Macroeconomics and finance

The general price level, temporary equilibrium and expectations

Plan

1. The setup
2. How demand varies with the general price level
3. Existence of a temporary competitive equilibrium
4. Extensions: nominal interest rate, economies with or without credit

The setup

Temporary equilibrium

1. The time frame: history, expectations, description of resource allocation
2. Resource allocation during the period: speed of search (models of unemployment with matching function on the labor market), price or wage flexibility, size of agents with respect to the overall market and competitive or monopolistic equilibrium...
3. Expectations: agents' horizons and subjectivity (vs. rationality) of expectations.
4. Dynamics. Going from one period to the next: adjustment costs, (Bayesian?) revision of expectations, accumulation of assets.

Description of consumer's behavior

Assets

1. physical vs. financial (role of intermediaries, rules in case of insolvency,...)
2. nominal vs. real

Examples: money, government bonds, firms debts, shares, derivatives.

Expectations

A priori random environment.

Expectations bear on future income, prices, returns...

News sequentially arrive, and lead to revisions, following a well defined a priori rule (for instance Bayes' rule).

Simple setup

Aggregation and study of a two periods model, with one asset ('money': numéraire, no dividends), one *non storable* consumption good. Certain expectations.

Typical agent (upper index to designate the agent, omitted here):

$$\begin{cases} \max\limits_{C_t, C_{t+1}, B_t, B_{t+1}} U(C_t,\ C_{t+1}) \\ p_t C_t + B_t = p_t Y_t + B_{t-1} \\ p^e_{t+1} C_{t+1} + B_{t+1} = p^e_{t+1} Y^e_{t+1} + B_t \\ C_t,\ C_{t+1} \geq\ 0, B_{t+1} \geq 0. \end{cases}$$

If debt is not allowed, the B's are constrained to be non negative.

Assumption

Assumption: *U is strictly quasi-concave, strictly increasing in C_t and C_{t+1}, and continuously differentiable on $\mathbb{R}^2_+$. Y_t and Y_{t+1} are (strictly) positive. For any positive couple C_1 and C_{t+1}:*

$$\lim_{c \to 0} U(c, C_{t+1}) < U(C_t, C_{t+1}),$$

$$\lim_{c \to 0} U(C_t, c) < U(C_t, C_{t+1}).$$

(the indifference curves are asymptote to the axes).

The typical consumer's program

$$\begin{cases} \max\limits_{C_t, C_{t+1}} U(C_t,\ C_{t+1}) \\ p_t C_t + p^e_{t+1} C_{t+1} = p_t Y_t + p^e_{t+1} Y^e_{t+1} + B_{t-1}, \end{cases}$$

with the demand for asset B_t derived from the first period budget constraint

$$p_t C_t + B_t = p_t Y_t + B_{t-1}.$$

The *subjective* real interest rate

$$1 + \rho_t^e = \frac{p_t}{p_{t+1}^e}.$$

The budget set can be rewritten as:

$$C_t - Y_t + \frac{1}{1 + \rho_t^e}(C_{t+1} - Y_{t+1}^e) = \frac{B_{t-1}}{p_t}.$$

Consumer behavior

FOC (or Euler equation)

$$\frac{U_1'(C_t,\ C_{t+1})}{U_2'(C_t,\ C_{t+1})} = \frac{p_t}{p_{t+1}^e} = 1 + \rho_t^e.$$

Together with the budget constraint, it yields the *continuous* consumption function:

$$C_t = \gamma \left(\frac{p_{t+1}^e}{p_t},\ Y_t + \frac{B_{t-1}}{p_t},\ Y_{t+1}^e \right),$$

with asset demand B_t (or demand for savings) following through

$$\begin{aligned} B_t &= p_t \left[Y_t + \frac{B_{t-1}}{p_t} - \gamma \left(\frac{p_{t+1}^e}{p_t},\ Y_t + \frac{B_{t-1}}{p_t},\ Y_{t+1}^e \right) \right] \\ &\equiv p_t \beta \left(\frac{p_{t+1}^e}{p_t},\ Y_t + \frac{B_{t-1}}{p_t},\ Y_{t+1}^e \right). \end{aligned}$$

