MSc Macroeconomics G022

Problem set # 9

Due week of December 13-December 17 : hand in at beginning of tutorial

Problem 1
Finding a linear equilibrium through the method of undetermined coefficients

Consider an overlapping generations model with a stationary population where agents live two periods, work when they are young and consume when old.

While young, an agent born at date t supplies a quantity of labor ℓ_t which is transformed unit for unit into the same quantity of perishable good. She saves her income using the only assets available in the economy, nominal bonds (or 'money') which serve as the numéraire. The price of the good at t is P_t. When old, the consumer consumes C_{t+1}. Her utility function, seen from date t, is

$$E_t C_{t+1} - \frac{L_t^2}{2},$$

where E_t stands for the mathematical expectation, taken conditionally on the information available at date t.

The stock of bonds at the outset of date t is denoted B_{t-1}. The nominal interest rate is equal to zero. The government issues a random (positive or negative) quantity of bonds ΔB_t which serves to buy (or sell) goods for public use.

1. Define a temporary equilibrium at date t. Show that it satisfies the equation

$$\frac{B_t}{P_t} = E_t \frac{P_t}{P_{t+1}^e}.$$

2. Log linearize the equation when the shocks are small around the stationary rational expectations equilibrium, using lower case letters to denote logarithms.

3. Suppose that the money shocks can be described through

$$b_t = \mu b_{t-1} + u_t$$

   where $\mu < 1$ and the deviations of u_t around its mean are small. Find the expression for the linearized stationary rational expectations equilibrium using the method of undetermined coefficients (hint: postulate an expression for p_t as a function of current and past values of b_t; substitute and identify). Compare with the general results of Blanchard and Khan.

Problem 2
An exact real business cycle model: the Blanchard Kyotaki example

Consider an economy with a representative consumer and n firms, indexed with $j = 1, \dots, n$, each producing a good imperfectly substitutable with the others. The representative consumer behaves competitively. Her preferences are given by the utility function

$$U = C^{\gamma} \left(\frac{M}{P} \right)^{1-\gamma} - \omega H,$$

where H denotes labor supply and P the price index. C is a measure of aggregate consumption

$$C = n^{\frac{1}{1-\theta}} \left(\sum_{j=1}^{n} C_j^{\frac{\theta-1}{\theta}} \right)^{\frac{\theta}{\theta-1}},$$

where θ is a parameter larger than 1. The typical firm j produces output Y_j using labor H_j, with a production function $Y_j = A H_j^{\alpha}$, with $0 < \alpha < 1$.

1. The microeconomic choice of goods variety by the consumer is described by the program

$$\begin{cases} \max_{(C_j)_{j=1,\dots,n}} \left(\sum_{j=1}^{n} C_j^{\frac{\theta-1}{\theta}} \right)^{\frac{\theta}{\theta-1}} \\ \sum_{j=1}^{n} P_j C_j = Z. \end{cases}$$

   One defines the price index as

$$P = \left(\frac{1}{n} \sum_{j=1}^{n} P_j^{1-\theta} \right)^{\frac{1}{1-\theta}}.$$

   Show that, for all price configurations and all Z, one then has

$$PC = Z.$$

   Show also that

$$C_j = \frac{1}{n} \left(\frac{P_j}{P} \right)^{-\theta} C.$$

2. Let $R = PwH + \Pi + M_0$ be the initial nominal income of the consumer, where Π stands for profits of the productive sector, which are fully transferred to the consumer, w is the real wage, and M_0 is the initial money holdings.

(a) Show that the consumer's behaviour implies the following demand equations

$$M = (1 - \gamma)R$$

$$C = \gamma \frac{R}{P}$$

(b) Compute the equilibrium real wage as a function of ω and γ, assuming that labor supply is positive at equilibrium.

(c) Compute the value of aggregate demand at equilibrium as a function of M_0, γ and P. Derive the expression of C_j as a function of the same parameters together with P_j.

3. Suppose that the firms have local monopolistic power on their markets, while they are small enough to consider the aggregate price and consumption indices as fixed independently of their own actions. What is the price charged by an individual firm, as a function of the quantity produced? Derive the aggregate price and quantity at the symmetric Nash equilibrium.

4. How is the equilibrium affected by a shock on the quantity of money M_0 when prices are flexible? When prices are completely rigid?