize the time series properties of the variables we model. Given a set of identifying restrictions, we will be able to identify both patterns of dynamic dependence and some contemporaneous correlations. Having characterized these properties of group-level consumption, income and interest rates, we map the pattern of correlations that emerges from the data to those implied by different theoretical models. In particular, we show how to relate the coefficients of our statistical model to several models that have been studied in the consumption literature. In this sense, our approach provides a unifying framework that can be used to assess and compare different pieces of evidence that have been accumulated in the literature. Moreover, our focus on specific properties of the time series processes for consumption and other variables of interest provides insight on the features of a model that might be able to fit the data when simpler versions are rejected.

We show what are the restrictions that the Euler equation for consumption imposes on our MA representation of grouped consumption, income and interest rates and we show how, if these restrictions are not violated, one can use estimates of the restricted MA process to estimate structural preference parameters, as is typically done in the Euler equation literature. Our procedure makes it explicit which time series features of the data lead to potential rejections of the over-identifying restrictions implied by Euler equations and which lead to specific estimates of preference parameters. Moreover, our approach allows us to identify aspects of individual preferences that, while not leading to violations of the orthogonality conditions implied by an Euler equation, might be important determinants of individual behaviour.

But our approach can also go beyond what is learned from the estimation of Euler equations. As is well known, the Euler equation is silent about contemporaneous correlations between consumption innovations and other sources of innovation to variables that are relevant for the dynamic problem solved by the consumer, namely income and interest rates. The Euler equation does not provide, without additional equations, a consumption function. And yet our approach can identify some of the contemporaneous correlations and we can check whether these estimated correlations are consistent with different versions of the model. Our contribution is therefore also directly related to a strand of the literature on permanent income with Rational Expectations that has looked at the implications of the life cycle- permanent income model for the time series behaviour of consumption and income. Some of the relevant papers in this literature include Sargent (1978), Flavin (1981), Campbell (1987) and Campbell and Deaton (1989), West (1988), Hansen, Roberds and Sargent (1991). These papers worked with versions of the model that would deliver a closed form solution for consumption as a function of permanent income and test the restrictions implied by this closed form solution on the bi-variate representations of consumption and income. Similar results, however, can be obtained by log-linearization. Many of these papers concluded that consumption was 'excessively smooth' in that consumption did not react enough to permanent innovations to income. We pursue, using micro data, a similar strategy. In particular, we study the restrictions imposed by the life cycle model on the parameters of our time series model that govern the contemporaneous correlation between income and consumption. Following Attanasio and Pavoni (2011), we stress that we can have situations in which the Euler equation for consumption is satisfied and there is no evidence of 'excess sensitivity' of consumption and yet one can observe that consumption does not move enough in reaction to changes in permanent income. Attanasio and Pavoni (2011) stress how this excess smoothness is related to the violation of the intertemporal budget constraint *with a single asset* and the presence of insurance of idiosyncratic shocks.

Our approach, therefore, can be seen as giving a unifying framework that uses simultaneously the implications of the data covariance structure to estimate structural parameters and test the orthogonality restrictions implied by the model (as in the Euler equation approach), and studying how innovations to income are reflected (or not) in changes in consumption.

The use of large T asymptotics is an important distinguishing feature of our approach. While we remove deterministic trends, we do not remove business cycle aggregate shocks. Indeed, our approach can be described as an attempt to model these aggregate shocks and is therefore based on large T asymptotics.[2] Without relying on large-T asymptotics, our approach would not be able to encompass the Euler equation approach, which, in general, cannot identify the parameters of interest without a long time horizon.

The data on which we apply our approach are from the UK Family Expenditure Survey. The FES is, to the best of our knowledge, the longest time series of cross section containing exhaustive and detailed information on consumption, its components and several other variables of interest. Unfortunately, the data do not have a longitudinal dimension, as each individual household appears only once. To study the dynamics of our system, therefore, we rely on synthetic cohort techniques, of the type proposed by Deaton (1985). As the theoretical model on the background of our analysis is the life cycle model, it is natural to study consumption in its relation to age and to divide the sample to form year of birth cohorts of individuals that are followed over time. To capture differences in covariance structures (and hence preference parameters) among different groups of the population we also form groups according to the education levels. We use surveys from 1974 to 2000 to form quarterly observations of consumption and income at the cohort level.

The lack of a longitudinal component to our data imposes an important limitation to our work, in that we are forced to ignore pure idiosyncratic variability and to focus instead on the

[2] Another strand of the literature has focused on the time series properties of disaggregated business cycles. These studies, including Watson and Engle (1983), Quah and Sargent (1994), Forni and Reichlin (1996), focus on the time series properties and aim at characterizing the number of common factors and modeling their dynamic effects on the sectors considered. Another set of papers that are related to ours are those by Cunha et al. (2004) and Cunha et al. (2005) as well as Blundell et al. (2004). In these papers, the authors use schooling, income and consumption data to identify the shocks that inform individual choices.

dynamics of group averages. In this sense, the group shocks become really the focus of our empirical study. It should be stressed once more, however, that even if we are forced to aggregate the data, we can control the aggregation directly and construct the non-linear transformation of the data implied by the theory before aggregating them. Having said this, however, it is clear that, should longitudinal data on consumption be available, an interesting extension of our study would be to estimate our model on such data.

We find that at the cohort level the restrictions implied by the Euler equation are not rejected, with no evidence of excess sensitivity. When we form finer groups according to the education level, we find substantial differences between households whose heads completed at most compulsory schooling and households whose heads attained higher education levels. For the latter groups, we still do not find evidence of excess sensitivity of consumption to income. The estimated elasticity of intertemporal substitution for higher educated households is higher than (but not statistically different from) one and we do find evidence of "excess smoothness". For the less educated group (that is, households heads with at most compulsory school completed) results are very different: we do find evidence of excess sensitivity of consumption, and our estimates of the elasticity of intertemporal substitution are close to (and not statistically different from) zero.

The rest of this paper is organized as follows. In section 2 we present our statistical model and discuss its identification and estimation. In section 3, we present a simple theoretical framework that can be used to interpret the results. We also show how to derive the so-called 'excess sensitivity' and 'excess smoothness' tests. After describing the data sources in section 4, in section 5 we report the estimates of our statistical models. In section 6 we show what interpretation they have in terms of the models presented in section 3. Section 7 concludes the paper.

2. The Methodology

As stressed in the introduction, the main aims of this study are two. First, we would like to identify innovations to consumption and other variables and model their covariance structure. Second, we want to use this covariance structure to shed some light on the plausibility of alternative theoretical models of consumption. As the micro data we work with lack any longitudinal dimension, we are forced to use average grouped data to estimate any dynamic model. This means that we are not able to model idiosyncratic persistence, but only persistence at the group level. The advantage of using grouped data lies in their longitudinal (or time) dimension, which allows to identify the time series moments of the variables of interest.

Our model is effectively a model of group shocks. However, as we construct group averages from our micro sample, the variables we observe are affected by two sources of variability. On the one hand, we have genuine group specific shocks. On the other, our sample averages are affected by "measurement error" arising from the limited size of our sample. The latter source of variability in this setting constitutes a nuisance which can, however, be controlled for given the information on the sample structure and given the information on within cells variability (see Deaton, 1985).

In this section we sketch the main features of our approach. In particular, we write down the statistical model that we estimate and discuss the identifying assumptions we make.

2.1. The statistical model

Let us consider a generic variable z_{ct}^{h}, where the index *h* denotes the individual household, the index *t* the time period and the index *c* the group to which household *h* belongs. Without loss of generality, such a variable can be written as:

$$(1) \qquad z_{ct}^{h} = \bar{z}_{ct} + \eta_{ct}^{h}$$

where $\bar{z}_{ct}$ denotes the mean of the variable *x* for group *c* at time *t*. Given a sample in which group membership is observable, the mean in equation (1) can be easily estimated by the sample means.

Notice that the variables in (1) can be non-linear transformations of the variables of interest. As we work with micro data, we can control the aggregation directly. This turns up to be important for at least four reasons. First, theoretical models often imply relationships among non linear functions of variables. Therefore, to give a structural interpretation to the results we present it will be crucial to control aggregation. Second, in the case of corners, the theoretical relationships one might want to consider at the aggregate level involve both means conditional on not being at a corner and overall means. Third, within cell heterogeneity, that is the variability of

η_{ct}^{h}, is not only useful to correct for the measurement error in the estimation of $\bar{z}_{ct}$, but can also be informative to study the evolution of the inequality over time. Fourth, we can define groups not only on the basis of the year of birth, but also on some other relevant time-invariant characteristics, as the education level of the head of the household.

As stressed above, because we lack panel data, we can only study dynamic models by using grouped observations. That means that we can only study the dynamics of $\bar{z}_{ct}$. Any purely idiosyncratic persistence, embedded in η_{ct}^{h}, cannot be recovered by our methodology. This might be a serious problem in evaluating the importance of precautionary savings or similar phenomena. Nothing much can be done about this except noticing that to get a handle on persistence at the individual level, a genuine panel dimension is needed as it will be necessary to observe the covariance between individual variables in subsequent time periods[3].

The first step of our procedure consists in removing all deterministic trends from the variables of interest. These are likely to be affected by time, group and age effects. As within the framework of a life cycle model groups are often formed on the basis of the year of birth of the household head, group effects are essentially cohort effects. This implies the impossibility of disentangling age, time and cohort effects. In what follows we label all deterministic trends in the data as 'age and cohort' effects[4].