Temporary competitive equilibrium

Definition temporary competitive *equilibrium at date t*: value of price p_t such that supply equals demand, both for the consumption good and for the asset.

$$\sum_i [C_t^i - Y_t^i] = 0$$

$$\sum_i [B_t^i - B_{t-1}^i] = 0$$

Walras' law.

$$\sum_i \gamma^i \left(\frac{p_{t+1}^{ei}}{p_t},\ Y_t^i + \frac{B_{t-1}^i}{p_t},\ Y_{t+1}^{ei} \right) - \sum_i Y_t^i = 0,$$

$$\sum_i p_t \beta^i \left(\frac{p_{t+1}^{ei}}{p_t},\ Y_t^i + \frac{B_{t-1}^i}{p_t},\ Y_{t+1}^{ei} \right) - \sum_i B_{t-1}^i = 0.$$

Existence of a temporary equilibrium (or determination of the price level)

Two polar reasons why existence might fail:

1. Keynesian unemployment: demand for good is less than potential supply, whatever the price. (Equivalently, savings are larger than the existing stock of asset).

2. Repressed inflation: demand for good is larger than potential supply, whatever the price, or the demand for asset (money) is smaller than the stock.

Behavior of demand when the price of the good (the asset is the numéraire) goes either to zero, or to infinity.

How demand varies with the general price level

Expectations

All the history is given and fixed: to avoid unnecessary notations, it is omitted from the arguments. The only explicit argument of the expectations is the current endogenous variable.

Here I shall only study the case where the Y^{ei}_{t+1}'s are given, independent from the endogenous variable.

$$p^{ei}_{t+1} = \psi^i(p_t)$$

Real balance effects

Unit elastic expectations: there is a constant ρ^{ei} such that

$$p_{t+1}^{ei} = \psi^i(p_t) = \frac{p_t}{1+\rho^{ei}}.$$

Consumer i maximizes $U^i(C_t, C_{t+1})$ on the budget constraint

$$C_t + \frac{1}{1+\rho^{ei}} C_{t+1} = Y_t^i + \frac{1}{1+\rho^{ei}} Y_{t+1}^{ei} + \frac{B_{t-1}^i}{p_t} = W^i.$$

Debtors: p_t goes to zero

$$B^i_{t-1} < 0$$

A decrease in price augments the debt burden and reduces intertemporal income. For p_t such that

$$\frac{B^i_{t-1}}{p_t} + \left[Y^i_t + \frac{1}{1+\rho^{ei}} Y^{ei}_{t+1} \right] < 0,$$

intertemporal income is negative, the consumer is bankrupt, the budget set is empty, and there is no solution to the consumer problem!

Creditors: p_t goes to zero

$$B_{t-1}^i \geq 0$$

Assume consumption is a normal good.
C_t^i goes to infinity when p_t goes to zero. This is Pigou *real balance effect*.

Debtors and creditors: p_t goes to infinity

When p_t goes to infinity, intertemporal income decreases to

$$Y_t^i + \frac{1}{1+\rho^{ei}} Y_{t+1}^{ei},$$

and C_t^i has a positive lower limit.

