Monetary policy in the short and in the long run

Policy: managing nominal assets in the short and in the long run

1. The quantity theory of money
2. Public deficit and seigniorage
3. Pensions

The quantity theory of money

The quantity theory of money describes the consequences on the economy of a fictitious(?) monetary manipulation (similar to a change of units): helicopter money (Milton Friedman).

The quantity theory says that such a manipulation leaves all the real magnitudes unchanged, while all the nominal magnitudes are multiplied by the same factor. It is satisfied in the *long run* in all economic models.

I am going to discuss its short run validity, which allows to review the famous Lucas critique of macroeconomic modeling.

Look out for errors!

Absence of money illusion

Consumers are free of money illusion *if, when their nominal assets holdings and all nominal prices are multiplied by a positive scalar* λ*, their demands for goods stay unchanged and their demands for nominal assets are multiplied by* λ*.*

The quantity theory

The *quantity theory of money* is a comparative statics property of the economic equilibrium, or of the set of equilibria when the equilibrium is not unique, which can be stated as follows

If initial (possibly negative) nominal holdings are multiplied by λ, all government actions staying unchanged in real terms, equilibria after the transformation are in a one-to-one correspondence with the initial equilibria which can be described as follows: real quantities are unchanged, while all nominal quantities, prices, wages, are multiplied by λ.

Application to the temporary equilibrium

Are consumers subject to money illusion?

$$\begin{cases} \max U(C_1,\ C_2) \\ \lambda p_1 C_1 + B_1 = \lambda p_1 Y_1 + \lambda B_0 \\ p_2^e C_2 = p_2^e Y_2^e + B_1 \\ C_1,\ C_2 \geq\ 0 \end{cases}$$

Absence of money illusion if and only if the solution (C_1, C_2) is independent of λ, while B_1 is proportional to λ, when λ varies in $\mathbb{R}_{++}$.

Money illusion and expectations

Except for very special utility functions, one needs $p_2^e = \psi(\lambda p_1) = \lambda p_1 \psi(1)$ for all positive λ and Y_2^e independent of current prices.

The expected price is an homogenous function of degree 1 of p_1.

Is the quantity theory of money valid?

Except for specific shapes of utility functions, a unit elasticity of the expected price with respect to the current price is a necessary and sufficient condition for the validity of the quantity theory of money.

Inconsistency with existence of a temporary equilibrium?

The error corrected

Expectations depend on the whole history, and in particular on past and current exogenous policy.

Absence of money illusion, as defined above, is not a plausible property.

A natural assumption under which the quantity theory holds in the short run is:

The expected price is homogenous of degree 1 with respect to $(p_1, B_0^i)_{i=1,\dots,I}$.

Quantity theory and interest rate policy

Interest rate paid on nominal assets:

$$\begin{cases} \max U(C_1,\ C_2) \\ p_1 C_1 + B_1 = p_1 Y_1 + B_0 \\ \psi(p_1, r) C_2 = \psi(p_1, r) Y_2 + (1 + r) B_1 \\ C_1,\ C_2 \geq\ 0. \end{cases}$$

Interest rate policy, with an equal rate for debtors and lenders in an economy with a short run nominal asset, is akin to helicopter money at date 2.

If the expectations satisfy $\psi(p_1, r) = (1 + r)\psi(p_1, 0)$, *the interest rate policy is neutral in the short run.*

The interest rate policy only has real effects when the inflation expectations are not adjusted in a way that leaves the real interest rate unchanged.

Monetary policy in the short run: channels

Maturity of nominal assets.

Price rigidities.

Interaction with expectations formation.

Public deficit and seigniorage

For once in this course, we separate cash (used for transactions) from a nominal asset that bears interest.

The government benefits from the privilege of emitting money, a monopoly. The public indirectly pays for the services given by money through seigniorage, or an implicit inflation tax.

Aim: develop a (long run) model where the mechanism of seigniorage is made explicit.

The setup

Agents: government an upper index g,
central bank upper index b,
representative private consumer upper index c.

Goods: a non storable physical good
a short run nominal asset
cash which serves for transactions.

Cash is the numeraire. A unit of nominal asset bought at date t entails its owner to receive $(1 + r_t)$ money units at the next date. The price of the physical good is p.

Long run, no bankruptcy.

The central bank balance sheet

Initially

$$B^b_{t-1} = M_{t-1}$$

Budget constraint

$$M_t = M_{t-1} - (1 + r_{t-1})B^b_{t-1} + B^b_t + r_{t-1}B^b_{t-1}.$$

(the profits of the bank are assumed to be transferred on the spot to the government).

