MSc Macroeconomics G022

Problem set # 6

Due week of November 22-November 26 : hand in at beginning of tutorial

Problem 1
Temporary competitive equilibrium

Consider an economy made of $i = 1, \ldots, I$ consumers-workers, in a two period setup. The current period is noted t, and the future $t+1$. All quantities relative to the second period are expected.

The economy has a non durable consumption good and a financial asset in fixed quantity which serves as the numeraire, and allows to borrow or lend from one period to the next. Consumer i has an inelastic labor supply and produces at home an output Y_t^i, $Y_t^i > 0$ for all i and t. The consumers enter period t with *non negative* asset holdings B_{t-1}^i. All consumers have the same utility functions

$$U(C_t, C_{t+1}) = \alpha \ln C_t + (1-\alpha) \ln C_{t+1},$$

where C_t and C_{t+1} denote respectively the non negative non durable consumption in period t and planned non durable consumption for period $t+1$, and $0 < \alpha < 1$.

The price of the non durable good in terms of the numeraire is p_t, and carries a superscript i when it is expected by consumer i.

1. Compute the demand of good of consumer i, supposing that her expectations (p_{t+1}^i, Y_{t+1}^i) are non random and that there are no borrowing constraints (there are no restrictions on B_t, which can take any positive or negative value), so that the consumer's program is to solve

$$\begin{cases} \max \quad U(C_t, C_{t+1}) \\ p_t C_t + B_t = p_t Y_t^i + B_{t-1}^i \\ p_{t+1}^i C_{t+1} + B_{t+1} = p_{t+1}^i Y_{t+1}^i + B_t \end{cases}$$

with respect to the non negative variables (C_t, C_{t+1}, B_{t+1}). Show that

$$C_t^i = \alpha \left(Y_t^i + \frac{B_{t-1}^i}{p_t} + \frac{p_{t+1}^i Y_{t+1}^i}{p_t} \right).$$

2. Define a temporary competitive equilibrium. Write the equation characterizing an equilibrium price.

3. Suppose that all the consumers expect the same constant inflation factor Π:
$$p_{t+1}^i = \Pi p_t,$$
while their output expectations Y_{t+1}^i do not depend on the current price. Discuss the existence of an equilibrium (hint: show that aggregate demand is continuous and decreasing in current price; show that the sign of $\sum_i[\alpha\Pi Y_{t+1}^i - (1-\alpha)Y_t^i]$ plays an important role).

4. Discuss the case where price expectations are adaptative, i.e.
$$p_{t+1}^i = \beta^i p_t + (1-\beta^i)p_{t-1},$$
with $0 \leq \beta^i < 1$.

5. (This question is optional) Suppose now that price expectations are uncertain: the (common) price expectations of the consumers is equal to Πp_t, with Π uniformly distributed on the interval $[\underline{\Pi}, \overline{\Pi}]$. Write the consumer's program when the consumers maximize their expected utilities. Show that the first order condition for B_t can be written as:
$$\frac{\alpha}{p_t Y_t^i + B_{t-1}^i - B_t} = (1-\alpha)E\left(\frac{1}{B_t + \Pi p_t Y_{t+1}^i}\right)$$
Show that when $Y_{t+1}^i = 0$ for all i the previous analysis of the case with certainty trivially applies.

Problem 2
Temporary equilibrium and bankruptcies

As in the preceding problem, consider an economy made of consumers-workers, with a two period horizon. The current period is denoted t, and the future $t+1$. All quantities relative to the second period are expected. The economy has a non durable consumption good and a durable numeraire which can be used as a support for credit.

There are two (categories of) consumers, respectively designated with a superscript c for creditor, and d for debtor. Consumer d has an inelastic labor supply and produces at home an output Y_t, $Y_t > 0$ for all t. From previous commitments, she owes B_{t-1} to consumer c, who has no other resources than this claim on consumer d. Similarly, during the current period, consumer d can borrow B_t from consumer c. All debt or credit operations cease at the end of period 2. The interest rate is zero.

The two consumers have identical utility functions
$$U(C_t, C_{t+1}) = \alpha \ln C_t + (1-\alpha)\ln C_{t+1},$$

where C_t and C_{t+1} denote respectively the non negative non durable consumption in period t and planned non durable consumption for period $t+1$, and $0 < \alpha < 1$.

The price of the non durable good in terms of the numeraire is p_t, and carries a superscript i when it is expected by consumer i. The exercise aims at redoing part of Problem I while discussing the impact of various possible bankruptcy regulations.

Preliminary question: Write the program of consumer d. What happens when

$$p_t Y_t + p^d_{t+1} Y_{t+1} - B_{t-1} < 0?$$

Show that consumer d's demand for good is

$$C^d_t = \alpha \left(Y_t + \frac{p^d_{t+1}}{p_t} Y_{t+1} - \frac{B_{t-1}}{p_t} \right),$$

whenever the quantity on the right hand side is nonnegative.

Bankruptcy rule 1

We start with a rule that makes bankruptcies the less disruptive possible for the functioning of the economy. Suppose that there is a central bank that monitors closely the activities of the debtors and guarantees their debts: it stands for them in case of default.

1. The debtor is declared bankrupt and left with zero current consumption and asset when $p_t Y_t + p^d_{t+1} Y_{t+1} - B_{t-1}$ is negative. All her properties then are confiscated by the bank, which pays the B_{t-1} due to consumer c. Define a temporary equilibrium.

   Show that the aggregate demand for good takes the form:

$$C^c_t + C^d_t = \alpha \frac{B_{t-1}}{p_t} + \alpha \max \left[0, \left(Y_t + \frac{p^d_{t+1}}{p_t} Y_{t+1} - \frac{B_{t-1}}{p_t} \right) \right].$$

2. Discuss the existence of a temporary equilibrium, along the lines of the lectures, in relationship to the variations of the expected price level p^d_{t+1} with the current price level p_t (hint: if expectations are continuous, aggregate demand is a continuous function of p_t; compute its limits when p_t goes to zero or to infinity, and compare with supply Y_t).

Bankruptcy rule 2

The previous rule has a number of awkward features. The creditors have no incentives to monitor the debtors (moral hazard), since the bank warrants their loans. The prospects of the debtors (price or production forecasts) are typically private informations, to which the bank does not have access: only current production may be observable, in which case bankruptcy rule 1 cannot be implemented.

1. As an alternative, suppose that the debtor declares herself bankrupt whenever $p_t Y_t - B_{t-1} < 0$, and then is allowed to substitute the delivery of Y_t to the creditor in payment of her debt. She then acts as usual, can borrow some more from the creditor...

   Show that the budget constraints of the two agents in the event of bankruptcy take the following form:

Agent c	Agent d
$p_t C_t^c + B_t = p_t Y_t$	$p_t C_t^d = B_t$
$p_{t+1}^c C_{t+1}^c = B_t$	$p_{t+1}^d C_{t+1}^d + B_t = p_{t+1}^d Y_{t+1}$

2. Define a temporary equilibrium. Compute the aggregate demand for good in the economy. Show that for $p_t < B_{t-1}/Y_t$, one has

$$C_t^c + C_t^d = \alpha Y_t + \alpha \frac{p_{t+1}^d}{p_t} Y_{t+1}.$$

3. Show that aggregate demand is a continuous function of current price provided that price expectations are also continuous. Discuss the existence of a temporary equilibrium.

4. In the preceding equilibrium, agent d can contract new debt and consume immediately after having defaulted on her previous commitments. Describe qualitatively the likely effect of sanctions, such as restrictions on her borrowing capacity or her consumption level?