CHAPTER 1

Expectations and the real balance effect

This chapter will examine, with the help of a simple microeconomic model, two propositions that play a significant role in neoclassical monetary theory.

The first proposition is that "money does not matter" – or, more precisely, that if the mere presence of money as a medium of exchange and as an asset is important for the smooth functioning of the economy, the quantity of money is unimportant. This is the *quantity theory* tradition, which claims that a change in the money stock will change all nominal values in the same proportion, but will have no effect on "real" variables. This old tradition still plays an important role in modern thinking. We wish to clarify the exact meaning of this theory and its domain of validity.

The second issue will be the belief, which is shared by many theorists, that a short-run Walrasian equilibrium in which money has positive exchange value usually exists. We shall investigate this question by looking at a simple model involving only outside money. "Money" is then printed money, and can be regarded as a part of private net wealth. In such a context, neoclassical theorists assume that the traders' price expectations are "unit elastic," so that expected prices vary proportionally with current prices. The essential short-run regulating mechanism is then the *real balance effect*. When money prices of goods are low, the purchasing power of the agents' initial money balances is large. This fact should generate, according to this viewpoint, an excess demand on the goods market at sufficiently low prices. Conversely, the purchasing power of the agents' initial cash balances becomes small when money prices are high, so that an excess supply of goods should eventually appear. Thus, by continuity, an equilibrium should exist in between.

It will be shown that this argument is wrong because it neglects the intertemporal character of the choices made by the agents. Explicitly modeling these choices will lead us to conclude that theoretically conceivable and empirically plausible circumstances exist in which the real balance effect is too weak to guarantee the existence of a short-run equilibrium in which money has positive value. As a matter of fact, the existence of such an equilibrium position essentially requires, as we shall see, that the price forecasts of some agent be substantially insensitive to current prices. The relative variations of current and expected prices then generate an *intertemporal substitution effect*, which reinforces the real balance effect, and is strong enough to equilibrate the market.

Such conditions on expectations are quite unlikely to prevail in reality. Indeed, the agents' price expectations are presumably very sensitive to the price levels that they currently observe, especially in periods of significant inflation or deflation. Expected rates of inflation may be biased upward when a significant inflation has been observed in the recent past and downward in the case of a deflation. It will be shown that in such circumstances, a short-run monetary equilibrium may not exist. The conclusion that will emerge, therefore, from our analysis is that the existence of a short-run Walrasian equilibrium in which money has positive value is somewhat more problematic in actual market economies than neoclassical economists used to believe.

In order to focus attention on the essentials, we will conduct the analysis within the framework of a simple model. Paper money will act as the *numéraire* and only store of value. Its stock, which can be viewed as the sum of the government's past deficits, will be assumed to be constant over time (outside money). Output, or equivalently, the stream of the agents' real income will be taken as exogenous. This hypothesis is in fact immaterial; analogous results would be obtained with variable output. In this chapter no attention will be paid to the services that actual money yields in our economies (medium of exchange, liquidity).[1] The main question will be whether an equilibrium exists either in the short run or in the long run in which the agents are willing to hold the outstanding quantity of money, and to investigate the properties of these equilibria (if any) in relation to the money stock.

[1] Liquidity services of money will be taken up in Chapter 4. The role of money in transactions will be considered in the "Notes on the Literature" section later in this chapter. See also the Conclusion of this book.

1.1 Classical and neoclassical views on money

We first proceed to a brief overview of the issues with which we shall be concerned. In order to fix the ideas, let us assume that there are l perishable goods, indexed by $h = 1, \ldots, l$, traded in each period at *money prices* $p = (p_1, \ldots, p_l)$, whose equilibrium values are to be determined by the market.[2] Fiat money, on the other hand, is the sole asset, and its stock is constant over time.

Classical economists (e.g., I. Fisher 1963) took the view that in order to find the level of equilibrium prices, one could reason in two steps. Markets for goods (the "real sector") would determine equilibrium *relative prices*, i.e., the ratios p_h/p_k, and quantities of goods exchanged in equilibrium. Then consideration of the money market would determine the level of equilibrium money prices, which would be in fact proportional to the money stock.The view that the real and money markets can be considered separately in this way is called the *classical dichotomy*. The proportionality of the money prices to the money stock is the essence of the quantity theory.

To be more precise, let us consider a specific agent a. In the classical approach, his array of excess demands for goods is written as a function of money prices alone, $z_a(p) = [z_{a1}(p), \ldots, z_{al}(p)]$. Equilibrium of the goods market would require that aggregate excess demand is zero, where the summation sign runs over the set of all agents a:

$$(1.1.A) \quad \sum_a z_a(p) = 0$$

Classical economists would assume that equation (1.1.A) displays the usual properties of an ordinary Walrasian system, that is, homogeneity of degree zero of excess demand functions; i.e., $z_a(\lambda p) = z_a(p)$ for every p and every positive λ, and what has been called *Say's Law* by Lange (1942) and Patinkin (1965): The value of aggregate excess demand $p\, \Sigma_a z_a(p)$ is zero for every price system p.[3] The structure of equations like (1.1.A) was understood to depend significantly on the mere presence of money in the economy. The main point is that these equations were assumed to be independent of the quantity of money, and of its distribution among traders.

One must next consider the money market. Let $m_a^d(p)$ be agent a's demand for (nominal) money when the price system is p. The classical view that only "real"money balances matter can be expressed here by

[2] All prices that are considered in this monograph are positive.

[3] In what follows, given two vectors x and y with l components, the notation $x\,y$ will stand for $\Sigma_h x_h y_h$.

the assumption that $m_a^d(p)$ is homogenous of degree 1 in p; i.e., $m_a^d(\lambda p) = \lambda m_a^d(p)$ for every p and λ. Then equilibrium of the money market requires that aggregate demand for money be equal to the amount of money M in circulation:

$$(1.1.B) \quad \sum_a m_a^d(p) = M$$

Writing down a system of equations like (1.1.A) and (1.1.B) seems to formulate in a consistent way classical views about the classical dichotomy and quantity theory. Modern competitive equilibrium theory tells us that (1.1.A) indeed has solution(s) in p under rather general conditions. Due to the homogeneity of degree zero of (1.1.A), such solutions are defined only up to a positive real number. Consideration of the real sector alone leads to the determination of equilibrium relative prices and real quantities exchanged. On the other hand, for any solution $\bar{p}$ of (1.1.A), there exists a unique λ such that $\Sigma_a m_a^d(\lambda\bar{p}) = M$, provided that aggregate demand for money at prices $\bar{p}$, i.e. $\Sigma_a m_a^d(\bar{p})$, is positive. Money prices are determined by the money market. Lastly, the homogeneity properties of the system imply that, say, a doubling of the quantity of money M leads to a doubling of equilibrium money prices and nominal money balances, leaving unchanged relative prices and real quantities exchanged.

Whether the system (1.1.A) and (1.1.B) is intended to represent the behavior of an economy with outside money in the short run or in the long run (i.e., along stationary states) has been the object of some debate. In particular, Patinkin (1965) claimed that (1.1.A) and (1.1.B) cannot describe the short-run determination of equilibrium prices. He argued that a change of the prices p that prevail at a given date alters the purchasing power of the money stocks $\bar{m}_a$ that the traders own at the outset of the period, and that it should accordingly influence in particular their short-run demands for goods. This is the *real balance effect*, which is clearly absent from the classical system (1.1.A), (1.1.B). A related criticism made by Patinkin is that agents face a budget constraint. If the system (1.1.A), (1.1.B) applies to the short period, it should therefore satisfy *Walras's Law*

$$\sum_a p z_a(p) + \sum_a m_a^d(p) = \sum_a \bar{m}_a = M$$

for every p. But adding this identity to the system leads to major drawbacks. Together with Say's Law, this identity implies $\Sigma_a m_a^d(p) = M$ for every p, in which case any solution of (1.1.A) fulfils (1.1.B): The level of money prices becomes indeterminate. Even if one puts aside Say's Law, Walras's Law contradicts the assumed homogeneity properties of the functions $z_a(p)$ and $m_a^d(p)$. For these imply that the left-hand side of the foregoing identity is homogenous of degree 1 in prices, whereas the right-hand side is a constant independent of them.