At the close of the period

$$B^b_t = M_t$$

The government balance sheet

Beginning of period debt B^g_{t-1}.

Budget constraint

$$p_t G_t = p_t T_t + r_{t-1} B^b_{t-1} - (1 + r_{t-1}) B^g_{t-1} + B^g_t.$$

G_t and T_t respectively are lump-sum expenditures and taxes, measured in units of good.

Overlapping generations setup

Taxes are paid when young.

$$\begin{cases} \max \; U(C_t^y, C_{t+1}^o) \\ p_t C_t^y + B_t^c + M_t^c = p_t(Y^y - T_t) \\ p_{t+1} C_{t+1}^o = p_{t+1} Y^o + (1 + r_t) B_t^c + M_t^c \\ M_t^c \geq k p_t (Y^y - T_t) \end{cases}$$

Clower constraint $0 < k < 1$

The demand for money for liquidity services is represented as an ad hoc reduced form. The coefficient k could be made dependent on the interest rate.

Supply demand equalities

Initial conditions of period t

$$M_{t-1} = M^c_{t-1},$$

$$B^g_{t-1} = B^b_{t-1} + B^c_{t-1}.$$

Feasibility (Walras' law)

$$C^y_t + C^o_t + G_t = Y^y + Y^o,$$

$$M_t = M^c_t,$$

$$B^g_t = B^b_t + B^c_t.$$

Some equilibrium properties

No sign constraint on B^c in the consumer's program. At any equilibrium, one must have:

$$r_t \geq 0.$$

When r_t is strictly positive, the Clower constraint binds.

Assume without loss of generality that

$$M_t = kp_t(Y^y - T_t)$$

Intertemporal consumer budget constraint

Substituting M_t and eliminating B^c yields

$$\frac{(1+r_t)p_t}{p_{t+1}}(C_t^y + T_t - Y^y) + C_{t+1}^o - Y^o + r_t \frac{M_t^c}{p_{t+1}} = 0.$$

Real interest rate $1 + \rho_t = (1+r_t)p_t/p_{t+1} = (1+r_t)/(1+i_t)$

Note the relationship between the nominal interest rate and the rate of inflation.

Long run equilibrium path

We look for trajectories along which real quantities stay constant over time, as well as the nominal interest rate and the rate of inflation.

From the (stationary) consumer budget constraint

$$(1+\rho)(C^y + T - Y^y) + C^o - Y^o + \frac{r}{1+r}(1+\rho)k(Y^y - T) = 0.$$

The consumers' supply curve in the plan (C^y, C^o) is similar to that drawn previously, once Y^y is replaced with $Y^y - T - k(Y^y - T)r/(1+r)$.

The term $k(Y^y - T)r/(1+r)$, which is equal to foregone income because cash balances do not bear interest, acts as a tax.

Government deficit

It is useful to rewrite the government budget constraint as

$$G = T + \frac{r}{1+r} k(Y^y - T) + D.$$

This defines the *generalized* deficit D as the excess of expenditures over regular taxes and the fictitious interest rate tax.