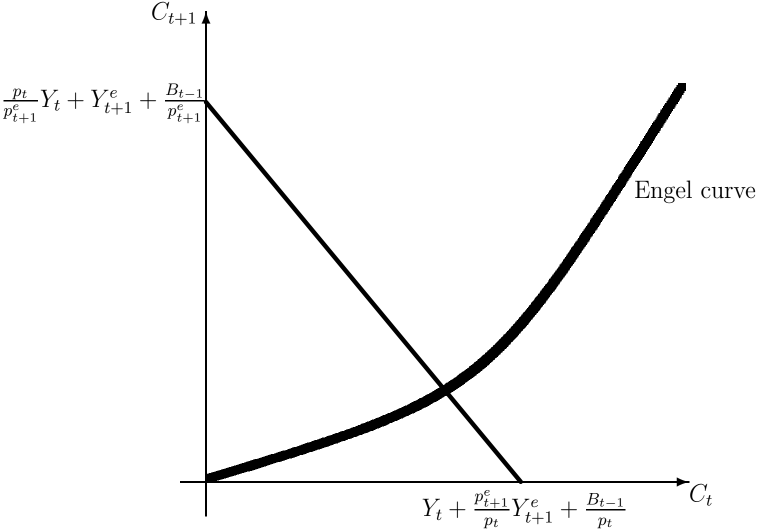

Figure 1: The demand for savings

Substitution effects

Define *substitution effects* as changes in demand induced by changes in the price level, in the absence of real balance effects, i.e. when we put initial cash balances B^i_{t-1} at zero.
They work through the expected real interest rate

$$1 + \rho^{ei}_t = \frac{p_t}{\psi^i(p_t)}.$$

The power of substitution

Diagram in the consumption plane: budget constraint rotating around the initial endowment point (Y_t^i, Y_{t+1}^{ei}).

When ρ goes to -1, C_{t+1} tends to Y_{t+1}^{ei} and C_t goes to $+\infty$;

When ρ goes to $+\infty$, C_t tends to Y_t^i and C_{t+1} goes to $+\infty$.

Existence of a temporary competitive equilibrium

Putting the two effects together

Demand behavior when prices decrease.

Proposition 1 (curing unemployment by deflation): *Assume $\psi^i(p)/p$ goes to infinity when p goes to zero and $B^i_{t-1} \geq 0$. Then demand $\gamma^i(\psi^i(p_t)/p_t,\ Y^i_t + B^i_{t-1}/p_t,\ Y^{ei}_{t+1})$ tends to infinity when p_t goes to 0.*

Proposition 1: comments

Substitution effect: the expected real interest rate $\rho = p/\psi(p) - 1$ tends to -1 when the price goes to 0: the budget constraint becomes horizontal.

Alternative: the real balance effect should be enough, under 'normality'. However income effects transfer wealth from the debtors (who may find themselves bankrupt) to the creditors.

Putting the two effects together: continued

Demand behavior when prices increase.

Proposition 2: *Assume that* $\psi^i(p)/p$ *tends to* 0 *when* p *tends to* ∞. *Then the demand for assets become non negative for* p_t *large enough.*

Proof : Otherwise, $C_{t+1} = B_t/\psi^i(p_t) + Y^{ei}_{t+1}$ stays smaller than Y^{ei}_{t+1}. C_t tends to Y^i_t. This gives a contradiction with the first order condition. ∎

Prices increase, continued

Proposition 3 (curing repressed inflation, continued): Suppose $\psi^i(p)/p$ tends to 0 when p tends to ∞, while $\psi^i(p)$ stays larger than a strictly positive number, say $\underline{p}$. Then the demand of financial assets $p_t \beta^i \left(\psi^i(p_t)/p_t,\ Y_t^i,\ Y_{t+1}^{ei}\right)$ goes to infinity with p_t.

Proof : Otherwise, B_t stays bounded. Then C_t tends to Y_t^i, and $C_{t+1} = B_t/\psi^i(p_t) + Y_{t+1}^{ei} \leq B_t/\underline{p} + Y_{t+1}^{ei}$ also is bounded. The marginal rate of substitution converges towards a finite value, and cannot stay equal to (or larger than) $p_t/\psi^i(p_t)$ which goes to infinity. ∎

Comments

The asset has value because one expects that it will keep being accepted as a means of payment by the future generations!

Note that the condition

$$\text{`}\psi^i(p)/p \text{ tends to 0 when } p \text{ goes to infinity'}$$

is crucial so that the substitution effect, the only active force when the value of the asset goes to zero, operates.

Temporary competitive equilibrium

Theorem: *Assume that for at least* **one** *agent,* $B^i_{t-1} \geq 0$*, and that* $\psi^i(p) \geq \underline{p} > 0$ *for all* p*. Furthermore, assume that for* **all** *agents* j*,* $j = 1, \ldots, I$*,* $\psi^j(p)/p$ *tends to* 0 *when* p *tends to* ∞*. Then there exists a temporary competitive equilibrium.*

Proof of Theorem

By Walras' law, one only needs to check that there is a price such that the demand for good is equal to the supply.

