MSc Macroeconomics G022

Problem set # 7

Due week of November 29-December 3: hand in at beginning of tutorial

Problem 1
Expectations dynamics and learning

Consider an economy made of generations of identical consumers, with the same utility function and labor supplies, who each live for two periods. The economy has a non durable consumption good and a durable financial asset in fixed quantity which serves as the numeraire. The typical consumer has an inelastic labor supply and produces at home an output Y^y when young and Y^o when old. She can save between the two periods by buying some of the financial asset. The typical consumer program when young is

$$\begin{cases} \max U(C^y, C^o) = \alpha \ln C^y + (1-\alpha) \ln C^o \\ p_t C^y + B_t = p_t Y^y \\ p^e_{t+1} C^o = p^e_{t+1} Y^o + B_t \\ C^y \geq 0, C^o \geq 0, \end{cases}$$

where p^e_τ denotes the price expectation of the generation born at date $\tau - 1$, made at date $\tau - 1$, on the price of the good at date τ. The history of the economy starts at date 1 with an old generation who holds a quantity B_0 of financial asset.

1. Define the temporary competitive equilibrium at date t. Compute the temporary equilibrium price and quantities as a function of the asset B_{t-1} held by the old generation and of the price expectations p^e_{t+1} of the young generation.

2. Define the competitive equilibria with perfect foresight of the economy. Give a geometrical construction of the set of equilibria in the plan (C^y, C^o). Discuss according to the values of $\alpha Y^o / (1-\alpha) Y^y$.

3. Study the dynamics of the sequence of temporary competitive equilibria in the situation where the price expectations is equal to the past price

$$p^e_{t+1} = p_{t-1},$$

starting with a given p_0. Can you give a quantitative estimate of the expectations errors along the trajectory? Does the economy converge to a perfect foresight equilibrium?

4. Same question as before with extrapolative expectations:

$$p^e_{t+1} - p_t = \lambda(p_t - p_{t-1}),$$

where $0 < \lambda < 1$.

Problem 2
Real competitive cycles and inventories

Consider an overlapping generations economy where agents live two periods, work when young and consume at both periods of their lives. There is a single non durable good that is produced and consumed, as well as a single durable good in fixed quantity which is carried forward from one period to the next without cost and serves as the numeraire. With standard notations, the typical generation program can be written as

$$\begin{cases} \max\limits_{C^y_t, C^o_{t+1}, B_t} U(C^y_t) + V(C^o_{t+1}) \\ p_t C^y_t + B_t = p_t Y \\ p_{t+1} C^o_{t+1} = B_t \\ C^y_t \geq 0 \; C^o_{t+1} \geq 0 \end{cases}$$

Here Y is the full employment production of good, supplied inelastically. The quantity of nominal asset B is given, held by the old generation at the initial date. We shall denote

$$R_t = 1 + \rho_t = \frac{p_t}{p_{t+1}}$$

the gross real interest rate at date t under perfect foresight.

1. We consider the situation where

$$U(C) = \ln(C)$$

and

$$V(C) = \begin{cases} \dfrac{C^b}{b} & \text{for } b \leq 1, b \neq 0 \\ \ln C & \text{for } b = 0 \end{cases}$$

Show that the equation of the supply curve (C^y, C^o), locus of the consumers' choice when R varies, is

$$C^y_t[1 + (C^o_{t+1})^b] = Y.$$

In the remainder of the problem, it is assumed that $Y = 2$.

Represent the various shapes of the supply curve in the plan $(C^y - Y, C^o)$ and discuss according to the value of b.

2. Define a stationary competitive equilibrium for this economy. Compute the associated price and allocation. Show that the slope of the supply curve at the stationary competitive equilibrium is equal to $-2/b$.

3. A competitive cycle of order 2 is defined by two (distinct) points (C_1^y, C_2^o) and (C_2^y, C_1^o) which satisfy

$$C_1^y[1 + (C_2^o)^b] = Y,$$

$$C_2^y[1 + (C_1^o)^b] = Y,$$

$$C_1^y + C_1^o = Y,$$

$$C_2^y + C_2^o = Y.$$

   Comment and justify this definition. Characterize geometrically such a cycle on the supply curve diagram in the plane of coordinates (C^y, C^o) : let A be the point of coordinates (C_1^y, C_2^o), and B that of (C_2^y, C_1^o). Show that feasibility (supply=demand) requires that the segment AB has slope equal to $+1$ (hint: $(C_1^y - C_2^y)/(C_2^o - C_1^o) = 1$), and that its middle belongs to the line $C^y + C^o = Y$.

4. Use a geometric argument to show that for $b < -2$, there exists a competitive cycle of order 2 in the economy.

5. (This question is optional) Suppose that there is a possibility to store some of the non durable good without cost up to a given (not too large) upper inventory limit I. Discuss the stabilizing role of inventories in a competitive world.