These arguments led Patinkin to the conclusion that any consistent monetary theory applicable to short-period problems must incorporate real balance effects. Modern neoclassical short-run macroeconomic models are built around the same idea.[4]

In order to make precise the structure of these models in our simple framework, let us consider our economy at a given date, say date 1, which we may call the "current period." Following Friedman (1956) and others, this line of theorizing emphasizes that money should be treated as a particular capital good. According to this viewpoint, the agents' demand functions at date 1 should depend in the present context on their initial money wealth (i.e., on their initial money holdings $\bar{m}_a$), on their current and expected real incomes (which we take here as exogenous), on the current prices of goods p_1, and on the prices that traders forecast for the future. Most theorists of this school postulate that expected prices are given by $(1 + \pi_a)^{t-1}p_1$ for $t = 2, 3, \ldots$, where π_a is the "expected rate of inflation" of trader a. What characterizes neoclassical models, however, is that the traders' expected rates of inflation are taken as exogenous in the short run; that is, they are assumed to be independent of the current price system p_1. A trader's excess demand for current goods and current demand for money can then be written $z_a(p_1,\bar{m}_a)$ and $m_a^d(p_1,\bar{m}_a)$, if one keeps implicit the influence of the exogenously given parameters π_a. A short-run monetary Walrasian equilibrium price system is then a solution of the following system of equations:

(1.1.C) $\sum_a z_a(p_1,\bar{m}_a) = 0$

(1.1.D) $\sum_a m_a^d(p_1,\bar{m}_a) = \sum_a \bar{m}_a$

According to this viewpoint, the foregoing system obeys Walras's Law

$$p_1\sum_a z_a(p_1,\bar{m}_a) + \sum_a m_a^d(p_1,\bar{m}_a) = \sum_a \bar{m}_a$$

for every p_1, as a consequence of the budget restraints that the agents face at date 1. Moreover, neoclassical theorists remark that a doubling of a trader's initial money holding $\bar{m}_a$ and of current prices p_1 – and thus of expected prices – leaves unaltered the "real" opportunities available to him, and thus should not change his "real" behavior if he is free of "money illusion." They postulate accordingly that *the short-run excess*

[4] For a good account of neoclassical macroeconomic models, see, e.g., Sargent (1979, Chap. 1).

demand functions for goods $z_a(p_1,\bar{m}_a)$ *are homogenous of degree 0, and the short-run money demand functions* $m_a^d(p_1,\bar{m}_a)$ *are homogenous of degree 1 in* $(p_1,\bar{m}_a)$.

The following propositions, which are immediate consequences of these assumptions, are central to neoclassical monetary theory.

First, the classical dichotomy is invalid in the short run. For Walras's Law implies that any solution of p_1 of (1.1.C) satisfies (1.1.D). Consideration of the real sector alone determines not only relative prices and real variables, but also the level of money prices and all nominal values, in contradiction to what the classicists claimed.

Second, quantity theory must be reformulated in order to be valid in the short run. It is clear that a change of the money stock that alters the distribution of initial money holdings $\bar{m}_a$ among agents has distributional effects and thus is likely to modify relative prices. On the other hand, an equiproprotionate change in initial money balances (every $\bar{m}_a$ being changed to $\lambda\bar{m}_a$) will change in the same proportion the equilibrium level of money prices and nominal money balances at the end of the period, but will leave unaltered relative prices and real variables, whenever the functions $z_a(p_1,\bar{m}_a)$ and $m_a^d(p_1,\bar{m}_a)$ are assumed to be homogenous of degree 0 and 1, respectively, with respect to $(p_1,\bar{m}_a)$. Indeed, under these homogeneity assumptions, if p_1 is a solution of (1.1.C) and (1.1.D) when initial money holdings are $\bar{m}_a$, λp_1 must be a solution of the same equations when every initial money stock $\bar{m}_a$ is multiplied by λ.

Finally, the essential short-run regulating mechanism in the neoclassical system (1.1.C), (1.1.D) is the real balance effect. To see this point more precisely, let us ignore the modifications of relatives prices, and consider accordingly the "macroeconomic" case where there is only one good ($l = 1$). In view of the assumed homogeneity properties of the functions z_a and m_a^d, each trader's excess demand for goods and demand for "real money balances" can be written:

$$z_a(p_1,\bar{m}_a) = z_a(1,\bar{m}_a/p_1)$$

and

$$m_a^d(p_1,\bar{m}_a)/p_1 = m_a^d(1,\bar{m}_a/p_1)$$

A variation of p_1 influences the trader's real behavior only through a variation of the initial real balance $\bar{m}_a/p_1$, that is, only through the real balance effect.

The neoclassical argument to assert the existence of a solution to (1.1.C), (1.1.D) then goes as follows. If the good is not inferior, aggregate excess demand in (1.1.C) is a decreasing function of p_1. If the price is low, initial real balances are large, and that should generate an excess demand on the good market. Conversely, if p_1 is large, initial real balances are low, and that should generate an excess supply of the good. By continuity, a unique value of p_1 should exist that achieves equilibrium of the good market, and thus by Walras's Law, of the money market as well. Furthermore, the unique short-run Walrasian monetary equilibrium is stable in any *tâtonnement* process.

That sort of argument is apparently viewed as theoretically valid by many macroeconomists today. As a matter of fact, the controversies have focused mainly on its empirical relevance. Many economists believe that real balance effects are weak, and thus can be neglected in practice. One may note, however, that the foregoing argument is only heuristic, and that it does not provide a consistent proof of the existence of a short-run monetary equilibrium, as Hahn (1965) pointed out some time ago. Moreover, it is based on quite restrictive assumptions on expectations. In particular, it neglects the potential short-run variations of the traders' expected rates of inflation π_a as a function of the current price system p_1, and thus ignores the corresponding intertemporal substitution effects, which seemed important to earlier writers such as Hicks (1946) or Lange (1945).

It has also been argued that, while Patinkin's critique of the classic views was apparently valid for a short period, the classical dichotomy and quantity theory retained their full force when applied to long-run (i.e., stationary) monetary phenomena (Archibald and Lipsey 1958, Samuelson 1968). According to this view, the system of equations (1.1.A), (1.1.B), is not intended to represent the result of an adjustment of prices within a given period nor over time. Rather, the functions $z_a(p)$ and $m_a^d(p)$ appearing in these equations should be interpreted as describing the stationary net trades and money stocks detained by the agents along stationary states, when prices remain constant and equal to p.

This brief review shows that, although a great deal of work has been done, a fully consistent integration of money and value theory in a neoclassical framework is still needed. The primary task of this chapter is to look at this matter. Sections 1.2–1.5 will be devoted to the study of the short-run behavior of the economy. Stationary states will be analyzed in Section 1.6. Finally, the question of the neutrality of changes of the money supply will be taken up in Section 1.7.

1.2 Structure of the model

Consider a simple exchange economy, where time is divided into infinitely many discrete periods. There are l consumption goods available in each period. The simplifying assumption that the agents' real income is fixed is expressed by the fact that each agent owns in each period an exogenously given endowment of consumption goods. The endowments cannot be stored, and must be traded and consumed within the period during which they are available. On the other hand, paper money is the only store of value, and its stock is constant over time. Thus, at any date, the traders (consumers) come to the market with their endowment of goods, and their (nonnegative) cash balances carried over from the past. The short-run competitive equilibrium of the markets at that date will determine Walrasian money prices of the goods $p = (p_1, \ldots, p_l)$, the consumers' net trades in the good markets and the nonnegative money balances they will hold until the next period.

The framework in which we choose to work is that of an overlapping generation model, without bequest (Samuelson 1958). There are accordingly various "types" of consumers. Each type is described by: the number of periods during which agents of this type live, the endowments of consumption goods that these agents own in each period of their lives, and their preferences among consumption streams during their lifetime. An important feature of the model is the fact that there are always "newborn" agents coming into the market at any date. Thus when an agent wishes to get rid of his cash balances at some time in his life, there will be always younger agents living in the same period for whom money may have value for saving purposes.

We will first consider this economy in a given period, which we call period 1, and study its properties in the short run (Sections 1.3–1.5). At this stage, there is no need to be specific about the characteristics of each "type" of agent nor about the demographic structure of the model. We shall make such a specification in Section 1.6, when studying stationary states of this economy. For the moment, what we need to know are the characteristics of every agent a living in the period under examination, i.e.:

(i) the number n_a of remaining periods for which the agent is going to live, including the current one;
(ii) the agent's preferences, represented by a utility function u_a which depends upon current and future consumption c_t, $t = 1, \ldots, n_a$, where c_t is a vector with l nonnegative components;
(iii) the agent's endowment of consumption goods, e_{at}, in every remaining period of his or her life, $t = 1, \ldots, n_a$, where again, e_{at} is a vector with l components;
(iv) the money stock $\bar{m}_a$ the agent owns at the outset of period 1, which is the result of past saving and consumption decisions.

From this point on we shall make the following traditional assumptions:

(a) *The utility function* u_a *is continuous, increasing, and strictly quasi-concave, for every* a;

(b) *All components of the endowment vector,* e_{at}, *are positive for every* a *and* t;

(c) *Each initial money stock* $\bar{m}_a$ *is nonnegative, and the total money stock* $M = \Sigma_a \bar{m}_a$ *is positive.*

1.3 Short-run demand functions

Consider a typical agent at date 1, the "current period." The agent's problem (dropping the subscript a for convenience) is to choose his current consumption of goods $c_1 \geqq 0$, current money holdings $m_1 \geqq 0$, and to plan future consumptions $(c_2, \ldots, c_n) \geqq 0$ and money holdings $(m_2, \ldots, m_n) \geqq 0$. If current money prices of goods are represented by the vector p_1, and if the agent expects the prices $p_2, \ldots, p_n$ to prevail in the future, this choice will be the solution of the following problem.

(1.3.I) *Maximize* $u(c_1, \ldots, c_n)$ *with respect to* $(c_1, \ldots, c_n) \geqq 0$ *and* $(m_1, \ldots, m_n) \geqq 0$ *subject to the current and expected budget constraints:*

$$p_1 c_1 + m_1 = p_1 e_1 + \bar{m}$$

$$p_t c_t + m_t = p_t e_t + m_{t-1} \qquad (t = 2, \ldots, n)$$

This decision-making problem has a solution, which is unique, when current and expected prices are positive. This solution gives rise to an excess demand for consumption goods $c_1 - e_1$ and a demand for money m_1 that are actually expressed on the market (plans for the future remain in the mind of the trader). These demands depend upon initial cash holdings $\bar{m}$, on current prices p_1, on the sequence of expected prices p_t, and on current and future endowments of goods $e_1, \ldots, e_n$.