Long run equilibria

The scarcity constraint is

$$C^y + C^o + G = Y^y + Y^o,$$

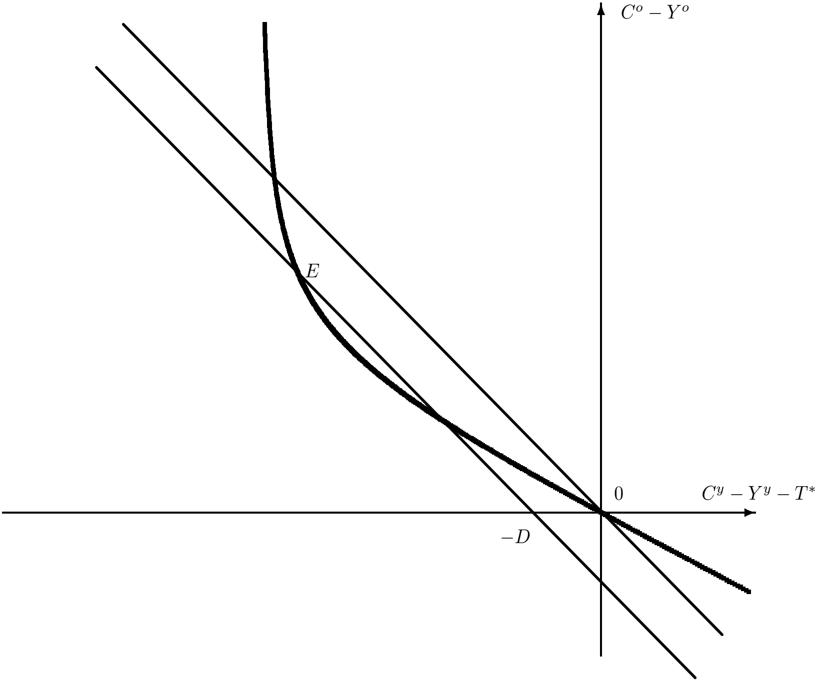
or equivalently

$$C^y + C^o = Y^y - T - \frac{r}{1+r}k(Y^y - T) + Y^o - D.$$

Using the budget constraints to eliminate $(C^o - Y^o)$, one gets:

$$\rho(C^y + T^* - Y^y) = D.$$

When $D = 0$, we find the standard two kinds of long run equilibria, autarkic ($C^y = Y^y - T^*$) and golden rule ($\rho = 0$). At the golden rule, the rate of inflation is equal to the nominal interest rate, and the interest rate tax is an inflation tax, also called *seigniorage*, income indirectly perceived by the government because of the services provided by cash.


$C^o - Y^o$
E
0
$C^y - Y^y - T^*$
$-D$

On pensions

Basic definitions

Pay as you go: the contributions of the workers are shared between the retirees (with little savings or deficits of the managing agency).

At the time when it is created, there is a big bonus for the retirees, who get pensions but did not contribute during their working lives. But in normal times there are tensions when the ratio of the number of contributors over the number of retirees decreases.

Fully funded: the contributions are saved in a fund and blocked until retirement.

Some differences with private savings: mutualization of length of life risk, taxes are typically paid at exit on pensions (rather than on current income: contributions are tax deductible), the contributor has a limited choice of funds and not much control on their managements.

Here I shall assimilate funded pensions schemes with private savings.

Elements of the debate

The before tax rate of return on the stock market (around 9% in the US in the past century) seems much larger than that of the PAYG (around 2.5%).

A move towards pension funds should make financial markets more liquid, facilitate capital accumulation, while possibly crowding out some current private investors. But who pays for the PAYG in the meanwhile?

Microeconomic vs macroeconomic, transition vs long run issues

Is a move towards a fully funded pension scheme increasing the economy capital stock and improving welfare in the long run?

Production and prices

There are constant returns to scale in production.
k is the stock of capital per head of the young generation at the beginning of the period; $f(k)$ is gross production per head of worker.
During production, capital gets consumed at rate δ, $\delta > 0$, per period: for an input of k entering the productive activities, one gets back $f(k) + (1-\delta)k$ at the end of the productive process.
Perfect competition:

$$w_t = f(k_t) - k_t f'(k_t), \; r_t = f'(k_t) - \delta. \tag{1}$$

Constant returns to scale imply

$$f(k_t) = w_t + (r_t + \delta)k_t. \tag{2}$$

Demographics and consumption

Population grows at a constant rate n, $n \geq 0$. All consumers live for two periods, consume at both dates. They only work when young, and then supply inelastically one unit of labor.

Savings can be done through two assets, physical capital s and government bonds b. In the absence of uncertainty both have the same rate of return r. *I do not make any distinction between private savings and public fully funded pension schemes*.

The PAYG pension scheme is parameterized through a number θ, which is the replacement rate along a stationary state: the pension is equal to a fraction θ of the wage. Since the PAYG budget is balanced, the contribution rate on the current workers wage slip is $\theta/(1+n)$.

The consumer program

$$\begin{cases} \max \quad u(c_t^y, c_{t+1}^o) \\ c_t^y + s_t + b_t = \left[w_t - \dfrac{\theta w_t}{1+n} \right] \\ c_{t+1}^o = s_t + b_t + \theta w_{t+1} + r_{t+1}(s_t + b_t), \end{cases} \tag{3}$$

In the absence of sign constraints on holdings of nominal assets b, the two budget constraints are equivalent to

$$c^y + \frac{c^o}{1 + r_{t+1}} = W_t$$

with

$$W_t = w_t + \theta \left(-\frac{w_t}{1+n} + \frac{w_{t+1}}{1 + r_{t+1}} \right). \tag{4}$$