It should be stressed that our first step regressions do not include time dummies: as business cycle shocks (either common across groups or not) are the focus of the study, we do not want to remove them. Removing group specific age polynomials, however, does remove deterministic trends from our data, which may, but do not need to, be interpreted as a combination of age and cohort effects. What we do not remove are business cycle fluctuations that are instead typically removed in studies that introduce time dummies in the first step regressions and focus on idiosyncratic shocks. Having estimated the parameters of such a regression, we interpret the cohort averages of the estimated residuals (z_{ct}) as deviations of the

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[3] Having said that, however, it should be stressed that even a relatively short genuine panel might be sufficient to estimate a model which requires big T asymptotics if repeated panels are available: one can then group the cross moments and follow their dynamics over time.

[4] While this label is arbitrary, disentangling the various effects is not the aim of the study which focuses, instead, on modelling the innovations to income and consumption at the business cycle frequency.

average group data from the deterministic trends present in the data. It is the cohort averages of these residuals that we model and study.

To take into account the possibility of stochastic trends (and because of the structural interpretation we give to the covariance structure that we estimate), we take the first differences of the cohort averages z_{ct} and study their time series properties. Hence we can rewrite equation (1) as:

$$\Delta z_{ct} = \Delta \bar{z}_{ct} + \Delta \eta_{ct} \tag{2}$$

where η_{ct} is the error induced by sample variability and that we will call "measurement error".

Consider three (possibly vectors of) observable variables, x, y and r[5]. For each of the cohort/education groups we consider, the model we estimate is the following:

$$\begin{aligned}
\Delta x_t &= \sum_{j=0}^{q_{xx}} \alpha_j^{xx} u_{t-j}^x + \sum_{j=0}^{q_{xy}} \alpha_j^{xy} u_{t-j}^y + \sum_{j=0}^{q_{xr}} \alpha_j^{xr} u_{t-j}^r + \eta_t^x - \eta_{t-1}^x \\
\Delta y_t &= \sum_{j=0}^{q_{yy}} \alpha_j^{yy} u_{t-j}^y + \sum_{j=0}^{q_{yr}} \alpha_j^{yr} u_{t-j}^r + \eta_t^y - \eta_{t-1}^y \\
r_t &= \sum_{j=0}^{q_{rr}} \alpha_j^{rr} u_{t-j}^r
\end{aligned} \tag{3}$$

where we dropped the cohort or group subscript c for ease of notation. The shocks u_t^x, u_t^y and u_t^r are assumed to be uncorrelated over time and with a diagonal variance-covariance matrix at each time t. We test and allow for structural breaks in the parameters and variances over time, a feature that turns out to be important for the real interest rate shocks, as discussed in section 4.1. The terms $\eta_s^{\cdot}$ represent the measurement error (or unobserved heterogeneity) present in the group specific variables, and we assume they are orthogonal to the unobservable random variables u. We can easily compute the time-varying variances of these terms using our micro data, and estimate system (3) conditionally on the $\eta_s^{\cdot}$. We describe the estimation method in section 4.

The variables x represent choice variables for the individual household: while in the basic specification we estimate x is non durable consumption, in general x can include several variables such as, for instance, different components of consumption, hours of work and participation rates by adding more equations to the system. These variables are affected by all the shocks present in the system.

The variables y are household (or cohort) specific but are assumed not to be determined by individual choice, at least at the frequency we are considering. In the specific model we estimate we consider y to be income, while in more general models it could include, for instance, wage rates for male and females, and they can be group specific. The y variables are affected by all the shocks in the system with the exception of the shocks specific to the variables in the first group (the choice variables). The r variables are also not determined by individual choices and, moreover, do not vary across individuals. We think of this kind of variables as prices. In the model we estimate below, we have a single variable of this type, the interest rate, but we could consider many, such as the prices of individual commodities.

The MA structure in equation (3) is quite general, but imposes some important and strong restrictions that are used to identify the model. In particular, for the contemporaneous effect of shocks we need to impose the triangular structure described above for identification. In particular we assume that shocks to individually determined variables, such as non-durable consumption, do not affect the variables that are assumed to be given to the individual households. This assumption is standard in the multivariate VAR literature. In addition to these restrictions, we make the normalization assumption that the coefficients on the own residuals are equal to one. For instance, we assume that all the components of the diagonal of the matrix $\alpha^{xx}{}_0$ are equal to one.

In system (3) the triangular restriction is imposed also on the lags of the relevant shock, but it is done only for ease of notation. As we discuss in the next section, theory imposes several (over-identification) restrictions on the lag coefficients which we will exploit to give a structural interpretation to our time series model.

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[5] For simplicity, we sketch the model for three univariate variables, but it can be easily extended to three vectors of variables.

3. A structural interpretation of the results

The starting point for a structural interpretation of the parameters in system (3) is the life cycle model, interpreted as a flexible parameterization of a dynamic optimization problem in which the decision unit is the household.

We start with the simplest version of the life cycle model as an example of a way in which a theoretical framework can be used to impose restrictions on the parameters of model (3). We then complicate the model to introduce a number of realistic elements. We neglect deterministic trends (including age effects) as well as family size effects from all the variables in our analysis. This implicitly assumes that the age and family size effects removed in the first step of our estimation procedure capture completely the effect of demographic variables and that these are considered as deterministic. As we focus on business cycle frequencies, we do not think that this assumption is particularly strong.[6]

3.1. A simple version of the life cycle model

A very simple version of the life cycle model implies the following system of equations for a generic individual:

$$(4) \qquad \log(\lambda_t) = E_t[\log(\lambda_{t+1})] + E_t r_{t+1} + k_t + \varepsilon_{t+1}$$

$$(5) \qquad \log(\lambda_t) = \log(U_x(x_t, z_t))$$

where the variable k is a function of the discount factor and of higher moments of the expectational error ε_{t+1}. U_x is the marginal utility of (non-durable) consumption, which is assumed to depend on consumption and a vector of observable and unobservable variables z. λ_t is the marginal utility of wealth and represents the effect of all present and future variables relevant for the optimization problem faced by the individual. r is the interest rate. The specification in equations (4) and (5) also assumes intertemporal separability, in that the marginal utility of consumption at t does not depend on variables from other time periods.

[6] If one thinks that the age polynomials used in the first step are not sufficient to remove the effect of demographic variables, and is willing to retain the assumption that they are deterministic, these variables can be used in the first step regressions.

If one considers the fact that equations (4) and (5) refer to a single generic household, it is clear why, even in such a simple framework, aggregating such an equation across groups of households would generate group specific fixed effects. These could arise if, for instance, there are systematic differences across groups in the discount factors, higher moments of the expectational errors, or in the unobserved components of z. Indeed, our results disaggregated by education level will show these effects to be important.

If we assume that the interest rate and k are constant over time and that the observable component of the vector z contains only deterministic variables that can be captured by the deterministic trends removed in our first step, equations (4) and (5) have very simple and strong implications for the model in (3).

First, one can simplify the model considerably eliminating the last equation (that refers to the interest rate). Furthermore, if the specification of the utility function is such that the marginal utility can be approximated by a linear function of log consumption (as it is the case, for instance, for a CRRA utility function), from equations (4) and (5) one can see that changes in log consumption can be related to the expectational error ε_{t+1} and therefore should not exhibit any serial correlation and should be uncorrelated with any information available at t. This is the celebrated 'random walk'[7] result, stressed by Hall (1978). In our model, it translates into restrictions on the coefficients on lags of all shocks of the consumption equation. The so-called tests of 'excess sensitivity' of consumption to predictable components of income take the form in our model of tests on the lagged income shocks being good predictors of consumption.

3.2. The elasticity of intertemporal substitution

A first and very important generalization of the model is to consider time varying interest rates. This extension is of particular interest as allows one to estimate the elasticity of intertemporal substitution.

Allowing for a time variable interest rate involves including the third equation in model (3), so that one can measure the correlation between innovations to interest rates and

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[7] Or martingale, to be precise.

consumption (and other variables)[8]. If we consider an asset whose rate of return is the same across groups and that is widely held, than equations (4) and (5) induce a set of additional restrictions on system (3). Let's define $A^{xx}(L) = \sum_{j=1}^{q} \alpha_j^{xx} L^j$, and analogously for $A^{xr}(L)$ and $A^{rr}(L)$. It is easy to show that an isoelastic utility function with a coefficient of relative risk aversion γ implies that:

$$\gamma(A^{xx}(L)u^x_{c,t+1} + A^{rx}(L)u^r_{c,t+1}) = A^{rr}(L)u^r_{c,t+1}. \tag{6}$$

As this has to hold for every possible realization of the residuals, the restrictions on the coefficients of system (3) are that:

$$A^{xx}(L) = 0 \text{ and} \tag{7}$$

$$\gamma A^{xr}(L) = A^{rr}(L). \tag{8}$$

The second set of restrictions implies that, as long as the interest rate is predictable, one can identify the coefficient of relative risk aversion.

Should one encounter a rejection of these restrictions, several alternative specifications are possible depending on the nature of the rejection. If a significant coefficient on the lagged 'consumption shocks' in the consumption equation were found, a possible explanation would be the possibility of an unobservable component in the vector of preference shifters *z*. Such a component, which, for lack of a better term we label 'unobserved heterogeneity', captures those aspects of preferences that are not directly modeled and that are likely to be important for consumption. The time series properties of consumption innovations would then be clearly affected by the time series properties of such a term. The fact that the restrictions about the proportionality of the coefficients on the interest rate lagged innovations is violated might be an indication of differences in interest rates and/or risk aversion across groups. If one more lag in the interest rate innovation enters the system even this possibility can be tested against more general misspecifications.