The solution of (1.3.I) displays very simple and straightforward homogeneity properties with respect to the initial money stock and the sequence of current and expected prices. Consider a change of $\bar{m}$ in $\lambda\bar{m}$, of p_1 and p_t in λp_1 and λp_t $(t = 2, \ldots, n)$, and call (1.3.I′) this new problem. No "real" change has been made in the constraints faced by the agent. In fact, it can easily be verified that $(c_1, \ldots, c_n)$ and $(m_1, \ldots, m_n)$ are solutions of (1.3.I) if and only if $(c_1, \ldots, c_n)$ and $(\lambda m_1, \ldots, \lambda m_n)$ are solutions of (1.3.I′), a property that we can call the *absence of money illusion*. We have obtained, in particular:

(1.3.1) *The excess demand for goods* $c_1 - e_1$ *arising from* (1.3.I) *is homogenous of degree* 0 *in the initial money stock* $\bar{m}$, *current prices* p_1 *and expected prices* $p_2, \ldots, p_n$. *The corresponding money demand* m_1 *is homogenous of degree* 1 *in the same variables.*

In order to complete our specification of a trader's behavior in the short run, it is necessary to describe how price expectations are formed. The agent's expectations are functions of his information on past history and on the current state of the economy. Since past history is fixed in a short-period analysis and cannot be altered by current events, we shall not explicitly mention its influence at the formal level. On the other hand, we shall assume that the only information our agent has on the current state of the economy is described by the current price system p_1, and shall write expected prices p_t as a function $\psi_t(p_1)$ $(t = 2, \ldots, n)$. Expected prices are thus independent of the agent's own actions. This formulation is warranted in a competitive framework, where the number of traders is implicitly assumed to be large, since in that case every agent can have only a negligible influence on market prices by varying his own decisions.

Let us call (1.3.II) the problem obtained from (1.3.I) by changing p_t in $\psi_t(p_1)$ for $t = 2, \ldots, n$:

(1.3.II) *Maximize* $u(c_1, \ldots, c_n)$ *with respect to* $(c_1, \ldots, c_n) \geqq 0$ *and* $(m_1, \ldots, m_n) \geqq 0$ *subject to:*

$$p_1 c_1 + m_1 = p_1 e_1 + \bar{m}$$

$$\psi_t(p_1) c_t + m_t = \psi_t(p_1) e_t + m_{t-1} \qquad (t - 2, \ldots, n)$$

The solution to this problem yields an excess demand for goods $c_1 - e_1$ and a demand for money m_1, which are expressed by the agent on the market in response to p_1. They depend upon the initial money stock $\bar{m}$ and upon current prices p_1 (and implicitly, on the trader's information on past history as well as on current and future endowments of goods). We can write them as $z_a(p_1,\bar{m}_a)$ and $m_a^d(p_1,\bar{m}_a)$ respectively, reintroducing finally the agent's subscript a.

We next discuss various properties of short-run demand functions. First, every agent must fulfill his current budget constraint, $p_1 z_a(p_1,\bar{m}_a) + m_a^d(p_1, \bar{m}_a) = \bar{m}_a$. Therefore aggregate excess demands satisfy Walras's Law:

$$p_1 \sum_a z_a(p_1 \bar{m}_a) + \sum_a m_a^d(p_1,\bar{m}_a) = \sum_a \bar{m}_a \quad \text{for every } p_1$$

On the other hand, short-run aggregate excess demand for goods does not in general satisfy Say's Law, $p_1 \Sigma_a z_a(p_1,\bar{m}_a) = 0$ for every p_1 since at some prices traders can find it profitable to save or dissave by adding to or subtracting from their initial cash balances $\bar{m}_a$.

Second, we must examine if the functions $z_a(p_1,\bar{m}_a)$ and $m_a^d(p_1,\bar{m}_a)$ are homogenous of degree 0 and 1 respectively with respect to current prices p_1 and initial cash balances $\bar{m}_a$, as neoclassical monetarists assume. In view of the absence of money illusion property stated in (1.3.1), the answer is in general no, unless the agents' expected prices $\psi_{at}(p_1)$ are unit elastic with respect to current prices, that is to say, $\psi_{at}(\lambda p_1) = \lambda\psi_{at}(p_1)$ for every p_1 and λ, and for every t. This is in particular the case if one assumes that expected prices are always equal to current ones, as Patinkin did (static expectations), or that every agent's expected rate of inflation π_a depends only in the short run on past history but not on current prices ($\psi_{at}(p_1) = (1 + \pi_a)^{t-1} p_1$ for every t), as neoclassical writers do. Such assumptions appear therefore to be highly specific: strict proportionality of expected prices with respect to current ones is quite unlikely, since expectations depend on the sequence of past prices as well. To sum up:

(1.3.2) *The functions* $z_a(p_1,\bar{m}_a)$ *and* $m_a^d(p_1,\bar{m}_a)$ *are homogenous of degree* 0 *and degree* 1, *respectively, with respect to* p_1 *and* $\bar{m}_a$ *if expected prices are unit elastic with respect to current prices* ($\psi_{at}(\lambda p_1) = \lambda\psi_{at}(p_1)$ *for every* p_1 *and* λ, $t = 2, \ldots, n_a$).

Finally, this formulation allows us to understand on a more precise basis the consequences of a change in the level of current prices on a trader's excess demand z_a. Let us ignore the complications that arise from the possible modifications of relative current prices and/or of relative expected prices, and look at the macroeconomic case where there is only one good ($l = 1$). Consider a variation of the current price from p_1 to λp_1, and the induced change of trader a's excess demand for the good:

$$\Delta z_a = z_a(\lambda p_1,\bar{m}_a) - z_a(p_1,\bar{m}_a)$$

We can split Δz_a in two parts:

$$\Delta z_a = \Delta' z_a + \Delta'' z_a$$

where $\Delta' z_a$ stands for the variation of excess demand that would occur when the current price changes from p_1 to λp_1, if the trader's expected prices had moved proportionally to p_1 – that is, from $\psi_{at}(p_1)$ to $\lambda\psi_{at}(p_1)$. The second term $\Delta'' z_a$ represents then the variation of excess demand that results from the change of expected prices from $\lambda\psi_{at}(p_1)$ to their true values $\psi_{at}(\lambda p_1)$, the current price being kept at the level λp_1.

The first term $\Delta' z_a$ represents the real balance effect. In view of the absence of money illusion property stated in (1.3.1), multiplying current and expected prices by λ, $\bar{m}_a$ being fixed, has the same consequence on excess demand for the current good as dividing the initial money balance $\bar{m}_a$ by λ, current and expected prices being kept at their initial level. What happens then is that the purchasing power of the initial money stock has been actually divided by λ. The real balance effect is thus measured by:

$$\Delta' z_a = z_a(p_1, \bar{m}_a/\lambda) - z_a(p_1, \bar{m}_a)$$

When the good is not inferior, an increase of p_1 generates, through the real balance effect, a decrease of the demand for the current good.

The second term,

$$\Delta'' z_a = z_a(\lambda p_1, \bar{m}_a) - z_a(p_1, \bar{m}_a/\lambda)$$

vanishes when $\psi_t(\lambda p_1) = \lambda \psi_t(p_1)$ for every t. This fact is indeed equivalent to (1.3.2) above and justifies the viewpoint of neoclassical macroeconomists, who exclusively consider real balance effects, since they assume explicitly or implicitly that price expectations are unit elastic.

The elasticity of price expectations with respect to current prices, however, differs typically from unity. In that case the second term $\Delta'' z_a$ must be taken into account. We shall call it, following Hicks and Lange, the *intertemporal substitution effect*, since it measures the consequence of the modification of expected prices relative to the current price. But the reader must keep in mind that it involves "income" as well as *stricto sensu* "substitution" effects, as in the traditional Slutsky equation.

Suppose now that the current price goes up from p_1 to λp_1, and that the elasticity of price expectations is greater than one, so that expected prices $\psi_{at}(\lambda p_1)$ are greater than $\lambda \psi_{at}(p_1)$. In that case, the intertemporal substitution effect is likely to favor an increase of current consumption, and thus to counteract the real balance effect. Conversely, when the elasticity of price expectations is less than unity, the intertemporal substitution effect is likely to reinforce the real balance effect.

