

How to finance pensions: pay as you go or fully funded?

## Basic definitions

**Pay as you go:** the contributions of the workers are shared between the retirees (with little savings or deficits of the managing agency).

At the time when it is created, there is a big bonus for the retirees, who get pensions but did not contribute during their working lives. But in normal times there are tensions when the ratio of the number of contributors over the number of retirees decreases.

**Fully funded:** the contributions are saved in a fund and blocked until retirement.

Some differences with private savings: mutualization of length of life risk, taxes are typically paid at exit on pensions (rather than on current income: contributions are tax deductible), the contributor has a limited choice of funds and not much control on their managements.

**Here I shall assimilate funded pensions schemes with private savings.**

## Elements of the debate

The before rate of return on the stock market (around 9% in the US in the past century) seems much larger than that of the PAYG (around 2.5%).

A move towards pension funds should make financial markets more liquid, facilitate capital accumulation, while possibly crowding out some current private investors. But who pays for the PAYG in the meanwhile?

Microeconomic vs macroeconomic, transition vs long run issues

Is a move towards a fully funded pension scheme increasing the economy capital stock and improving welfare in the long run?

# Plan

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## Production and prices

There are constant returns to scale in production.

$k$  is the stock of capital per head of the young generation at the beginning of the period;  $f(k)$  is gross production per head of worker.

During production, capital gets consumed at rate  $\delta$ ,  $\delta > 0$ , per period: for an input of  $k$  entering the productive activities, one gets back  $f(k) + (1 - \delta)k$  at the end of the productive process.  
Perfect competition:

$$w_t = f(k_t) - k_t f'(k_t), \quad r_t = f'(k_t) - \delta. \quad (1)$$

Constant returns to scale imply

$$f(k_t) = w_t + (r_t + \delta)k_t. \quad (2)$$

## Demographics and consumption

Population grows at a constant rate  $n$ ,  $n \geq 0$ . All consumers live for two periods, consume at both dates. They only work when young, and then supply inelastically one unit of labor.

Savings can be done through two assets, physical capital  $s$  and government bonds  $b$ . In the absence of uncertainty both have the same rate of return  $r$ . *I do not make any distinction between private savings and public fully funded pension schemes.*

The PAYG pension scheme is parameterized through a number  $\theta$ , which is the **replacement rate** along a stationary state: the pension is equal to a fraction  $\theta$  of the wage. Since the PAYG budget is balanced, the contribution rate on the current workers wage slip is  $\theta/(1+n)$ .

## The consumer program

$$\begin{cases} \max & u(c_t^y, c_{t+1}^o) \\ c_t^y + s_t + b_t = & \left[ w_t - \frac{\theta w_t}{1+n} \right] \\ c_{t+1}^o = s_t + b_t + & \theta w_{t+1} + r_{t+1}(s_t + b_t), \end{cases} \quad (3)$$

In the absence of sign constraints on holdings of nominal assets  $b$ , the two budget constraints are equivalent to

$$c^y + \frac{c^o}{1+r_{t+1}} = W_t$$

with

$$W_t = w_t + \theta \left( -\frac{w_t}{1+n} + \frac{w_{t+1}}{1+r_{t+1}} \right). \quad (4)$$

## Consumption functions

$C^y(r, W)$  and  $C^o(r, W)$  are solutions of

$$\begin{cases} \max & u(c^y, c^o) \\ & c^y + \frac{c^o}{1+r} = W \end{cases} \quad (5)$$

**Assumption** *Consumptions at the two dates are normal goods: they are increasing functions of the intertemporal wealth of the consumer.*



## The government

Let  $g_t$  be public spending per head of young agent. Its budget constraint is

$$g_t = -\frac{b_{t-1}(1+r_t)}{1+n} + b_t.$$

## Balanced equilibrium trajectories

A balanced growth path is a trajectory of the economy along which quantities per head stay constant and the scarcity constraints are satisfied.

$$g + c^y + \frac{c^o}{1+n} + s_t = f(k_t) + (1 - \delta)k_t. \quad (6)$$

A balanced growth path is an equilibrium if it is associated with (constant) prices and interest rates such that consumption and production correspond to the agents optimal competitive decisions, given the prices.