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[8] One can either assume that the interest rate is the same for all groups or allow for differences in intertemporal prices induced, for instance, by differences in marginal tax rates across groups. The latter approach, however, involves the necessity of measuring group specific interest rates.

3.3. Excess smoothness

The restrictions we have discussed so far are derived from the orthogonality conditions implied by the Euler equation for consumption derived from equations (4) and (5). These conditions, together with a set of intertemporal budget constraints (and initial and terminal conditions for assets) pin down the allocation of consumption over the life cycle. In situations in which it is possible to derive a closed form solution for consumption (as is the case, for example, with quadratic utility and constant interest rates) then the solution imposes restrictions on the coefficients of system (3) that relate income shocks to consumption. When a closed form solution for consumption that pins down the relationship between income and consumption innovations is not available, one can rely on approximate solutions, of the type developed by Campbell (1994) and used, among others, by Blundell, Pistaferri and Preston (2008) and Attanasio and Pavoni (2011).

The restrictions that the life cycle permanent income model imposes on the contemporaneous correlation between consumption and income relates to the fact that consumption should react to news about permanent income in a way that is mediated by the intertemporal budget constraint and that depends on the information that current income shocks give about future income. These are the type of restrictions that were studied by Flavin (1981), Campbell and Deaton (1989), West (1988), Quah (1990) and Hansen, Roberds and Sargent (1991) (HRS) among others. HRS, in particular, stress that given the Euler equation, the intertemporal budget constraint imposes testable restrictions on the response of the (change of) non-durable consumption to shocks to income whose violation has been interpreted as 'excess smoothness' of consumption. It is worth comparing the HRS approach to the specification we have proposed.

HRS show that , in a simple version of the permanent income model, the model gives rise to a representation of the following type:

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(9) \quad \begin{aligned}
\Delta y_t &= w_t^y + \beta_{111} w_{t-1}^y + \beta_{120} w_t^c + \beta_{121} w_{t-1}^c \\
\Delta c_t &= \qquad\qquad\qquad\qquad w_t^c + \beta_{221} w_{t-1}^c \\
\\
& E[w_t^y] = E[w_t^y w_{t-1}^y] = E[w_t^c] = E[w_t^c w_{t-1}^c] E[w_t^y w_t^c] = 0 \\
& Var(w_t^y) = \sigma_{wy}^2;\ Var(w_t^c) = \sigma_{wc}^2
\end{aligned}
$$

HRS stress that an implication of the theory is that this representation is *not* a Wold representation for the joint time series of consumption and income. Such a representation, assuming that consumption is a martingale (imposing the Euler equation) can be used to test the restrictions imposed by the intertemporal budget constraint. The test proposed does not require the specification of the information set observed by the consumer.

The structure of this representation is quite similar to ours, except that we have the opposite triangular structure, that is in the system (3) consumption is allowed to depend on all shocks, while income is not allowed to depend on the consumption shock. Under special circumstances, the two representations are equivalent. If, for instance, all the lag coefficients in our consumption equation are zero, one can map one specification into another. We need such a restriction so that, in our simple model, consumption is a martingale. However, in more general circumstances in which the HRS specification allows for lags in the consumption equation (maybe originated by temporal non-separabilities), the two specifications impose different restrictions on the data.

In a recent contribution, Attanasio and Pavoni (2011) discuss how violations of the intertemporal budget constraint can arise in a situation in which the Euler equation is satisfied but consumption is partly insured in a model with moral hazard and hidden assets. In particular, Attanasio and Pavoni (2011) stress the difference between the restrictions that imply the lack of correlation between predicted income and predicted consumption and the restrictions that involve the contemporaneous correlation of income and consumption. The latter can arise even with an Euler equation holding, if the intertemporal budget constraint *with a single asset* is violated, maybe because it ignores state contingent transfers that insure part of permanent shocks.

In our model, the intertemporal budget constraint with a constant interest rate (and the Euler equation) implies that:[9]

$$\text{(10)} \qquad \sum_{j=0}^{q} \alpha_j^{xy} z^j \equiv \alpha^{xy}(z) = \alpha^{xy}(z) \equiv \sum_{j=0}^{q} \alpha_j^{yy} z^j$$

where $z = (1/1+r)$ and r is the interest rate. Campbell (1987), West (1988) and Campbell and Deaton (1989), Hansen, Roberds and Sargent (1991) report results on versions of this test obtained from aggregate time series on income, consumption and saving that imply that consumption responds too little to innovations in income, a result that has been labelled the excess smoothness of consumption. In our framework, this result would imply that the left-hand-side of (10) would be less than the right hand side.

4. Data and estimation

The data used in the estimation are drawn from the UK Family Expenditure Survey (FES) from 1978, first quarter to 2005, first quarter. In this survey, about 8,000 families in the UK are interviewed each year and they are asked to fill diaries in which they record all the expenditures they make for two weeks. The survey records also information on demographic and labour supply variables for each member of the family. From these figures, it is possible to reconstruct total family income. We use these data to construct consumption and income grouped data at a quarterly frequency.

The FES has been widely used in the research on consumption. It has several advantages, among which we should mention the fact that it is available over a long time period, which, as we discussed above, is crucial for our identification strategy. Moreover, the quality of the data seems, at least until very recently, very high. Tanner (1998) and more recently Brewer et al. (2006) show that, when aggregated, the FES reproduces closely the dynamics of National Account consumption data.

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[9] In some of our models, the coefficient on lagged income shocks are zero. This implies that consumption is a martingale. Hansen Roberds and Sargent (1991) stress that without imposing the martingale property implied by the Euler equation for consumption the intertemporal budget constraint does not impose restrictions on the time series properties of savings and consumption.

We select a sub-sample of the FES. In particular, we select all married or cohabitating couples, living in England, Scotland or Wales, whose head is an employee. We further selected families in order to define two seven-year-of-birth groups: the first cohort consists of families whose head was born between 1947 and 1953, while the second one whose head was born between 1954 and 1960. Estimates for the second (and younger) cohort are carried out starting from 1980, first quarter. Table 1 reports more details.

The variables we use in estimation are: non-durable consumption and disposable income, as well as prices for non-durable consumption computed using the weights available from the FES. Non-durable consumption is defined as the sum of: food, alcohol and tobacco, fuel, clothing, transportation costs and services. We also use the education level of the head of the households to define finer groups: hence each cohort is further divided into two education groups, "compulsory" and "more than compulsory" schooling.

In order to remove deterministic trends and seasonal effects, for each cohort/education group, we regress (the logarithm of) each variable of interest on a second order polynomial in age and on quarterly dummies, as well as on the family-specific McClements equivalence scale.

$$(11) \qquad \tilde{z}_{ct}^{h} = \delta_c + \sum_{i=1}^{3} \alpha_{ci} q_{ti} + f^{c}(t-c) + \beta_c \cdot eqscale_{ct}^{h} + z_{ct}^{h}$$

where $f^{c}(t-c)$ is the group specific polynomial in age (obtained as time - year of birth), q's are quarterly seasonal dummies, *eqscale* is the equivalence scale and δ_c the group specific intercepts. Equation (4) is estimated by OLS[10]. The cohort average of the residual, z_{ct}, reflects both genuine time variation in group averages and measurement error arising from the limited sample sizes in computing the group averages, as we discuss in the next subsection.

Having obtained a cohort/education level measure of consumption and income depurated from deterministic trends, we conduct a preliminary investigation of their time series properties. In particular we perform standard time series unit root tests such as the augmented Dickey-Fuller, the GLS Dickey-Fuller, and the Kwiatkowski, Phillips, Schmidt, Shin (KPSS) tests. They all give indication of the existence of a unit root in all the series considered, i.e. non-durable

[10] More efficient estimates could be obtained by controlling for the heteroskedasticity induced by different cell sizes and within cell variances.

expenditure and disposable income for both cohorts. This justifies our decision to model the first differences of these series.

The interest rate used in the estimation is the 3-month treasury bill rate, from which we subtract the inflation rate in the consumer price index constructed from the FES, to obtain a real rate. The real interest rate has also been de-trended, using the Hodrick-Prescott filter with a smoother set to 1600 as we deal with quarterly data[11].

4.1. Estimation

There are several ways in which one can estimate the model (3). By making assumptions on the distribution of the shocks that enter the system (3) it is possible to compute the likelihood function associated with a given sample and estimate the parameters of interest by maximizing such a function. It would be also possible to avoid making specific functional form assumptions, and use a method of moment estimator. In particular, one could compute variances, covariances and autocovariances of the series of interest and minimize the distance between the sample moments and those implied by the parameters of the model. While the latter method is potentially more attractive, we experienced a variety of numerical problems in its implementation, especially when trying to correct for the presence of measurement error, which we discuss below. For this reason we adopted the former method. The estimates we present assume that the residuals in system (3) are Gaussian. The assumption of normality of the residuals can certainly be criticized. Indeed, Abowd and Card (1989) report some evidence against normality within their framework. We should stress, however, that we are modelling time series, rather than cross sectional variability.

Before turning to the estimates, we need to take into account what we labelled "measurement error", i.e. the terms $\eta_s^{\cdot}$ in system (3). As the sample used in estimating the cohort averages is refreshed in each period, the error induced by sample variation implies an MA(1) structure in the *changes* in the variable.

[11] We also estimated our models using a first-step deterministically de-trended interest rate: this procedure does not affect our results.

Given that we know the cell size and we can estimate the within cell variance, we have a substantial amount of information on the measurement error that we can use to correct the variance/covariance matrix of our system.