The following example permits a simple representation of the properties of short-run demand functions. Let us assume that there is only one real good ($l = 1$), and consider a typical consumer who is planning for the current period and the next one only. If current and expected prices are p_1 and p_2, the consumer's current and expected budget constraints are (dropping the subscript a for simplicity):

$$p_1 c_1 + m_1 = p_1 e_1 + \bar{m}$$

$$p_2 c_2 + m_2 = p_2 e_2 + m_1$$

It is convenient to rewrite these constraints by eliminating the variables m_1 and m_2. By adding the two equalities and by taking into account the fact that $m_2 \geqq 0$, one gets the intertemporal budget constraint:

(i) $$p_1 c_1 + p_2 c_2 \leqq p_1 e_1 + p_2 e_2 + \bar{m}$$

The fact that m_1 must be nonnegative yields the liquidity constraint:

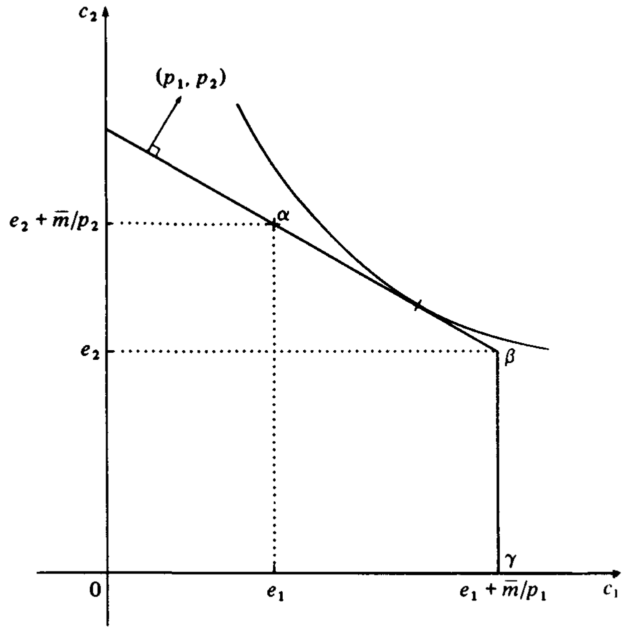

FIGURE 1.1.

(ii) $\quad p_1c_1 \leqq p_1e_1 + \bar{m}$

The optimum current and future consumptions of the agent are thus obtained by maximizing the utility function $u(c_1,c_2)$ under the two constraints (i) and (ii). The associated demand for money is then given by $m_1 = \bar{m} + p_1e_1 - p_1c_1$, or by $m_1 = p_2c_2 - p_2e_2$.

The consumer's opportunity set described by the constraints (i) and (ii), as well as the result of the utility maximization, are pictured in Figure 1.1, in the plane (c_1,c_2). There, the line going through the points α and β represents the intertemporal budget constraint (i), and the vertical line $\beta\gamma$ represents the constraint (ii).

The absence of money illusion property stated in (1.3.1) can easily be verified in Figure 1.1. Consider a change of $\bar{m}$, p_1, p_2 in $\lambda\bar{m}$, λp_1, and λp_2. The change obviously leaves unaltered the coordinates of the points α and β. This means that the opportunity set described by (i) and (ii) is unchanged. The optimum values of c_1 and c_2 are accordingly the same. This is exactly what was stated in (1.3.1).

The decomposition between a real balance effect and an intertemporal substitution effect of the impact of an increase of p_1 on the trader's

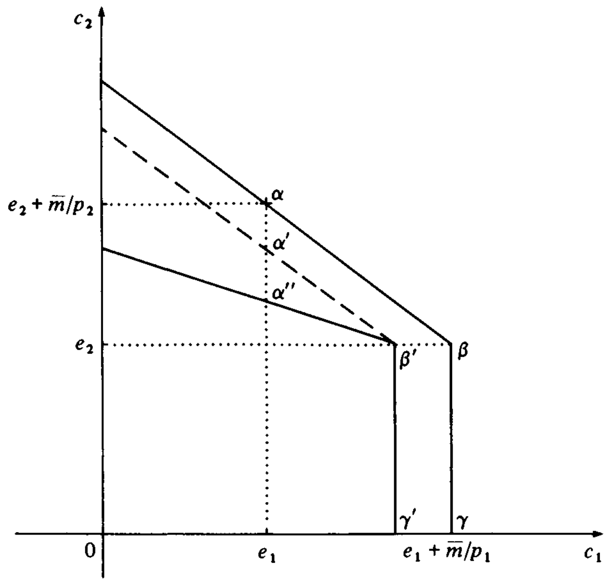

FIGURE 1.2.

behavior is easily visualized by using Figure 1.2. When price expectations are unit elastic, an increase of the current price p_1 causes a horizontal displacement toward the left of the lines $\alpha\beta$ and $\beta\gamma$, which then become $\alpha'\beta'$ and $\beta'\gamma'$, the slopes of these lines being unchanged. This generates a pure income or real balance effect. When price expectations are not unit elastic, there is in addition a rotation of the intertemporal budget line $\alpha'\beta'$ around the point β', downward if the elasticity exceeds 1, upward if the elasticity is less than 1. This rotation generates what we called the intertemporal substitution effect.

1.4 The existence of a short-run Walrasian monetary equilibrium

A short-run Walrasian monetary equilibrium in period 1 obtains when the current price system p_1 achieves equality of supply and demand in the goods and the money markets. By taking into account the demand functions $z_a(p_1,\bar{m}_a)$ and $m_a^d(p_1,\bar{m}_a)$ constructed in Section 1.3, this leads us to define such an equilibrium price system as a solution of a system of equations such as (1.1.C) and (1.1.D):

(1.1.C) $\sum_a z_a(p_1,\bar{m}_a) = 0$

(1.1.D) $\sum_a m_a^d(p_1,\bar{m}_a) = \sum_a \bar{m}_a$

These equations obey Walras's Law, as in the neoclassical system. By contrast, we do not assume here any specific homogeneity properties.[5]

The neoclassical conclusions about the invalidity of the classical dichotomy in the short run apply equally well here, since by Walras's Law any solution of (1.1.C) fulfills (1.1.D): the equilibrium of the real sector determines not only relative prices, but also the level of money prices. But the main question is to find the conditions under which this system of equations has indeed a solution.

In order to study this problem, it is most convenient to look at the simple case in which there is only one good ($l = 1$), so that equations (1.1.C) and (1.1.D) are in fact equivalent. The usual argument to assert the existence of a solution to this system goes as follows:

(i) If the price p_1 is low enough, there is an excess demand for the good (or equivalently, an excess supply for money);

(ii) If conversely p_1 is large, there is an excess supply of the good (equivalently, an excess demand for money).

Then, by continuity, there would exist a value of p_1 achieving equilibrium on both markets.

Neoclassical macroeconomists focus attention on the real balance effect by restricting price expectations to be unit elastic. Moreover, they usually claim, as we recalled in Section 1.1, that the real balance effect is strong enough in the present context to bring about the above properties of the aggregate excess demand function. We shall see that this argument is wrong, and that there are theoretically conceivable and empirically plausible circumstances in which the real balance effect is too weak to equilibrate the market. What is actually needed is a strong intertemporal substitution effect in order to reinforce it.

In order to see this point more precisely, let us consider the simple example given in Section 1.3, where there is only one good and where every consumer is planning only one period ahead. The choices open to a typical agent were represented in the plane (c_1,c_2) in Figure 1.1, which forms the basis of Figure 1.3.

It is clear from Figure 1.3 that the agent's demand for current consumption c_1 will exceed his endowment e_1 if and only if the slope of the normal to the intertemporal budget line $\alpha\beta$, that is p_2/p_1, exceeds the

[5] In view of our discussion of the neoclassical system in Section 1.1, it would seem that the short-run quantity theory no longer holds, and that money is not in general neutral in the short run. The issue is in fact more subtle. We shall go back to it in Section 1.7 below.

marginal rate of substitution u'_2/u'_1 evaluated at the point α.[6] If we assume that a typical trader's utility function can be written $w(c_1) + \delta w(c_2)$, where w is strictly concave and differentiable, and δ is a parameter between 0 and 1, this fact can be expressed by:

$$c_1 - e_1 > 0 \qquad \text{if and only if} \qquad \frac{p_2}{p_1} > \delta \, \frac{w'(e_2 + \bar{m}/p_2)}{w'(e_1)}$$

It is quite easy by using this simple result to design examples where there is a persistent disequilibrium on the good market for all values of the current price.

Example 1: Persistent excess demand

Assume that a typical trader's expectations are biased upwards, so that the ratio p_2/p_1 is greater than or equal to the marginal rate of substitution at the endowment point (e_1, e_2), that is $p_2/p_1 \geqq \delta\,[w'(e_2)/w'(e_1)]$ for all p_1. Since w' is a decreasing function, the trader's demand for consumption c_1 will then always exceed his endowment e_1. If *all* traders' expectations are biased upward in this way, there will be an aggregate excess demand on the good market at all values of the current price p_1, and no short-run Walrasian equilibrium in which money has positive value can exist.

The phenomenon just described may occur in particular when price expectations are unit elastic with respect to the current price, that is, when the ratio p_2/p_1 is independent of p_1. The real balance effect is then the sole regulating mechanism of the economy, but it is too weak to bring the market into equilibrium. This conclusion can be valid for small expectational "inflationary bias," since the ratio p_2/p_1 need not be very large. In particular, the phenomenon occurs in the case of static expectations ($p_2 = p_1$ for every p_1), if the marginal rate of substitution at the endowment point is less than or equal to 1.

Example 2: Persistent excess supply

A similar story can be told in the case of "deflationary" expectations. Assume for instance, that the marginal rate of substitution evaluated at various points of the vertical line going through the endowment point in Figure 1.3 is bounded below by some positive number ν. If the consumer's expectations are biased downward, so that $p_2/p_1 < \nu$ for every p_1, the

[6] It is assumed for the simplicity of the argument that every agent's money endowment $\bar{m}$ is positive.