## Back to pensions: stationary equilibria

One exogenous parameter  $\theta$ , one endogenous unknown  $k$ .

Wages and interest rates are functions of  $k$  from the profit maximization first order conditions. Substituting in the expression of life time income  $W$ , one obtains  $W$  as a function of  $(k, \theta)$ . The consumption functions can be expressed with the same arguments:

$$C^y(r, W) \rightarrow D^y(k, \theta)$$

$$C^o(r, W) \rightarrow D^o(k, \theta)$$

The scarcity constraint then is

$$g + \underbrace{D^y(k, \theta) + \frac{D^o(k, \theta)}{1+n}}_{D(k, \theta)} + (n + \delta)k = f(k). \quad (7)$$

Given the replacement rate  $\theta$ , all the stationary equilibria capital stocks are the solutions of (7).

$$Z(k, \theta) = D(k, \theta) + (n + \delta)k - f(k) = -g$$

# The golden rule

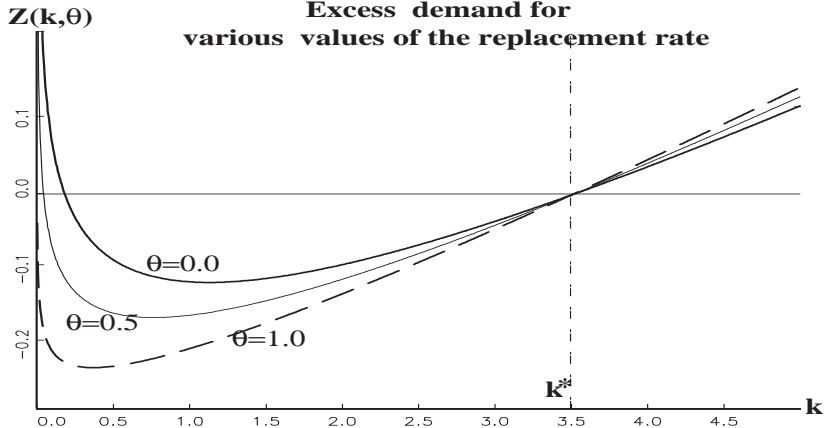
The golden rule capital stock  $k^*$  is the value of the capital stock which maximizes the stationary net output per head: it is a long run optimum.

$$n + \delta = f'(k^*) = r^* + \delta$$

The contribution to the PAYG pension is akin to forced savings and its return is identical to the population growth rate.

At the golden rule, the PAYG return is equal to that of the financial assets. The PAYG scheme has no impact on the equilibrium trajectory: with perfect financial markets, any forced savings can be undone without cost by the private agents.

**Figure 1**  
**Excess demand for**  
**various values of the replacement rate**



## Comparative statics

Efficiency: the capital stock is at most equal to its golden rule value; equivalently the rate of interest is larger than or equal to the rate of population growth. I shall limit my attention to the region  $k \leq k^*$  in all the following

$$W = w + \theta \left( -\frac{w}{1+n} + \frac{w}{1+r} \right)$$

At fixed  $k$ ,  $k \leq k^*$ , the consumers life time wealth decreases with the PAYG replacement rate  $\theta$ .

## Comparative statics: capital stock

A marginal decrease in the replacement rate **decreases** the equilibrium stock of capital if and only if

$$\frac{\partial Z}{\partial k}(k, \theta) > 0$$

The initial (first round) extra savings generated by the reduction of the PAYG scheme **crowds out** private savings to such an extent that it reduces the equilibrium capital stock.

Importance of the shape of the aggregate demand function of the overall population  $D(k, \theta)$ .