More specifically, we can estimate variances and covariances of the $\eta_s^{\cdot}$ terms consistently from the within cell variability and the cell size. We will assume that the idiosyncratic shocks are independent of $\Delta \bar{z}_{ct}$, the changes in the group-level innovations we are interested in modelling.

Hence we compute the time *t* variance of the cohort/education group averages $\eta_s^{\cdot}$ using the cross-sectional variance for each point in time, corrected for the cell size:

$$(12)\qquad Var_t\left(\eta_t^{\cdot}\right)=\frac{1}{N_t}Var_t\left(\eta_{it}^{\cdot}\right)\qquad\qquad t=1,\ldots T$$

Analogously, the covariance terms are:

$$(13)\qquad Cov_t\left(\eta_t^{x},\eta_t^{y}\right)=\frac{1}{N_t}Cov_t\left(\eta_{it}^{x},\eta_{it}^{y}\right)$$

We filter each series of variances or covariances with an MA smoother with a 12-period span. As we have about 12,000 observations for each cohort, we treat the estimated variances and covariances as known in the estimation of our system.

5. Results

In this section, we report our main estimation results. A structural interpretation of the coefficients we estimate is given in the next section. Before we present estimates of the statistical models we discussed in Section 2, we present briefly some of the time series properties of the data we use in estimation.

5.1. Univariate models

We start by reporting, in tables 2 to 4, estimates of univariate MA models for consumption, income, and the interest rate. In particular, table 2 shows estimates of univariate parsimonious MA representations for the logarithm of non durable expenditure for each cohort, both without and with correction for measurement error. The coefficient on the lag zero shock (α_0^{xx}) is constrained to be one, and it is not shown in the tables.

The main interest of this exercise is to check the validity of our assumptions; in particular we want to test whether, conditional on time-varying unobserved heterogeneity, the variance of the MA disturbance is constant over time[12]. To this purpose, we split the sample in two periods, from 1978 to 1993 (first quarter) and from 1993(2) to 2005(1), and estimate two separate models for each cohort[13]. We find that an MA(1) process fits the data well for both cohorts (upper and lower panel, respectively), when both including measurement error (left panel, columns (i) and (ii)) and disregarding unobserved heterogeneity (right panel, columns (v) and (vi)). We then restrict each model to test if *a)* the MA coefficient is constant over time (columns (iii) and (vii)) and *b)* in addition, the (square root of the) variance of the MA disturbance is also constant over time (columns (iv) and (viii)). The P-values for these likelihood ratio tests are reported in each corresponding column: for example, for cohort 1 and when correcting for measurement error the p-value of restricting the MA coefficient to be constant across periods is 62 per cent, while the p-value for restricting both the coefficients and the (square roots of the) variances to be constant is 51 per cent. The restrictions are also not rejected for the second cohort, with p-values of 78 and 69 per cent respectively.

In the right panel of table 2 we repeat the tests without explicitly taking into account the unobserved heterogeneity: while we find that for both cohorts the MA coefficient is constant over the two sample periods (with p-values of 49 and 74 per cent respectively), the test of constancy of the coefficient and of the variance displays a p-value lower than 5 per cent (1 and 3

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[12] For brevity, in this section we present our results splitting the sample into two cohorts, without further grouping according to the education level of the head.

[13] The second, and younger, cohort is estimated over the period 1980(1)-1993(1) and 1993(2)-2005(1). Diagnostic tests on the residuals as well as on the squared residuals of these and subsequent MAs are reported in the appendix, and do not indicate departures from randomness.

per cent respectively). For both cohorts, comparison of the likelihoods in columns (vii) and (viii) results in a p-value lower than 1 per cent.

Hence, we find that when allowing for time-varying unobserved heterogeneity the assumption of a stable cohort-specific MA process for non durable expenditure is not rejected by the data.

Table 3 reports results analogous to those in Table 2, but for (the first difference of the logarithm of) disposable income. Also in this case the data always indicate that disposable income is an MA(1) with a negative coefficient. The tests of constancy of the MA parameter and variances are not rejected when we explicitly model measurement error (left panel), and also, for cohort 2 only, in the case in which we do not include the measurement error term. For cohort 1, instead, the hypothesis of constant variances is rejected when we do not include unobserved heterogeneity.

A negative autocorrelation of the first difference of income has been found in the US literature both with aggregate data (for example by Watson, 1986) and with micro data (MaCurdy, 1982, Abowd and Card, 1989 and, more recently, Meghir and Pistaferri, 2004)[14]. Even after correcting for unobserved heterogeneity, our coefficient is larger in absolute value than what reported in these studies.

Table 4 reports estimates for univariate MA models for the real interest rate. Preliminary tests based on the squared residuals of a linear MA with constant variance (reported column v in the Table) indicate the residuals being not independent[15]. To overcome this problem, we introduce a number of structural breaks, choosing the time periods on previous evidence. The real interest rate in the short run is influenced both by the nominal interest rate and from the (expected) inflation rate. To take into account the "great moderation" that took place in the mid eighties we insert a structural break in 1985, first quarter. In addition, at the end 1992 the Bank of England started its inflation-targeting policy, hence we include a second structural break at the end of 1992. When allowing for two breaks at 1984(4) and 1993(1), we find the interest rate dynamics is best captured by a three-lag specification. In Table 5 we report unrestricted estimates

[14] Other studies, such as Blinder and Deaton (1985) using US aggregate data, find a positive autocorrelation in the differentiated series of income.

[15] Q- and normality tests on the residuals and squared residuals are shown in the Appendix for all the models estimated.

for each sub-period: 1978(1)-1985(1) in column (i), 1985(2)-1993(1) in column (ii), and 1993(2)-2005(1) in column (iii). The p-value of the restrictions imposed by specification (iv), which imposes constancy of the MA coefficients in the three sample periods, is 90%. Further imposing a constant variance to the shock, as shown in column (v), leads instead to a rejection of the restrictions at any significance level[16]. Hence, in the subsequent analysis we will model the interest rate as an MA process with constant MA coefficients and two breaks in the variance.

5.2. Multivariate models

Having estimated the univariate models, we now move on to the estimation of the main model discussed in Section 2, equation (3). In particular, for each cohort, we consider two variables: non-durable expenditure and disposable income. In addition, we have an equation for the real interest rate. We thus perform estimates of the three-equation system for each cohort separately; these systems all include correction for the measurement error. In both cases we start from a six-lag specification, constraining to zero one coefficient at a time to find a parsimonious specification: this specification is shown in table 5 for cohort 1 and 2. Given the interpretation of the results in terms of excess sensitivity we give in the next section, we paid particular attention to the coefficients on lag 'income shocks' in the specification for consumption. Hence for each specification we report in table 8 the p-value of the Likelihood ratio test on the restriction to zero of the one-lag income shock in the consumption equation, and we comment on it in the next section. In addition, we do not attempt to constrain the coefficients of the interest rate shock in the consumption equation: those coefficients will be used to identify the elasticity of intertemporal substitution and will be further analysed in subsequent sections.

For both cohorts we do not reject the hypothesis that lagged 'income' shocks are equal to zero in the consumption equation: the p-value of the test that the coefficient is zero is 0.17 and 0.51 for the first and second cohort respectively[17]. In the tables we only report the estimates for the first lag, but we also tried with additional lags obtaining the same result. Similarly, we also tried to add to the consumption equation lagged values of the 'consumption' shock and failed to

[16] The same tests, with virtually identical results, have been carried on a six-lag specification.
[17] These numbers are reported in table 8, in the next section.

reject the hypothesis that these were equal to zero. For this reason, in the table we report the results of a specification that does not contain lagged income and consumption shocks.

As for the interest rate shocks, we selected a six-lag specification for cohort 1, while for the second cohort a three-lag specification is preferred. Allowing for six-lag specification in the interest rate in the equations for cohort 2 does not affect our results. In the consumption equations for the two cohorts, the interest rate shocks are only estimated significantly at lag one for the first cohort, and at lag zero and one for the younger one. This evidence is in line with the univariate models for consumption presented in the second and fourth columns of Table 3, which did not show any prolonged dynamics.

For both cohorts, the contemporaneous shock to income has a significant effect on consumption.

As for the income equations, consistently with the evidence on the univariate models, we do not find a very long dynamics. In addition, we fail to reject the hypothesis that, in the income equation, all the coefficients to the interest rate shocks are zero, hence in our parsimonious specification disposable income does not depend on the interest rate. Once again, this evidence is not inconsistent with the evidence on earnings presented by other authors for the US.

We proceed by estimating the same systems for the two education groups, that is compulsory and more than compulsory education (lower and higher education in tables 6 and 7). The main differences with the overall estimates are: i) the prolonged dynamics for the interest rate in cohort 1 is found to significant only in the higher education group, and, more interestingly, ii) the one-lag income shock enters the consumption equation for both cohorts in the lower education group, while it is found to be non significantly different from zero for the higher educated.

6. A structural interpretation of the results

6.1. Excess sensitivity

As we previously discussed, we can easily test whether changes in log consumption exhibit "excess sensitivity" to past (i.e. predictable) income innovations; in other words, we test whether the lagged income group-level shocks are good predictors of consumption.

To summarize the evidence already reported in tables 5-7, we report in table 8 the p-values of the zero restriction on the lagged income shock in the consumption equation. For both cohorts, without further grouping, the corresponding p-values are 17 and 51 per cent, indicating that neither cohort, as a whole, displays excess sensitivity to income. When we distinguish among higher and lower educated heads of households, we find that for the latter group the p-values are lower than 5 per cent (4 and 2 per cent in cohort 1 and 2 respectively). On the other hand, the p-values are higher than 80 per cent for both cohorts in the higher education group.