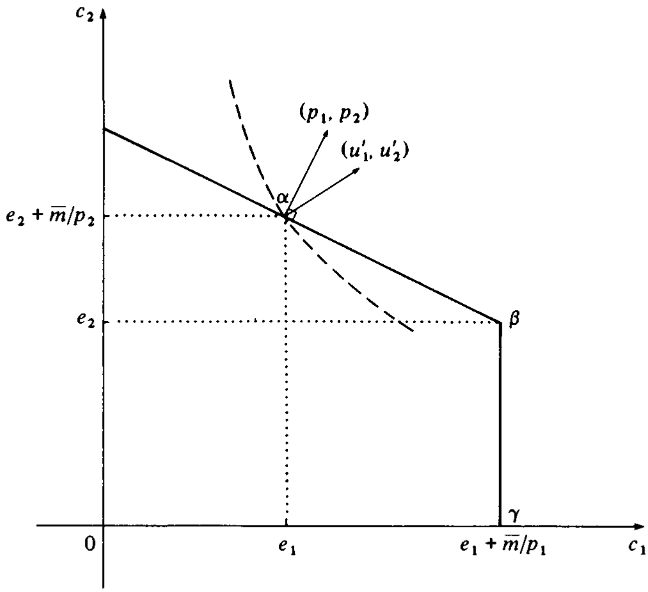

FIGURE 1.3.

agent's demand for current consumption c_1 is less than his endowment e_1 for every p_1. If all traders' price expectations are biased downward in this way, there is an aggregate excess supply on the good market for all values of the current price system p_1, and no short-run Walrasian equilibrium can exist. Again, the phenomenon can occur when price expectations are unit elastic with respect to the current price. The real balance effect, which is then the only regulating mechanism of the economy, is too weak in that case to equilibrate the market.

Many other examples can be designed along these lines. For instance, consider a utility function of the type $c_1^\lambda + \delta c_2^\lambda$, where λ and δ are parameters between 0 and 1. Then straightforward computations show that $c_1 - e_1 > 0$ if and only if $p_1 < f(p_2)$ where:

$$f(p) = (1/\delta e_1^{1-\lambda}) p^\lambda (pe_2 + \bar{m})^{1-\lambda}$$

The function f increases from 0 to infinity when p varies from 0 to infinity, and therefore has an inverse. Then an excess demand on the good market will exist at all current prices p_1 if $p_2 > f^{-1}(p_1)$ for every p_1, an excess supply if $p_2 < f^{-1}(p_1)$ for all p_1.

These examples can of course be transposed to the case of several goods. For instance, consider the case where a typical trader's utility function is of the form $w(c_1) + \delta w(c_2)$, where w is strictly concave and δ between 0 and 1. Assume moreover that the trader's real income is constant over time ($e_1 = e_2$) and that he has static expectations ($p_2 = p_1$). It is then straightforward to check that the value of the trader's excess demand for current consumption, $p_1(c_1 - e_1)$, always exceeds $\bar{m}/2$. If all traders satisfy these conditions, the goods markets cannot be brought simultaneously into equilibrium. The reader will easily design other examples along these lines.

The above examples show that *the real balance effect may be too weak,* and that *it must be reinforced by a strong intertemporal substitution effect if one wishes to be able to equilibrate the market.* As we shall see, *this essentially requires in the present model that the expected prices of at least one agent be substantially insensitive to current prices.*

To verify this point, let us return to the simple case where there is only one good and where all traders are planning only one period ahead, and let us assume that there is a particular consumer whose price expectations are "insensitive" to the current price, in the sense that two positive real numbers ϵ and η exist such that $\epsilon \leqq \psi(p_1) \leqq \eta$, for all p_1.

We hope that an excess supply appears on the good market when its price p_1 is large. Let us consider accordingly what happens when p_1 tends to $+\infty$. Money balances being necessarily nonnegative, every agent's excess demand for the good, $c_1 - e_1$ must not exceed his initial real balance $\bar{m}/p_1$, which tends to 0 as p_1 tends to infinity. Intuitively, it suffices that the demand c_1 of our "insensitive" agent eventually becomes less than his endowment e_1 in order that an excess supply appears on the good market at the aggregate level. This is precisely what happens. Indeed, if we look at Figure 1.3 and view it as describing the behavior of the insensitive trader, we see that the point β tends to the endowment point (e_1, e_2) and that the intertemporal budget line tends to be almost vertical. The intertemporal substitution effect favors future consumption against current consumption. The insensitive trader's planned consumption c_2 actually goes to $+\infty$, as well as his current demand for money $m_1 = \psi(p_1)\, c_2 - \psi(p_1)\, e_2$. There is eventually an aggregate excess demand for money, and thus, by Walras's Law, an aggregate excess supply of the good when p_1 tends to $+\infty$.

Let us consider next what happens when the price p_1 goes to 0, and look again at Figure 1.3. For the "insensitive" trader, the point β in the figure goes to the far right, whereas α stays at a finite distance. The insensitive agent's intertemporal budget line becomes almost horizontal as p_1 decreases: the intertemporal substitution effect favors current consumption relative to future consumption. Since $\bar{m}$ is positive, the agent's demand for current consumption actually goes to infinity, and this is sufficient to generate an excess demand on the good market at the aggregate level.

We can therefore conclude that, by continuity, a short-run monetary equilibrium does exist in such a case.[7]

In order to describe a formal result along this line on the existence of a short-run equilibrium, we introduce a few definitions. Let us say that whenever $n_a \geqq 2$, agent a's price expectations are *continuous* if the functions $\psi_{at}(p_1)$ are continuous in p_1, for every t. An agent's price expectations are said to be *bounded* if there are two vectors ϵ and η, with all their components positive, such that $\epsilon \leqq \psi_{at}(p_1) \leqq \eta$ for every current price system p_1 and every t. The last condition, which is of course incompatible with the neoclassical assumption of unit elastic price expectations, ensures the presence of a strong intertemporal substitution effect, which reinforces the real balance effect. It is the key condition for the following existence theorem.[8]

(1.4.1) *Assume (a), (b), and (c) of Section 1.2. Assume moreover that every agent's price expectations are continuous, and that there is at least one agent* a, *with* $n_a \geqq 2$ *and* $\bar{m}_a > 0$ *whose price expectations are bounded. Then, there exists a short-run Walrasian monetary equilibrium.*

The general idea that underlies this result and the arguments that led to it is that one needs a strong intertemporal substitution effect that reinforces the real balance effect in order to be sure of the existence of a short-run monetary equilibrium. This essentially requires that some agent's price forecasts display a substantial degree of insensitivity to large variations of current prices. The insensitive traders are then there to act as a flywheel, and to stabilize the market process. That such conditions can be met in actual market economies is hardly to be expected. Price forecasts are indeed somewhat volatile, and are presumably quite sensitive to the level of current prices. The traders' expected rates of inflation may be biased upward when a significant inflation has been observed in the recent past, and downward in the case of a deflation. The examples we gave show that a short-run equilibrium may not exist

[7] What was actually used in this simple example, as the careful reader may have noticed, was that the insensitive trader's expected price is bounded below by $\epsilon > 0$, and that the ratio $\psi(p_1)/p_1$ tends to 0 when p_1 goes to $+\infty$. In the more realistic case where $l \geqq 2$ and $n \geqq 3$, matters are less simple, and one has to resort to the condition of the text.

[8] A formal proof of the theorem is given in Appendix B.

in such circumstances. The general conclusion that emerges from this analysis is accordingly that *the existence of a short-run Walrasian equilibrium, in which money has positive value is somewhat problematic in actual market economies, contrary to what neoclassical economists like to believe.*

1.5 The expected utility of money[9]

It is common practice among monetary theorists to write down a utility function for each agent depending on consumption and "real" balances, on the ground that money, for instance, renders services as a liquid asset and a medium of exchange. Short-run demand functions are then obtained as the result of maximizing such utility functions under the relevant budget constraints. The aim of this section is to describe a method of general applicability that justifies this procedure, provided that the indirect utility of money balances is correctly derived.

Consider again a typical consumer (dropping his subscript a for convenience) who is faced in period 1 by a price system p_1. We are trying to construct an index that would describe his preferences among current consumption $c_1 \geqq 0$ and money balances $m_1 \geqq 0$. Since m_1 represencs a stock, the usefulness of which depends on its purchasing power in the future, such an index will certainly depend upon expected prices, and thus on current prices inasmuch as they determine price expectations. More precisely, consider the maximum level of utility that the consumer can expect to achieve over the remainder of his lifetime if he now chooses c_1 and m_1. This is the result of the following decision problem:

(1.5.III) *Given* $c_1 \geqq 0$, $m_1 \geqq 0$ *and* p_1, *maximize* $u(c_1, c_2, \ldots, c_n)$ *with respect to* $(c_2, \ldots, c_n) \geqq 0$ and $(m_2, \ldots, m_n) \geqq 0$, subject to the expected budget constraints:

$$\psi_t(p_1)c_t + m_t = \psi_t(p_1)\, e_t + m_{t-1} \qquad (t = 2, \ldots, n)$$

The maximum value of the utility function depends upons c_1, m_1 and on current prices through their influence on price expectations (and implicitly on past history, as well as on future real incomes). Let $v(c_1,m_1,p_1)$ be this maximum. It can be interpreted as the *expected utility* of (c_1,m_1) when the price system p_1 is currently quoted.

This expected utility $v(c_1,m_1,p_1)$ is indeed the index that we were looking for. It is in fact easy to verify:

(1.5.1) *Maximizing* $v(c_1,m_1,p_1)$ *with respect to* $c_1 \geqq 0$ *and* $m_1 \geqq 0$ *subject to the current budget constraint* $p_1c_1 + m_1 = p_1e_1 + \bar{m}$, *yields an excess*

[9] This section may be skipped on a first reading without any loss of continuity.

demand $c_1 - e_1$ *and a demand for money* m_1 *which are equal to* $z(p_1\bar{m})$ *and* $m^d(p_1,\bar{m})$.