Consumption functions

$C^y(r, W)$ and $C^o(r, W)$ are solutions of

$$\begin{cases} \max \quad u(c^y, c^o) \\ c^y + \dfrac{c^o}{1+r} = W \end{cases} \tag{5}$$

Assumption *Consumptions at the two dates are normal goods: they are increasing functions of the intertemporal wealth of the consumer.*

The government

Let g_t be public spending per head of young agent. The government budget constraint is

$$g_t = -\frac{b_{t-1}(1 + r_t)}{1 + n} + b_t.$$

Balanced equilibrium trajectories

A balanced growth path is a trajectory of the economy along which quantities per head stay constant and the scarcity constraints are satisfied.

$$g + c^y + \frac{c^o}{1+n} + s_t = f(k_t) + (1-\delta)k_t. \tag{6}$$

$$s_t = (1+n)k_{t+1}. \tag{7}$$

A balanced growth path is an equilibrium if it is associated with (constant) prices and interest rates such that consumption and production correspond to the agents optimal competitive decisions, given the prices.

Back to pensions: stationary equilibria

One exogenous parameter θ, one endogenous unknown k.

Wages and interest rates are functions of k from the profit maximization first order conditions. Substituting in the expression of life time income W, one obtains W as a function of (k, θ). The consumption functions can be expressed with the same arguments:

$$C^y(r, W) \to D^y(k, \theta)$$

$$C^{\o}(r, W) \to D^{\o}(k, \theta)$$

The scarcity constraint then is

$$g + \underbrace{D^y(k, \theta) + \frac{D^o(k, \theta)}{1+n}}_{D(k,\theta)} + (n+\delta)k = f(k). \tag{8}$$

Given the replacement rate θ, all the stationary equilibria capital stocks are the solutions of (8).

$$Z(k, \theta) = D(k, \theta) + (n+\delta)k - f(k) = -g$$

The golden rule

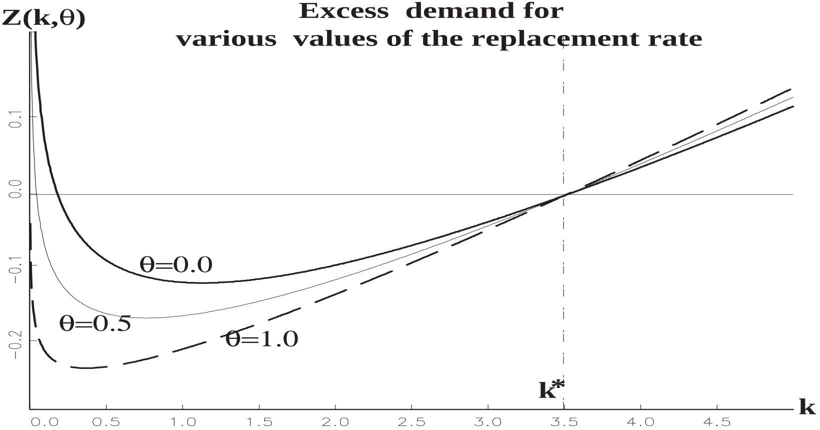
The golden rule capital stock k^* is the value of the capital stock which maximizes the stationary net output per head: it is a long run optimum.

$$n + \delta = f'(k^*) = r^* + \delta$$

The contribution to the PAYG pension is akin to forced savings and its return is identical to the population growth rate.

At the golden rule, the PAYG return is equal to that of the financial assets. The PAYG scheme has no impact on the equilibrium trajectory: with perfect financial markets, any forced savings can be undone without cost by the private agents.

Figure 1
Excess demand for various values of the replacement rate



Comparative statics

Efficiency: the capital stock is at most equal to its golden rule value; equivalently the rate of interest is larger than or equal to the rate of population growth. I shall limit my attention to the region $k \leq k^*$ in all the following

$$W = w + \theta \left(-\frac{w}{1+n} + \frac{w}{1+r} \right)$$

At fixed k, $k \leq k^*$, the consumers life time wealth decreases with the PAYG replacement rate θ.

Comparative statics: capital stock

A marginal decrease in the replacement rate decreases the equilibrium stock of capital if and only if

$$\frac{\partial Z}{\partial k}(k, \theta) > 0$$

The initial (first round) extra savings generated by the reduction of the PAYG scheme crowds out private savings to such an extent that it reduces the equilibrium capital stock.

Importance of the shape of the aggregate demand function of the overall population $D(k, \theta)$.