While these findings can be interpreted in various ways, which cannot be effectively distinguished in this framework, it is noteworthy that we find excess sensitivity only for the less educated individuals, who hold less wealth to be used as a collateral and might be more subject to liquidity constraints than the higher educated.[18] This type of finding is also consistent with the evidence reported by Blundell, Pistaferri and Preston (2008) who show that a higher fraction of transitory shocks is reflected in the consumption of low education households than in that of high education households in the US. Alternatively, as emphasised by Carroll (2001) and Carroll and Kimball (2001), impatient consumers who hold little financial wealth (with respect to uncertain human wealth) will engage in precautionary saving, hence behaving like liquidity constrained individuals. Attanasio et al. (1999) indicate that low education households might be more impatient. The elasticity of intertemporal substitution may also play a role: individuals who are less willing to reallocate expenditure between periods to take advantage of interest rates movements end up accumulating less financial wealth, and hence they could be more exposed to liquidity constraints.

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[18] This argument has been first highlighted by Zeldes (1989). Recent statistics about the wealth distribution in UK are provided in the Wealth and Asset Survey (www.statistics.gov.uk).

6.2. The elasticity of intertemporal substitution

In Table 9 we report the estimated elasticity of intertemporal substitution implied by the restrictions in equation (8) and by the estimates of the coefficients we have fitted in all our specifications. To obtain those estimates we estimate a restricted version of our system, in which we impose the constraints in (8). We also use the log-likelihood of the restricted model to construct likelihood ratio tests of the restrictions that we impose.

For the first and elder cohort, we find that the elasticity is estimated at 1.15 with a standard error of 0.28, so that it is statistically different from zero and not from one. For the second younger cohort, we find a lower elasticity equal to 0.71, and, although less precisely estimated with a standard error of 0.39, is also statistically different from zero (with a p-value of 3.5 per cent) and not from one. The restrictions imposed by equation (8) are tested by comparing the value of the unrestricted and restricted likelihoods: the corresponding p-values are reported in the last two rows of table 9 and, for the two cohorts as a whole, indicate that the restrictions imposed by (8) are not rejected. These estimates of the elasticity parameter are consistent with those reported by Attanasio and Weber (1993) and other studies that used the FES.

When we disaggregate each cohort into more homogeneous educational attainments, we find that both the higher educated groups display an elasticity of intertemporal substitution higher than one (1.46 and 1.14 for cohort 1 and 2 respectively): in both cases the standard errors indicate that the parameter is statistically different from zero and not different from one. The p-values of the likelihood ratio tests of the restrictions imposed by equation (8) indicate the restrictions are not rejected.

Results for the compulsory education groups are rather different. The estimated elasticity is 0.6 for the elder cohort with a standard error equal to 0.37: hence the significance level is 5 per cent. For the younger cohort, the estimated parameter is equal to 0.53 and it not statistically different from zero. It is important to notice that in this case the restrictions imposed by (8) are rejected, with p-values of 4 and 0.5 per cent.

A number of studies emphasise the presence of heterogeneity in the preference parameters: Blundell, Browning and Meghir (1994), Attanasio and Browning (1995), and more recently Crossley and Low (2009) and Alan and Browning (2010) all find a non-constant elasticity of intertemporal substitution. All these studies, with the exception of Alan and Browning (2010), find the elasticity of intertemporal substitution is increasing in consumption/wealth. Mankiw and Zeldes (1991) and Attansio, Banks and Tanner (2002) find different elasticities among stock-holders and non stock-holders. Guvenen (2006) shows how introducing heterogeneity in elasticities of intertemporal substitution into a calibrated model improves the fit of the model, and provides an example of how policy conclusions based on a calibrated model with constant elasticity can be misleading.

6.3. Excess smoothness

Using the results we obtained estimating various versions of system (3), and following Attanasio and Pavoni (2011), we can test the restriction in equation (11) against the alternative of excess smoothness. In particular we test the hypothesis that $\alpha^{xy}(z) - \alpha^{yy}(z) = 0$ against the alternative that the same difference is less than 0. We evaluate the terms $\alpha^{\cdot y}(z)$ at a quarterly interest rate equal to 1 per cent. We report these results in Table 10.

Of particular interest are the results derived from the estimates in table 5 (cohort 1 and 2) and for the highly educated in table 6 (cohort 1) and 7 (cohort 2) where the restrictions implied by the Euler equations are imposed (it should be stressed that, as we mentioned above, these restrictions are not rejected). For the first (and elder) cohort the excess smoothness test equals -0.35, imprecisely estimated with a standard error of 0.22 and a P-value of about 5 per cent. For the second (and younger) cohort the excess smoothness test is equal to -0.22, with an estimated standard error equal to 0.12 (the P-value is 3.7 per cent). When disaggregating the two cohorts, the excess sensitivity test is equal to -0.23 and -0.14 for the high educated groups, with a standard error lower than 0.1 in both cases; the P-values are 5 and 0.8 per cent, respectively. Therefore, we reject the hypothesis of no excess smoothness for this educational group.

Although the restrictions implied by the Euler equation are not imposed, it is interesting to notice that in the low education group of cohort 1 the excess smoothness parameter is estimated to be very small with a large standard error, which explains the relatively high standard error found for that cohort as a whole.

Our results are in line with those of Attanasio and Pavoni (2011) who find, when using non-durable consumption and a broad definition of disposable income which includes smoothing mechanisms, comparable results.

7. Conclusions

In this paper we have analyzed the time series properties of group-level consumption expenditure and income. The methodology we propose consists in estimating multivariate moving average systems for synthetic panels constructed from time series of repeated cross sections. This approach has the advantage of allowing to explicitly take into account the measurement error present in the individual measures of consumption and income. Data are drawn from the UK Family Expenditure Survey.

When we define groups based on the year of birth, we cannot reject the hypothesis that lagged consumption, income and interest rate shocks have no effects on consumption changes. This evidence is coherent with the results of 'no-excess sensitivity' of consumption reported on micro data by Attanasio and Weber (1993, 1995), Blundell et al. (1994) and Attanasio and Browning (1995). However, when we define cohort-education groups, we find that those households whose head has at most completed compulsory schooling do display excess sensitivity of consumption to lagged income. Households whose head has a higher education, on the other hand, do not display any excess sensitivity.

We also use the estimated parameters to identify the elasticity of intertemporal substitution using micro data: when we aggregate our data to form year-of-birth groups, we find estimates consistent with the estimates reported by Attanasio and Weber (1993). When disaggregating according to the educational level, we find important differences in the estimated elasticities: while the better educated cohorts display an elasticity of intertemporal substitution greater than –but not statistically different from – one, estimates for the lower educated are in the range of 0.5 and not statistically different from zero. In addition, the restrictions imposed by our estimation procedure to estimate the elasticity of intertemporal substitution are rejected for the lower educated, while the higher educated easily pass the test.

Our approach can be also used to assess the extent to which innovation to income (or interest rates) are reflected into consumption. The Euler equation approach is typically completely silent about this. Given our estimates of the parameters of the time series model for consumption, income and the interest rate, we do find some evidence of 'excess smoothness' of consumption. That is, we find that consumption reacts to permanent innovations to income in a way that is not consistent with the life cycle model. Our evidence is in line with that in Attanasio and Pavoni (2011) who report evidence of excess smoothness and interpret it, following Hansen, Roberds and Sargent (1991) as a violation of the intertemporal budget constraint *with a single asset*. Attanasio and Pavoni (2011) also find that using income definitions that include smoothing mechanisms, such as social assistance and net taxes, results in milder evidence of "excess smoothness".

The analysis in this paper was done on grouped data, mainly because of data availability. The English FES provides a long time series of cross section that can be used to study the time series properties of grouped level data. The availability of individual level data is useful for a variety of reasons, including the fact that one controls the process of aggregation. However, the lack of a longitudinal dimension to the data forces us to use grouped data to study the dynamics of our data. This means that we potentially loose dynamic effects that are purely idiosyncratic as they are washed out by the averaging at the cohort level. The availability of individual level longitudinal data could be very useful to investigate how much of the dynamics of individual consumption we were unable to observe because of the nature of the data available to us.

References

Abowd, J. and D. Card (1989), "On the Covariance Structure of Earnings and Hours Changes", *Econometrica*, 57, 411-445.

Alan, S. and M. Browning (2010) "Estimating Intertemporal Allocation Parameters using Synthetic Residual Estimation", *The Review of Economic Studies*, forthcoming.

Alan, S., O.P. Attanasio and M. Browning (2009) "Estimating Euler Equations with Noisy Data: Two Exact GMM Estimators", *Journal of Applied Econometrics*, Vol. 24, No. 2, pp. 309-324.

Attanasio, O.PJ. Banks, C. Meghir and G. Weber (1999): "Humps and Bumps in Lifetime Consumption", *Journal of Business Economics and Statistics, Vol* 17, pages 22-35

Attanasio, O.P, J. Banks and S. Tanner (2002) "Asset Holding and Consumption Volatility", Journal of Political Economy, University of Chicago Press, vol. 110(4), pages 771-792, August.

Attanasio, O.P. and M. Browning (1995) "Consumption over the Life-Cycle and over the Business Cycle", *American Economic Review*, 85, 1118-1137.

Attanasio, O.P. and H. Low (2004) "Estimating Euler Equations", *Review of Economic Dynamics*, Elsevier for the Society for Economic Dynamics, vol. 7(2), pages 405-435, April.

Attanasio, O.P. and N. Pavoni (2011): "Risk Sharing in Private Information Models with Asset Accumulation: Explaining the Excess Smoothness of Consumption", forthcoming in *Econometrica*.