In order to prove this proposition, consider the unique solution of problem (1.3.II), $(c_1^*, \ldots, c_n^*)$ and $(m_1^*, \ldots, m_n^*)$. One has by definition, $c_1^* - e_1 = z(p_1, \bar{m})$, $m_1^* = m^d(p_1,\bar{m})$ and $p_1c_1^* + m_1^* = p_1e_1 + \bar{m}$. Moreover, the program of future consumptions $(c_2^*, \ldots, c_n^*)$ and of money holdings $(m_2^*, \ldots, m_n^*)$ is the solution of (1.5.III) corresponding to c_1^*, m_1^* and p_1. Therefore, by definition of the expected utility:

$$u(c_1^*, \ldots, c_n^*) = v(c_1^*,m_1^*,p_1)$$

Consider now other values of current consumption and of money holdings, (c_1,m_1), which differ from (c_1^*,m_1^*) and fulfill the current budget constraint, $p_1c_1 + m_1 = p_1e_1 + \bar{m}$. Note that this necessarily implies that c_1 is different from c_1^*. One can associate to (c_1,m_1) the solution $(c_2, \ldots, c_n)$, $(m_2, \ldots, m_n)$ of problem (1.5.III). By definition of the expected utility index:

$$u(c_1, \ldots, c_n) = v(c_1,m_1,p_1)$$

Since the program $(c_1, \ldots, c_n)$, $(m_1, \ldots, m_n)$ fulfills the constraints of problem (1.3.II), and since c_1^* differs from c_1, one must have:

$$u(c_1^*, \ldots, c_n^*) > u(c_1, \ldots, c_n)$$

or equivalently:

$$v(c_1^*,m_1^*,p_1) > v(c_1,m_1,p_1)$$

which proves the claim.

The above construction of the expected utility index $v(c_1,m_1,p_1)$ justifies accordingly the "introduction of money balances in the utility function." Current prices enter the utility function too, since they determine price expectations. We can thus discuss precisely the validity of the neoclassical claim, stating that "only real money balances enter the utility function." The natural counterpart of this statement in the present context would be to say that the expected utility index $v(c_1,m_1,p_1)$ is homogenous of degree 0 with respect to money balance m_1 and current price p_1. If we had let expected prices $p_2, \ldots, p_n$ vary independently of the current price system p_1 in problem (1.5.III), we would have obtained an expected utility index v depending upon c_1, m_1 and the sequences $p_2, \ldots, p_n$ of expected prices, which would have been indeed homogenous of degree 0 in the money balance m_1 and expected prices $p_2, \ldots, p_n$. But once the dependence of price expectations upon current prices is recognized, the conclusion is that $v(c_1,m_1,p_1)$ is *not* in general homogenous of degree 0 with respect to (m_1,p_1) unless price expectations are *unit elastic* with

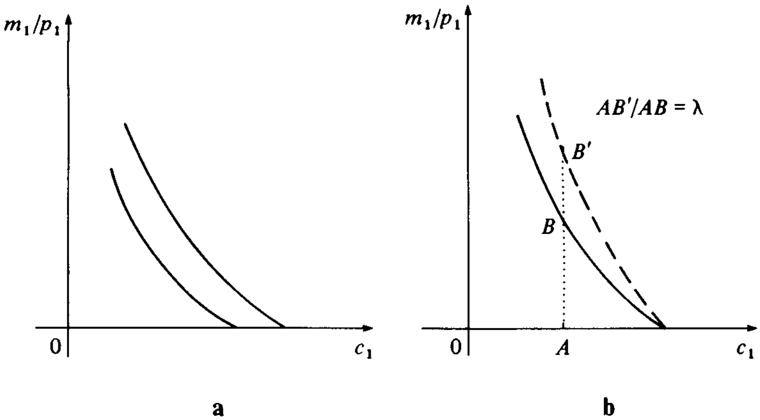

FIGURE 1.4.

respect to current prices. This conclusion is the analog of results (1.3.1) and (1.3.2) on short-run demand functions.

It might be useful to illustrate the concept of an expected utility index by means of an example. Let us consider the case in which there is only one real good ($l = 1$) and in which the typical consumer plans for the current period and the next one only ($n = 2$). If the consumer's money balance is m_1, and if his price forecast is $\psi(p_1)$, he has no freedom of choice in the future, since his consumption c_2 must then equal $e_2 + m_1/\psi(p_1)$. In that case, the expected utility index is:

$$v(c_1,m_1,p_1) = u(c_1,e_2 + m_1/\psi(p_1))$$

Since the expression $m_1/\psi(p_1)$ can be rewritten (m_1/p_1) $[p_1/\psi(p_1)]$, the expected utility index v can be viewed as a function of the current consumption c_1 and the "real balance" m_1/p_1, and of the current price p_1 inasmuch as it determines the ratio $\psi(p_1)/p_1$. Therefore, given p_1, the expected utility index defines a set of indifference curves in the plane $(c_1, m_1/p_1)$ which have the usual shape, as shown in Figure 1.4a.

The reader will easily verify that indifference curves do cut the axis Oc_1. Indeed, the expression of the marginal rate of substitution between current consumption and current real balances at any point of coordinates $(c_1,0)$ is $[p_1/\psi(p_1)]$ (u_2'/u_1'), where the ratio u_2'/u_1' is evaluated at the point (c_1,e_2). This marginal rate of substitution is positive.

It is useful to examine at this stage the effect upon the indifference curves of, say, an increase of the current price from p_1 to p_1'. Consider an indifference curve corresponding to a given level of the expected utility index before the increase of the current price has taken place

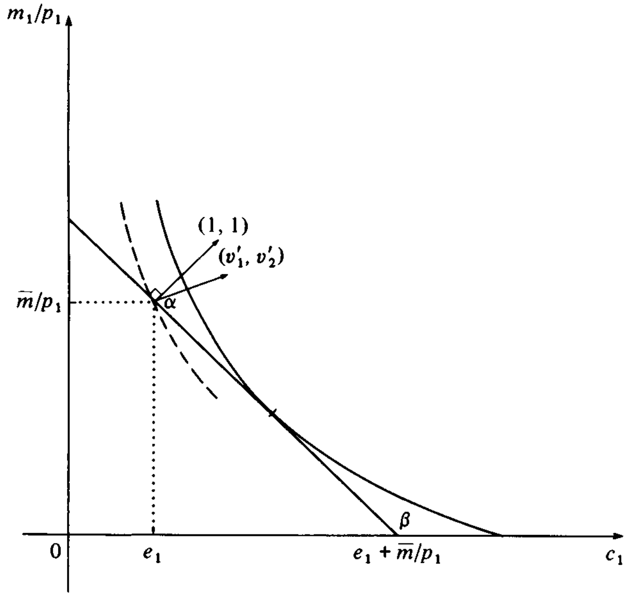

FIGURE 1.5.

(see the plain curve in Figure 1.4b). The new indifference curve associated to the same level of utility is then obtained by applying an affine transformation to the old one, using the axis $0c_1$ and the ratio $\lambda = [\psi(p_1')/p_1'] [p_1/\psi(p_1)]$ (see the dashed curve in Figure 1.4b). In the neoclassical case, that is, when price expectations are unit elastic, λ is equal to 1, and the indifference curves are unaltered. But when the elasticity of price expectations is greater than unity, for instance, the increase of the current price leads to an "upward affine transformation" of the indifference curves, since then λ is greater than 1. Otherwise the transformation takes place in the other direction.

The result stated in (1.5.1) is obvious in this simple case: given p_1, the trader's optimum consumption c_1 and real balance m_1/p_1 is obtained by maximizing the expected utility index subject to the current budget constraint. This budget constraint can be written:

$$c_1 + (m_1/p_1) = e_1 + (\bar{m}/p_1)$$

and is thus represented in the plane $(c_1, m_1/p_1)$ by a line that is perpendicular to the vector (1,1) and goes through the point $(e_1, \bar{m}/p_1)$. The result of the maximization of the expected utility index is shown in Figure 1.5.

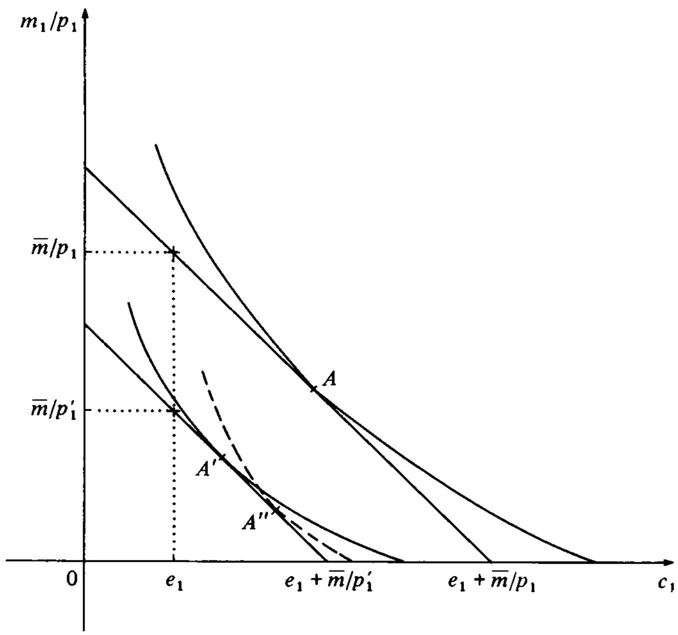

FIGURE 1.6.