Excess demand, (i.e. demand - supply), is a continuous function of price.

It tends to infinity when p goes to zero from Proposition 1. It is equal to $-p\sum_i(B_t^i - B_{t-1}^i)$, from the sum of the budget constraints (Walras' law), and therefore becomes negative when p goes to infinity from Propositions 2 and 3.

Extensions

Economy without credit

Credit is a source of unstability.

Sufficient conditions for existence are less stringent in an economy without credit:

> *For at least* **one** *agent,* $B^i_{t-1} \geq 0$*, and* $\psi^i(p) \geq \underline{p} > 0$ *for all* p*.*
>
> *For* **one** *agent* j*,* $\psi^j(p)/p$ *tends to* 0 *when* p *tends to* ∞*.*

Nominal interest rate

r_t denotes the nominal interest rate, bearing both on creditors and debitors.

$$\begin{cases} \max\limits_{C_t, C_{t+1}, B_t, B_{t+1}} U(C_t,\ C_{t+1}) \\ p_t C_t + B_t = p_t Y_t + (1 + r_{t-1}) B_{t-1} \\ p^e_{t+1} C_{t+1} + B_{t+1} = p^e_{t+1} Y^e_{t+1} + (1 + r_t) B_t \\ C_t,\ C_{t+1} \geq\ 0, B_{t+1} \geq 0. \end{cases}$$

$$1 + \rho^e_t = (1 + r_t) \frac{p_t}{p^e_{t+1}}.$$

Bankruptcies and credit: a general person to person setup

Short run credit (still a single nominal asset)

Let B_t^{ij} be the (non negative) sum that agent j commits to give to agent i at the beginning of period $t+1$, for $i \neq j$, with the notational convention $B^{ii} = 0$.

Suppose that there is a chance that j may not honor his debt. Suppose that then all creditors receive the same share $(1 - r_j)$ of the sum that is due.

Suppose that there is a market on debts issued by agent k and let q_k be their price.

How to determine the extent of bankruptcies at the current date?

The budget constraints of a typical agent, say i, can be written as:

$$p_t C_t + \sum_j q^j B_t^{ij} - q^i \sum_{j \neq i} B_t^{ji} = p_t Y_t^i + \sum_j B_{t-1}^{ij}(1 - r_t^j) - \sum_j B_{t-1}^{ji}$$
$$p_{t+1} C_{t+1} = p_{t+1} Y_{t+1}^{ei} + \sum_j B_t^{ij}(1 - r_{t+1}^j) - \sum_j B_t^{ji}$$

Example of a rule: all previous commitments must be met before proceeding to new borrowing or lending

$$1 - r^i = \min\left(1\ ,\ \frac{p_t Y_t^i + \sum_j B_{t-1}^{ij}(1 - r^j)}{\sum_{j \neq i} B_{t-1}^{ji}}\right)$$

Non-linear system; chains of bankruptcies.

Other features

Possibility of putting a claim on future incomes.

Non convexities, discontinuities in behavior, associated with changes in expectations and/or fixed costs of bankruptcies.

Credit multipliers (Kyotaki Moore; Bernanke Gertler)

Summary: temporary equilibrium

Is the price level able to make supply equal to demand?

With positive money balances, the real balance effect (Pigou) theoretically makes the demand for good as large as one wishes when prices go to zero. But there is not much empirical support for this property (debate on fiscal stimulus).

The important driving force seems to be the impact on the subjective expected real interest rate, which works in case of sufficiently inelastic expectations.

How to pin down expectations?

The temporary equilibrium outcomes crucially depend on the arbitrary expectations.

We need an explicit dynamic structure to give some theoretical foundations to the formation of expectations.

Minimal errors in the 'long run': *perfect foresight* in a deterministic model, *rational expectations* when there is uncertainty.