Attanasio, O.P. and G. Weber (1993), "Consumption Growth, the Interest Rate and Aggregation", *Review of Economic Studies*, 60, 631-49.

Attanasio, O.P. and G. Weber (1995), "Is Consumption Growth Consistent with Intertemporal Optimization? Evidence from the Consumer Expenditure Survey", *Journal of Political Economy*, 103, 1121-1157.

Blinder, A.S. and Deaton, A.S. (1985) "The Time-Series Consumption Revisited", *Brookings Papers on Economic Activity*, 465-521.

Blundell, R., L. Pistaferri and I. Preston (2008) "Consumption Inequality and Partial Insurance", *American Economic Review*, American Economic Association, vol. 98(5), pages 1887-1921, December.

Blundell, R., M. Browning and C. Meghir (1994), "Consumer Demand and the Lifetime Allocation of Consumption", *Review of Economic Studies*, 61, 57-80.

Brewer, M., A. Goodman, J. Shaw and L. Sibieta (2006), *Poverty and Inequality in Britain: 2006*, IFS Commentary 101 (http://www.ifs.org.uk/comms/comm101.pdf), London: IFS

Campbell, J.Y. (1987) "Does Saving Anticipate Declining Labor Income? An Alternative Test of the Permanent Income Hypothesis", *Econometrica*, 55(6), 1249-73.

Campbell, J.Y. (1994) "Inspecting the mechanism: An analytical approach to the stochastic growth model," *Journal of Monetary Economics*, 33, pp. 463-506.

Campbell, J. Y. and A.S. Deaton (1989): "Why Is Consumption So Smooth?", *Review of Economic Studies* 56, pp. 357–374.

Carroll, C.D. (2001) "Death to the Log-Linearized Consumption Euler Equation! (And Very Poor Health to the Second-Order Approximation)", The B.E. Journal of Macroeconomics, Berkeley Electronic Press, vol. 0(1).

Carroll, C.D. and M.S. Kimball (2001) "Liquidity Constraints and Precautionary Saving", NBER Working Papers 8496, National Bureau of Economic Research, Inc.

Crossley T.F. and H. W. Low (2009) "Is the Elasticity of Intertemporal Substitution Constant?", mimeo, available at http://www.econ.cam.ac.uk/faculty/low/pp09/constanteis_r2.pdf

Cunha, F., Heckman, J., and Navarro, S. (2004). "Separating heterogeneity from uncertainty an Aiyagari-Laitner economy", Paper presented at the Goldwater Conference on Labor Markets in Arizona, March, 2004.

Cunha, F., J. Heckman and Navarro, S. (2005). "Separating Uncertainty from Heterogeneity in Life Cycle Earnings," NBER Working Papers 11024, National Bureau of Economic Research, Inc.

Deaton, A. (1985) "Panel Data from Time Series of Cross-Sections", *Journal of Econometrics*, 30, 109-26.

Flavin, M.A. (1981) "The Adjustment of Consumption to Changing Expectations about Future Income", *Journal of Political Economy*, 89(5), 974-1009.

Forni, M., and L. Reichlin (1996) " Dynamic Common Factors in Large Cross-Sections" *Empirical Economics*, 21, 27-42.

Guvenen, F. (2006) "Reconciling conflicting evidence on the elasticity of intertemporal substitution: A macroeconomic perspective", *Journal of Monetary Economics*, Elsevier, vol. 53(7), pages 1451-1472, October.

Hall, R.E. (1978) "Stochastic Implications of the Life-Cycle Permament Income Hypothesis: Theory and Evidence", *Journal of Political Economy*, 86, 971-87.

Hansen, L.P., W. Roberds and T.J. Sargent (1991) "Time Series Implications of Present Value Budget Balance and of Martingale Models of Consumption and Taxes", in Lars P. Hansen and Thomas J. Sargent, *Rational Expectations Econometrics*, Boulder. Westview, 121-61.

MaCurdy, T.E. (1982), "The Use of Time Series Processes to Model the Error Structure of Earnings in a Longitudinal Data Analysis", Journal of Econometrics, 18, 83-114.

Mankiw N.G. and S.P. Zeldes (1991) "The consumption of stockholders and nonstockholders", *Journal of Financial Economics*, Elsevier, vol. 29(1), pages 97-112, March.

Meghir, C. and L. Pistaferri (2004): "Income variance dynamics and heterogeneity", *Econometrica,* 72, 1-32.

Quah, D. (1990), "Permanent and Transitory Movements in Labor Income: An Explanation for "Excess Smoothness" in Consumption", *Journal of Political Economy.* 98, pp. 449-475

Quah, D. and T.J. Sargent (1994), "A Dynamic Index Model for Large Cross Sections". In: Stock J., Watson M. (eds) Business Cycles, indicators and forecasting. NBER and University of Chicago.

Sargent, T.J. (1978) "Rational Expectations, Econometric Exogeneity and Consumption", *Journal of Political Economy*, 86(4), 673-700.

Tanner, S. (1998), 'How Much Do Consumers Spend? Comparing the FES and National Accounts' in Banks, J. and P. Johnson (eds), *How Reliable is the Family Expenditure Survey?*

Trends in Incomes and Expenditures Over Time, IFS Report 57 (http://www.ifs.org.uk/comms/r57.pdf), London: IFS.

Watson, M.W. (1986) "Univariate Detrending Methods with Stochastic Trends", *Journal of Monetary Economics*, 18, 49-75.

Watson M.W. and R.F. Engle, (1983) "Alternative Algorithms for the Estimation of Dynamic Factor, MIMIC, and Time Varying Coefficient Regression Models", *Journal of Econometrics*, 23, 385-400.

West, K.D. (1988) "The Insensitivity of Consumption to News About Income", *Journal of Monetary Economics*, 21, 17-34.

Zeldes, S.P. (1989) "Consumption and Liquidity Constraints: An Empirical Investigation", *Journal of Political Economy*, University of Chicago Press, vol. 97(2), pages 305-46, April.

Tables

Table 1 – Cohort definition

Cohort	Year of Birth	Observation period	Median age in first year	Median age in last year	Mean Cell Size
1	1947-53	1978(1)-2005(1)	28	55	122
2	1954-60	1980(1)-2005(1)	23	52	118

Note: data drawn from the FES, 1978(1) – 2005(1)

Table 2 – Parsimonious estimates: change in non-durable consumption.

	With correction for measurement error				**Without correction for measurement error**			
	1978(2)-1993(1) (i)	**1993(2)-2005(1)** (ii)	**1978(2)-2005(1) break** (iii)	**1978(2)-2005(1) no break** (iv)	**1978(2)-1993(1)** (v)	**1993(2)-2005(1)** (vi)	**1978(2)-2005(1) break** (vii)	**1978(2)-2005(1) no break** (viii)
Cohort 1								
u_{t-1}^{ND}	-0.429	-0.684	-0.544	-0.523	-0.689	-0.796	-0.728	-0.746
(se)	*(0.109)*	*(0.168)*	*(0.160)*	*(0.104)*	*(0.120)*	*(0.084)*	*(0.070)*	*(0.058)*
$\sigma(u_{93-05}^{R})$		3.847	3.254			5.765	5.822	
(s.e.)		*(0.946)*	*(0.831)*			*(0.639)*	*(0.650)*	
$\sigma(u_{78-93}^{R})$	2.264		2.572		3.959		3.975	
(s.e.)	*(0.573)*		*(0.473)*		*(0.331)*		*(0.338)*	
$\sigma(u_{78-05}^{R})$				2.692				4.881
(s.e.)				*(0.537)*				*(0.316)*
Log L	-168.21	-152.32	-321.00	-321.21	-167.69	-152.18	-320.58	-324.43
P-value			0.624	0.506			0.489	0.010
Cohort 2								
u_{t-1}^{ND}	-0.184	-0.588	-0.217	-0.197	-0.521	-0.713	-0.568	-0.597
(se)	*(0.069)*	*(0.164)*	*(0.138)*	*(0.052)*	*(0.117)*	*(0.111)*	*(0.083)*	*(0.081)*
$\sigma(u_{93-05}^{R})$		3.686	2.821			5.444	5.458	
(s.e.)		*(1.026)*	*(1.061)*			*(0.775)*	*(0.741)*	
$\sigma(u_{80-93}^{R})$	2.441		2.359		3.763		3.775	
(s.e.)	*(0.914)*		*(0.964)*		*(0.359)*		*(0.372)*	
$\sigma(u_{80-05}^{R})$				2.430				4.657
(s.e.)				*(0.637)*				*(0.368)*
Log L	-145.43	-150.03	-295.71	-295.83	-142.68	-149.43	-292.41	-295.70
P-value(a)			0.777	0.689			0.744	0.028

Note: parsimonious univariate MA estimates of the change in log non-durable consumption, using cohort data – without/with correction for measurement error. Standard errors in parentheses.

(a) P-value for the Likelihood ratio test when testing the zero restrictions versus the unrestricted models (given by column (i) plus (ii) and (v) plus (vi) respectively)

Table 3 – Parsimonious estimates: change in log disposable income.