It is clear from this diagram that whenever $\bar{m}$ is positive, the trader's optimum consumption c_1 exceeds his endowment e_1 if and only if the marginal rate of substitution between real balances and consumption evaluated at the point $(e_1, \bar{m}/p_1)$ is less than 1. When the utility function u is of the form $w(c_1) + \delta w(c_2)$, where w is strictly concave, differentiable, and where δ is a parameter between 0 and 1, this condition reads:

$$\delta \frac{p_1}{\psi(p_1)} \cdot \frac{w'[e_2 + \bar{m}/\psi(p_1)]}{w'(e_1)} < 1$$

This is of course the same condition that was obtained in the previous section when directly analyzing the trader's intertemporal optimization program.

The relative impacts of the real balance and of the intertemporal substitution effects, which were discussed in Section 1.3, also appear quite clearly on the diagram. Consider an increase of the current price from p_1 to p_1'. The budget line then moves down since it now goes through the point $(e_1, \bar{m}/p_1')$ (Figure 1.6). If price expectations were unit elastic, the indifference curves in the plane $(c_1, m_1/p_1)$ would be unchanged, and the trader's optimum mix of consumption and real balance would move from

A to A', as shown in Figure 1.6. This move corresponds to the real balance effect, which is a pure "income effect," and thus is likely to reduce both the demands for current consumption and for real balances. But when price expectations are not unit elastic, indifference curves are modified as was described above. The point representing the trader's optimum decision moves accordingly from A' to, say, A'' on the new budget line (see Figure 1.6). This move corresponds to the intertemporal substitution effect. When the elasticity of price expectations exceeds 1, there is an upward affine transformation of the indifference curves. The intertemporal substitution effect is thus likely to lead to an increase of current consumption in that case, and thus to counteract the real balance effect.

To conclude, the arguments developed in this section show that "the introduction of money balances in a trader's utility function" is a valid procedure, provided, however, that the utility of money is derived from the trader's intertemporal decision program which lies underneath, as in problem (1.5.III). Once this is done, one can of course choose to work exclusively with the resulting expected utility index, since it embodies all the information that was contained in the trader's intertemporal choice problem.[10] But this analysis points to one of the great dangers of this procedure. For it is greatly tempting to start directly with a utility function that depends upon current consumption, money balance and current prices, and to forget the trader's underlying intertemporal choices. This is in fact what many neoclassical monetarists have done and still do. It is clear that such a neglect renders a precise study of intertemporal substitution effects quite difficult, the importance of which we have stressed at length while studying market equilibrating forces.

1.6 Stationary states

Our goal in this section is to study (monetary) stationary states of the model, that is, sequences of short-run Walrasian monetary equilibria in which the price system is constant over time. Our findings will confirm the view that the classical dichotomy and quantity theory are valid propositions when applied to stationary states. More precisely, we shall show that a stationary equilibrium price system p may be defined as a solution of a system of equations such as the classical system (1.1.A), (1.1.B), displaying the same properties.

[10] In this respect, a useful exercise for the reader would be to transpose the arguments developed in the previous section about the existence, or the nonexistence, of a short-run monetary equilibrium, by using the expected utility index constructed in this section in the plane $(c_1, m_1/p_1)$.

In order to achieve this goal, we need a more detailed specification of the dynamic structure of the model. To fix ideas, we choose to work within the convenient framework of an overlapping generation model without bequest and with a constant population. This is merely a convenience, however, as the conclusions that we shall reach do not depend in an essential way on the specific structure of that sort of model. There are various "types" of agents. Agents of type i are characterized by:

(i) the number $n_i \geqq 2$ of periods of their lifetimes;
(ii) the profile of their real income during their lives, which is described by the endowment of goods $e_{i\tau}$, a vector with l components, that they own in the τth period of their lives, $\tau = 1, \ldots, n_i$;
(iii) their preferences over consumption streams, which are represented by a utility function $u_i(c_{i1}, \ldots, c_{in_i})$, where $c_{i\tau}$, a vector with l nonnegative components, is their consumption in the τth period of their lives.

It is assumed further that, in each period, a "newborn" agent of each type comes into the market. At the same date, an agent of each type who arrives at the end of his life leaves the market. Thus, in each period, there are n_i agents of type i who participate in market activity, each in a different period of his life. The characteristics of the agents of a given type are supposed to be independent of time, that is, they are independent of the date of their "birth."

In this context, a trader a living in a given period, say period 1, is described by his type i and his "age" τ (which means that he is in the τth period of his life). It is then easy to deduce from the characteristics of his type and a knowledge of his past decisions, the short-run characteristics of this agent, as they were introduced in Section 1.2. Indeed, n_a is then equal to the number of periods he still has to live including the present one, i.e., to $n_i - \tau + 1$. The utility function u_a is obtained from u_i by keeping fixed its $\tau - 1$ first arguments at the level of the trader's past consumptions. The endowment vectors e_{at} are equal to his current and future endowments of goods. Lastly, his money balance $\bar{m}_a$ at the outset of the period is equal to the amount of money he decided to keep at the preceding date. Of course, $\bar{m}_a = 0$ for a "newborn" trader ($\tau = 1$), since by assumption there are no bequests in this model.

In the present section we shall use the following assumptions, which are natural counterparts of the assumptions made in Section 1.2 on the short-run characteristics of every agent.

(a) *The utility function* u_i *is continuous, increasing, and strictly quasi-concave, for every* i;

(*b*) *The endowment vectors* $e_{i\tau}$ *have all their components positive, for all* i *and* τ;

(*c*) *The total stock of money* M *is positive.*

The notion of a sequence of short-run equilibria is now intuitively clear. Consider this economy at a given date, say date 1. The economy's past history, in particular past equilibrium prices and the traders' past decisions, are then given. We just saw how these data determine the short-run characteristics of every agent living at date 1. In order to apply the analysis developed in Sections 1.3 and 1.4, all that is needed is to know how price forecasts are formed. This involves the specification of: (i) the information a trader of type i and age τ has in any period about the economy's past history, and (ii) the functional relationship linking his price forecasts for the remaining periods of his life with his information on past history and with current prices. For the purpose of this study, it is not necessary, however, to be precise on these points. It is enough to remark that such a specification implies a relation between expected prices and current prices for every trader, and that the notion of a short-run equilibrium analyzed in the preceding sections applies directly here. Picking up a short-run equilibrium price system at date 1 determines in particular the traders' actions at that date. One can then repeat this procedure at date 2, and so on, and define a sequence of short-run equilibria in this manner.

A *stationary state* in this economy is by definition a sequence of short-run equilibria in which prices remain constant over time. It should be emphasized that, although the analysis carried out in the previous sections casts serious doubts about the existence of a short-run Walrasian equilibrium in general, stationary states are of interest on their own. For they can arise as stationary states of dynamic processes that differ from the one we just described, e.g., from disequilibrium processes in which prices do not clear markets at every date.

It is reasonable to assume that whenever a trader observes at some date that the price system has been the same in the past and in the current period, he believes that the same price system will prevail in the future.[11] This assumption – which implies that traders have "rational" or "correct" expectations along stationary states – permits a very simple derivation of the equations that must be satisfied by stationary equilibrium prices.

[11] This assumption postulates implicitly that a trader's information about the past contains at least past prices. It should be noted that the assumption is a statement on the dependence of expected prices with respect to current *and* past prices. It is therefore compatible with conditions saying that expected prices are to some extent intensive to, or even independent of, current prices, like the conditions used in (1.4.1).

Indeed, along a stationary state in which the price system is p, the consumptions $(c_{i\tau})$ and money holdings $(m_{i\tau})$ of a trader of type i during his lifetime will be solutions of the following program:

(1.6.IV) *Maximize* $u_i(c_1, \ldots, c_{n_i})$ *with respect to* $(c_1, \ldots, c_{n_i}) \geqq 0$ *and* $(m_1, \ldots, m_{n_i}) \geqq 0$, *subject to the budget constraints:*

$$pc_\tau + m_\tau = pe_{i\tau} + m_{\tau-1} \qquad (\tau = 1, \ldots, n_i)$$

(*with the convention* $m_0 = 0$).

When all components of p are positive, (1.6.IV) has a solution that is unique. We can thus write the excess of consumption over endowments $c_{i\tau} - e_{i\tau}$ and the money stocks $m_{i\tau}$ obtained from (1.6.IV) as functions of p, that is, $z_{i\tau}(p)$ and $m_{i\tau}(p)$, for every τ. We shall note $z_i(p) = \Sigma_\tau z_{i\tau}(p)$ and $m_i(p) = \Sigma_\tau m_{i\tau}(p)$.

Consumers forecast correctly the future along a stationary state. Accordingly, in any period, what a newborn trader plans to do in the future in the τth period of his life is precisely what the agent of the same type and of age τ is actually doing in the same period. Therefore $z_i(p)$ and $m_i(p)$ represent the aggregate excess of consumption over endowment of goods, and the aggregate money stock of all agents of type i in each period along the stationary state. It follows that p is a stationary equilibrium price system if and only if it is a solution of the following equations, which express the fact that all markets clear:

$$\text{(1.6.A)} \quad \sum_i z_i(p) = 0$$

$$\text{(1.6.B)} \quad \sum_i m_i(p) = M$$

where the summation sign runs over all types i of agents.