	With correction for measurement error				Without correction for measurement error			
	1978(2)-1993(1)	**1993(2)-2005(1)**	**1978(2)-2005(1) break**	**1978(2)-2005(1) no break**	**1978(2)-1993(1)**	**1993(2)-2005(1)**	**1978(2)-2005(1) break**	**1978(2)-2005(1) no break**
	(i)	(ii)	(iii)	(iv)	(v)	(vi)	(vii)	(viii)
Cohort 1								
u_{t-1}^{Y}	-0.287	-0.744	-0.447	-0.463	-0.576	-0.826	-0.641	-0.687
(se)	*(0.051)*	*(0.110)*	*(0.114)*	*(0.066)*	*(0.147)*	*(0.081)*	*(0.074)*	*(0.054)*
								4.246
$\sigma(u_{93-05}^{R})$		3.346	2.788			4.905	5.058	*(0.238)*
(s.e.)		*(0.498)*	*(0.538)*			*(0.433)*	*(0.480)*	
$\sigma(u_{78-93}^{R})$	2.084		2.405		3.464		3.489	
(s.e.)	*(0.370)*		*(0.376)*		*(0.268)*		*(0.271)*	
$\sigma(u_{78-05}^{R})$				2.481				4.246
(s.e.)				*(0.356)*				*(0.238)*
Log L	-158.91	-144.21	-304.91	-304.98	-159.67	-144.42	-305.99	-309.40
P-value			0.167	0.156			0.151	0.005
Cohort 2								
u_{t-1}^{Y}	-0.295	-0.714	-0.456	-0.470	-0.522	-0.809	-0.615	-0.641
(se)	*(0.067)*	*(0.134)*	*(0.121)*	*(0.067)*	*(0.138)*	*(0.101)*	*(0.074)*	*(0.075)*
$\sigma(u_{93-05}^{R})$		3.242	2.764			4.405	4.558	
(s.e.)		*(0.606)*	*(0.487)*			*(0.363)*	*(0.421)*	
$\sigma(u_{80-93}^{R})$	2.734		3.079		3.900		3.947	
(s.e.)	*(0.621)*		*(0.532)*		*(0.299)*		*(0.307)*	
$\sigma(u_{80-05}^{R})$				2.985				4.239
(s.e.)				*(0.428)*				*(0.256)*
Log L	-145.49	-140.03	-287.02	-287.08	-144.55	-139.26	-286.07	-286.30
P-value(a)			0.224	0.211			0.104	0.083

Note: parsimonious univariate MA estimates of the change in log disposable income, using cohort data – without/with correction for measurement error. Standard errors in parentheses.
(a) P-value for the Likelihood ratio test when testing the zero restrictions versus the unrestricted models (given by column (i) plus (ii) and (v) plus (vi) respectively)

Table 4 – Parsimonious estimates: real interest rate

	1978(1)-1984(4) (i)	1985(1)-1993(1) (ii)	1993(2)-2005(1) (iii)	1978(1)-2005(1) (iv)	1978(1)-2005(1) (v)
u_{t-1}^{R}	0.908	0.678	0.933	0.821	0.681
(s.e.)	*(0.146)*	*(0.227)*	*(0.159)*	*(0.099)*	*(0.089)*
u_{t-2}^{R}	0.704	0.518	0.667	0.575	0.532
(s.e.)	*(0.230)*	*(0.211)*	*(0.184)*	*(0.121)*	*(0.100)*
u_{t-3}^{R}	0.786	-0.169	0.358	0.287	0.325
(s.e.)	*(0.177)*	*(0.195)*	*(0.154)*	*(0.108)*	*(0.077)*
$\sigma(u_{93-05}^{R})$	-	-	0.323	0.326	
(s.e.)			*(0.033)*	*(0.032)*	
$\sigma(u_{82-92}^{R})$	-	0.747	-	0.724	-
(s.e.)		*(0.163)*		*(0.123)*	
$\sigma(u_{78-81}^{R})$	1.169			1.267	0.792
(s.e.)	*(0.177)*			*(0.174)*	*(0.037)*
Log L	-42.5	-37.2	-13.9	-95.6	-128.0
P-value[(b)]				0.905	0.00

Note: parsimonious univariate MA estimates of the real interest rate: columns (i)-(iii) report unrestricted estimates for the three subperiods 78(1)-84(4); 85(1)-93(1); 93(2)-05(1). Column (iv) reports the restricted model with variance breaks, column (v) the restricted model with a single variance.

[(a)] Refers to whole sample period, 1978(1)-2005(1)

[(b)] P-value for the Likelihood ratio test when testing the zero restrictions imposed by specification (iv) versus the unrestricted model (columns i+ii+iii), and by specification (v) versus specification (iv).

Table 5 – Consumption, income and interest rate, cohort 1 and 2, parsimonious

	Cohort 1			Cohort 2		
	Non durable	Income	Interest rate	Non durable	Income	Interest rate
u_t^x	1			1		
(se)						
u_t^Y	0.364	1		0.378	1	
(se)	*(0.182)*			*(0.162)*		
u_{t-1}^Y		-0.307			-0.401	
(se)		*(0.240)*			*(0.149)*	
u_t^R	-0.898		1	0.218		1
(se)	*(0.621)*			*(0.769)*		
u_{t-1}^R	1.162		0.865	0.730		0.779
(se)	*(0.726)*		*(0.106)*	*(0.891)*		*(0.115)*
u_{t-2}^R	0.707		0.723	0.195		0.545
(se)	*(0.852)*		*(0.149)*	*(0.822)*		*(0.133)*
u_{t-3}^R	-0.454		0.426	0.169		0.215
(se)	*(0.859)*		*(0.203)*	*(0.647)*		*(0.133)*
u_{t-4}^R	0.283		0.134			
(se)	*(0.701)*		*(0.173)*			
u_{t-5}^R	-0.708		0.035			
(se)	*(0.402)*		*(0.016)*			
u_{t-6}^R	0.160		-0.105			
(se)	*(0.564)*		*(0.126)*			
$\sigma(u_{93-05}^Z)$	0.864[(a)]	1.955[(a)]	0.333	1.104[(a)]	2.851[(a)]	0.329
(s.e.)	*(0.237)*	*(0.424)*	*(0.032)*	*(0.221)*	*(0.491)*	*(0.040)*
$\sigma(u_{82-92}^Z)$			0.716			0.714
(s.e.)			*(0.136)*			*(0.138)*
$\sigma(u_{78-81}^Z)$			1.384			1.471
(s.e.)			*(0.197)*			*(0.218)*
Z= X, Y, R						
Log L	-496.3			-463.8		

Note: Maximum likelihood estimates of the system (3) in the text, with correction for measurement error. Sample: 1978(1)-2005(1). $\sigma(u_{t1-t2}^Z)$ is the estimated square root of the variance of the unobserved component for the period t1-t2, with Z=X, Y, and R. [(a)] Refers to whole sample period.

Table 6 – Consumption, income and interest rate, cohort 1, by education

	Low education			High education		
	Non durable	Income	Interest rate	Non durable	Income	Interest rate
u_t^x	1			1		
(se)						
u_t^Y	1.048	1		0.351	1	
(se)	*(0.259)*			*(0.130)*		
u_{t-1}^Y	-0.644	-0.459			-0.438	
(se)	*(0.261)*	*(0.176)*			*(0.135)*	
u_t^R	-1.093		1	-1.032		1
(se)	*(0.540)*			*(0.606)*		
u_{t-1}^R	1.533		0.840	0.946		0.882
(se)	*(0.691)*		*(0.102)*	*(0.788)*		*(0.132)*
u_{t-2}^R	0.064		0.588	1.154		0.736
(se)	*(0.851)*		*(0.104)*	*(0.768)*		*(0.182)*
u_{t-3}^R	-0.320		0.272	-0.501		0.451
(se)	*(0.620)*		*(0.080)*	*(0.799)*		*(0.258)*
u_{t-4}^R				0.375		0.134
(se)				*(0.795)*		*(0.315)*
u_{t-5}^R				-1.381		0.035
(se)				*(0.778)*		*(0.034)*
u_{t-6}^R				0.438		-0.075
(se)				*(0.715)*		*(0.048)*
$\sigma(u_{93-05}^Z)$	0.873[(a)]	3.077[(a)]	0.329	0.889[(a)]	2.675[(a)]	0.330
(s.e.)	*(0.265)*	*(0.692)*	*(0.034)*	*(0.325)*	*(0.543)*	*(0.033)*
$\sigma(u_{82-92}^Z)$			0.774			0.724
(s.e.)			*(0.104)*			*(0.099)*
$\sigma(u_{78-81}^Z)$			1.418			1.378
(s.e.)			*(0.205)*			*(0.209)*
Z= X, Y, R						
Log L	-569.1			-552.2		

Note: Maximum likelihood estimates of the system (3) in the text, with correction for measurement error. Sample: 1978(1)-2005(1). $\sigma(u_{t1-t2}^Z)$ is the estimated square root of the variance of the unobserved component for the period t1-t2, with Z=X, Y, and R.
(a) Refers to whole sample period

Table 7 – Consumption, income and interest rate, cohort 2, by education

	Low education			High education		
	Non durable	Income	Interest rate	Non durable	Income	Interest rate
u_t^x	1			1		
(se)						
u_t^Y	0.709	1		0.324	1	
(se)	*(0.164)*			*(0.088)*		
u_{t-1}^Y	-0.430	-0.508			-0.520	
(se)	*(0.167)*	*(0.109)*			*(0.110)*	
u_t^R	0.348		1	-0.342		1
(se)	*(0.523)*			*(0.904)*		
u_{t-1}^R	0.986		0.767	0.978		0.773
(se)	*(0.656)*		*(0.103)*	*(1.212)*		*(0.106)*
u_{t-2}^R	-0.368		0.501	0.317		0.539
(se)	*(0.697)*		*(0.119)*	*(1.405)*		*(0.126)*
u_{t-3}^R	-0.028		0.223	0.498		0.198
(se)	*(0.570)*		*(0.091)*	*(0.995)*		*(0.131)*
$\sigma(u_{93-05}^Z)$	0.809[a]	3.998[a]	0.330	0.871[a]	4.335[a]	0.330
(s.e.)	*(0.391)*	*(0.572)*	*(0.036)*	*(0.270)*	*(0.638)*	*(0.041)*
$\sigma(u_{82-92}^Z)$			0.761			0.712
(s.e.)			*(0.099)*			*(0.140)*
$\sigma(u_{78-81}^Z)$			1.464			1.469
(s.e.)			*(0.243)*			*(0.201)*
Z= X, Y, R						
Log L	-512.4			-539.2		

Note: Maximum likelihood estimates of the system (3) in the text, with correction for measurement error. Sample: 1978(1)-2005(1). $\sigma(u_{t1-t2}^Z)$ is the estimated square root of the variance of the unobserved component for the period t1-t2, with Z=X, Y, and R.
[a] Refers to whole sample period.

Table 8 – Excess sensitivity

	Total	Low education	High education
Cohort 1	0.173	0.040	0.842
Cohort 2	0.511	0.021	0.809

Note: P-values of the Likelihood ratio tests on the coefficients on past income shocks in the consumption equation in the models in tables 6, 7, 8.

Table 9 – Elasticity of intertemporal substitution

	Total	Low education	High education
Cohort 1	1.148	0.599	1.458
(se)	*(0.279)*	*(0.375)*	*(0.440)*
Cohort 2	0.709	0.539	1.141
(se)	*(0.390)*	*(0.499)*	*(0.535)*
P-values[(a)]			
Cohort 1	0.998	0.005	0.150
Cohort 2	0.913	0.044	0.917

Note: estimates of the elasticity of intertemporal substitution obtained estimating by maximum likelihood the restricted versions of the models in tables 6, 7, 8 and 9: the restrictions imposed are given by equation (10) in the text.

[(a)] P-value of the likelihood ratio test of the restriction imposed by (10) on the unrestricted models in table 6, 7, and 8.

Table 10 – Excess smoothness

	Total	Low education	High education
Cohort 1	-0.353	0.015	-0.233
(se)	*(0.221)*	*(0.128)*	*(0.097)*
Cohort 2	-0.215	-0.243	-0.139
(se)	*(0.120)*	*(0.105)*	*(0.088)*

Note: Estimates of excess smoothness obtained from the maximum likelihood estimates of the models in table 5, 6, and 7. The discounted sum of the income coefficients has been computed at a quarterly interest rate equal to 1%.

Appendix

We report, for all the model estimates presented in the main text, diagnostic tests based on the residuals of such models. In particular, we compute the portmanteau statistic using both the residuals and the squared residuals, to detect non linearities. We also report a D'Agostino-Pearson test of linearity based on skewness and kurtosis in order to detect, although indirectly, departures from linearity.

A1 – Q- and normality test on the residuals of univariate consumption models

Cohort 1	with correction			no correction		
	78(2)-93(1)	93(2)-05(1)	78(2)-05(1)	78(2)-93(1)	93(2)-05(1)	78(2)-05(1)
Column in Table 3	(iii)	(iii)	(iv)	(vii)	(vii)	(viii)
Residuals						
Q(12)	0.929	0.640	0.759	0.927	0.617	0.750
Q(24)	0.284	0.937	0.896	0.284	0.930	0.898
Q(36)	0.283	0.911	0.912	0.289	0.906	0.914
Squared residuals						
Q(12)	0.439	0.742	0.233	0.486	0.702	0.209
Q(24)	0.270	0.781	0.422	0.305	0.748	0.397
Q(36)	0.169	0.863	0.428	0.196	0.837	0.425
Normality	0.510	0.983	0.513	0.540	0.982	0.620
Cohort 2	with correction			no correction		
	80(2)-93(1)	93(2)-05(1)	80(2)-05(1)	80(2)-93(1)	93(2)-05(1)	80(2)-05(1)
Column in Table 3	(iii)	(iii)	(iv)	(vii)	(vii)	(viii)
Residuals						
Q(12)	0.817	0.499	0.630	0.800	0.498	0.664
Q(24)	0.972	0.582	0.844	0.968	0.590	0.870
Q(36)	0.875	0.639	0.962	0.873	0.656	0.972
Squared residuals						
Q(12)	0.819	0.425	0.200	0.828	0.472	0.208
Q(24)	0.448	0.354	0.240	0.443	0.438	0.286
Q(36)	0.342	0.348	0.582	0.351	0.424	0.629
Normality	0.415	0.216	0.435	0.447	0.213	0.449

Note: Residuals from models in table 3 in the text.

A2 – Q- and normality test on the residuals of univariate income models

Cohort 1	with correction			no correction		
	78(2)-93(1)	93(2)-05(1)	78(2)-05(1)	78(2)-93(1)	93(2)-05(1)	78(2)-05(1)
Column in Table 4	(iii)	(iii)	(iv)	(vii)	(vii)	(viii)
Residuals						
Q(12)	0.037	0.762	0.420	0.033	0.714	0.434
Q(24)	0.011	0.979	0.635	0.006	0.971	0.550
Q(36)	0.003	0.982	0.820	0.002	0.978	0.790
Squared residuals						
Q(12)	0.961	0.424	0.362	0.961	0.523	0.458
Q(24)	0.931	0.484	0.511	0.908	0.603	0.630
Q(36)	0.966	0.300	0.452	0.955	0.352	0.544
Normality	0.219	0.056	0.044	0.253	0.032	0.036

Cohort 2	with correction			no correction		
	80(2)-93(1)	93(2)-05(1)	80(2)-05(1)	80(2)-93(1)	93(2)-05(1)	80(2)-05(1)
Column in Table 4	(iii)	(iii)	(iv)	(vii)	(vii)	(viii)
Residuals						
Q(12)	0.322	0.596	0.750	0.259	0.537	0.728
Q(24)	0.527	0.527	0.567	0.467	0.467	0.548
Q(36)	0.366	0.534	0.547	0.314	0.476	0.525
Squared residuals						
Q(12)	0.215	0.973	0.205	0.204	0.949	0.202
Q(24)	0.603	0.998	0.632	0.596	0.995	0.629
Q(36)	0.768	0.998	0.569	0.754	0.994	0.557
Normality	0.109	0.126	0.077	0.114	0.207	0.080

Note: Residuals from models in table 4 in the text.

A3 – Q- and normality test on the residuals of univariate interest rate models

	Model (v)	Model (iv)	Model (iv)	Model (iv)
Time period	78(2)-05(1)	78(2)-85(1)	85(2)-93(1)	93(2)-05(1)
Residuals				
Q(12)	0.593	0.807	0.820	0.878
Q(24)	0.732	0.780	0.447	0.937
Squared residuals				
Q(12)	0.0001	0.493	0.856	0.669
Q(24)	0.0004	0.851	0.819	0.581
Normality	0.0005	0.332	0.838	0.196

Note: Residuals from models in table 5 in the text.

A4 – Q- and normality tests on residuals from cohort 1, by education

	Lower education					Higher education				
	ndur	y	r 78-81	r81-93	r93-05	ndur	y	r 78-81	r81-93	r93-05
Residuals										
Q(12)	0.357	0.983	0.503	0.398	0.903	0.436	0.229	0.548	0.730	0.837
Q(24)	0.725	0.977		0.103	0.880	0.350	0.553		0.491	0.844
Q(36)	0.604	0.943				0.398	0.616			
Squared residuals										
Q(12)	0.022	0.217	0.556	0.282	0.320	0.001	0.598	0.756	0.897	0.442
Q(24)	0.059	0.693		0.318	0.274	0.061	0.448		0.829	0.403
Q(36)	0.221	0.642				0.306	0.365			
Normality	0.979	0.041	0.744	0.667	0.570	0.653	0.012	0.677	0.626	0.228

A5 – Q- and normality tests on residuals from cohort 2, by education

	Lower education					Higher education				
	ndur	y	r 78-81	r81-93	r93-05	ndur	y	r 78-81	r81-93	r93-05
Residuals										
Q(12)	0.874	0.692	0.909	0.531	0.782	0.973	0.305	0.875	0.801	0.837
Q(24)	0.914	0.439		0.144	0.852	0.996	0.641		0.448	0.911
Q(36)	0.573	0.432				0.986	0.516			
Squared residuals										
Q(12)	0.992	0.312	0.9997	0.082	0.584	0.679	0.419	0.9996	0.529	0.605
Q(24)	0.945	0.415		0.084	0.493	0.918	0.659		0.509	0.439
Q(36)	0.923	0.247				0.745	0.877			
Normality	0.811	0.637	0.007	0.600	0.389	0.331	0.759	0.008	0.661	0.257

A6 – Q- and normality tests on residuals from cohort 1 and 2, total

	Cohort 1					Cohort 2				
	ndur	y	r 78-81	r81-93	r93-05	ndur	y	r 78-81	r81-93	r93-05
Residuals										
Q(12)	0.498	0.411	0.484	0.742	0.761	0.631	0.640	0.880	0.802	0.857
Q(24)	0.798	0.678		0.504	0.800	0.819	0.419		0.457	0.919
Q(36)	0.680	0.862				0.973	0.420			
Squared residuals										
Q(12)	0.011	0.214	0.778	0.873	0.413	0.119	0.166	0.9996	0.563	0.623
Q(24)	0.121	0.241		0.832	0.383	0.041	0.594		0.546	0.471
Q(36)	0.196	0.182				0.175	0.543			
Normality	0.632	0.019	0.692	0.567	0.231	0.159	0.097	0.008	0.698	0.242