Before looking at the properties of these equations, a comment is in order. Although the functions appearing in the equations (1.6.A) and (1.6.B) do not depend on the quantity of money, their structure depends crucially on the presence of money in the economy. Indeed, if money did not exist as a store of value, the constraints in (1.6.IV) should read $pc_t = pe_{it}$ for every t, which would lead to an entirely different system of equations. For instance, in the case of a single consumption good, the unique solution would be the autarkic one, $c_{it} = e_{it}$.

We proceed now to show that (1.6.A) and (1.6.B) display all the properties of the classical system that we discussed in Section 1.1. First, examination of (1.6.IV) leads to the immediate conclusion that:

The functions $z_i(p)$ *are homogenous of degree 0, and the functions* $m_i(p)$ *are homogenous of degree 1 in prices.*

As a matter of fact, this is nothing other than the absence of money illusion property (1.3.1), since a newborn agent has no money ($m_0 = 0$).

On the other hand, it is easy to verify that:

> *Walras's Law reduces here to Say's Law, that is,* $p\Sigma_i z_i(p) = 0$ *for every* p.

Indeed, if one takes into account that it is never optimal for an agent to keep a positive money balance at the end of his life, summation of the constraints in (1.6.IV) yields $pz_i(p) + m_i(p) = m_i(p)$ for every p and every i, hence, the result.

We have therefore proved that the equations (1.6.A) and (1.6.B) satisfy all the properties of the classical system. The classical dichotomy and quantity theory are thus valid propositions when applied to stationary states in an economy with a constant money stock. The real sector [equation (1.6.A)] determines relative prices and real variables, independently of the quantity of money. The money sector [equation (1.6.B)] in turn determines the level of money prices and of nominal variables, which is proportional to the total money stock.[12]

It remains to be seen whether such a system of equations has a solution under reasonable conditions. Since (1.6.A) looks like an ordinary Walrasian system, one can expect that it has a solution under standard assumptions. As a matter of fact:[13]

(1.6.1) *Assume (a) and (b) of the present section. Then* (1.6.A) *has a solution* p, *which has all its components positive and is defined up to a positive real number.*

Given any solution $\bar{p}$ of (1.6.A), the level of money prices must be determined by looking at the money market equation (1.6.B). Since the functions $m_i(p)$ are homogenous of degree 1 in prices, this is done by solving in λ the equation $\Sigma_i m_i(\lambda\bar{p}) = M$, or equivalently, $\lambda\Sigma_i m_i(\bar{p}) = M$. This equation has a positive solution, which is then unique, if and only if $\Sigma_i m_i(\bar{p})$ is positive, or equivalently, if and only if there is a type i of traders such that $m_i(\bar{p}) > 0$.

[12] It should be intuitively clear to the reader that these conclusions do not depend on the specific structure of the model that we consider. The homogeneity properties of the functions that appear in the market clearing equations (1.6.A), (1.6.B) should hold in any model in which the traders are free of money illusion. On the other hand, the identities, $pz_i(p) + m_i(p) = m_i(p)$ for every p, which give rise to Say's Law, reflect the budget constraints that a trader or a group of traders face in any period, and the fact that money balances are constant along a stationary state. These properties should hold in any well-specified economy with a constant money stock.

[13] A formal proof of this statement is given in Appendix B.

The existence of a stationary equilibrium will therefore be guaranteed if there is a type i of agents who have enough incentives to save when the price system is stationary. Intuitively, this will be the case if they experience a fall in income during their lifetimes, and if their preferences for present consumption are not too strong.

In order to make precise this intuition, let us assume that there is a type i of consumers that satisfies the following two assumptions:

(d) *The utility function* u_i *is of the form* $\Sigma_\tau \delta_i^{\tau-1} w_i(c_{i\tau})$, *where* w_i *is strictly concave and* $0 < \delta_i \leqq 1$;

(e) *There are* τ *and* τ' *with* $\tau < \tau'$ *such that* $e_{i\tau} \geqq e_{i\tau'}$, *with strict inequality for some component.*

Our goal is to show that under these assumptions, the system (1.6.A), (1.6.B) has a solution if the parameter δ_i is close to 1. We shall show in the first place that the result is true when there is no preference for present consumption, that is, when $\delta_i = 1$. Then by continuity, the result is still true when δ_i is close to 1.

It is first clear that *under assumptions (d) and (e),* $m_i(p)$ *is positive for every stationary price system* p *when* δ_i *is equal to* 1. The proof of this claim is quite simple. Let us suppose on the contrary that there exists a p such that $m_i(p) = 0$. This would mean that the solution of problem (1.6.IV) associated with this specific p, say (c_τ^*, m_τ^*), is such that $m_\tau^* = 0$ for every τ. This implies of course $pc_\tau^* = pe_{i\tau}$ for every τ. But this is impossible, since $pe_{i\tau} > pe_{i\tau'}$. As a matter of fact, the consumption program obtained by replacing c_τ^* and $c_{\tau'}^*$ by

$$c_\tau = c_{\tau'} = \tfrac{1}{2}(c_\tau^* + c_{\tau'}^*)$$

is certainly feasible: The agent can achieve it by saving $m_\tau = p(e_{i\tau} - c_\tau) > 0$ at age τ, by keeping this amount of money until age τ', and by spending it at that time. Given the specific form of the utility function and the fact that $\delta_i = 1$, this new program yields a higher level of utility, since by strict concavity of w_i:

$$w_i(c_\tau) + w_i(c_{\tau'}) > w_i(c_\tau^*) + w_i(c_{\tau'}^*)$$

Hence a contradiction, which proves the claim.

The result we just proved implies that if there is a type i of agents that satisfies (d) and (e), then $\Sigma_i m_i(\bar{p})$ is positive for every solution $\bar{p}$ of (1.6.A) when δ_i is equal to 1. In that case, the system (1.6.A), (1.6.B) indeed has a solution. It is intuitively clear that by continuity, (1.6.A), (1.6.B) still has a solution if δ_i is close to unity. This heuristic argument accordingly justifies the following proposition:[14]

[14] A rigorous proof of this proposition is given in Appendix B.

(1.6.2) *Assume (a), (b), and (c) of the present section. Assume moreover that there is a type* i *of consumers that satisfies (d) and (e). Then the system* (1.6.A), (1.6.B) *has a solution if* δ_i *is close enough to 1.*

Remark. In this section, it was explicitly specified that a stationary equilibrium is one in which money has positive value. Under our assumptions on utility functions and endowments, it can be shown using standard methods of equilibrium theory that a stationary equilibrium exists in which money has zero value. In the case of a single good it corresponds to the autarkic state, where every consumer consumes his own endowment in every period of his life.

1.7 The neutrality of money

We will end this chapter with a few remarks about the "neutrality" of money in the simple model under consideration. More precisely, we wish to look at the impact on equilibrium real and nominal magnitudes, both in the short run and in the long run, of a change of the money stock M that is implemented by the Government through lump-sum transfers – the textbook favorite: "money rain."

We already gave an answer in the preceding section, which concerned stationary equilibria. A once-and-for-all change of the money stock M leaves unaltered the set of "real" quantities that are exchanged along stationary equilibria, and generates a proportional variation of stationary equilibrium money prices and of nominal cash holdings. In this sense, money is "neutral in the long run."

The study of the short-run effects of a change of the money supply requires a little more care. Let us consider accordingly our simple economy at date 1, as we did in Sections 1.2–1.5, and let us assume that the Government implements at that date a *once-and-for-all* change of the outstanding stock of money from M to $\lambda_1 M$, where λ_1 is an arbitrary positive real number, by making a lump-sum subsidy to, or by levying a lump-sum tax on, each trader living at that date. Let $\Delta \bar{m}_a$ be the resulting variation of trader a's initial money holding $\bar{m}_a$.

It is first clear that if the money creation – or destruction – does not respect the initial distribution of cash balances, that is, if the $\Delta \bar{m}_a$ values are not proportional to the $\bar{m}_a$ values, the change of the money supply generates a "distribution effect" among traders and thus is likely to modify short-run equilibrium "real" variables.[15] The only hope, therefore,

[15] Such "distribution effects" are partly responsible for the lack of neutrality of money in some monetary models. For instance, if one considers an overlapping generation model without bequests, and if money is injected into the economy

to get neutrality is to consider the Ricardian experiment of a scalar change of the traders' initial money balances, that is:

$$\bar{m}_a + \Delta\bar{m}_a = \lambda_1\bar{m}_a \qquad \text{for all } a$$

What is the impact of this change on Walrasian equilibrium prices at date 1? The answer given by neoclassical theorists is that a new short-run equilibrium price system p_1 is a solution of the equations (1.4.C), (1.4.D), where each trader's initial money holding $\bar{m}_a$ is changed in $\lambda_1\bar{m}_a$, that is:

$$(1.7.C) \quad \sum_a z_a(p_1,\lambda_1\bar{m}_a) = 0$$

$$(1.7.D) \quad \sum_a m_a^d(p_1,\lambda_1\bar{m}_a) = \sum_a \lambda_1\bar{m}_a$$

As we recalled in Section 1.1, the neoclassical argument would then be that the functions z_a and m_a^d that appear in this system are homogenous of degree 0 and 1 respectively, in regard to current prices and initial money holdings, and therefore that changing the money supply in this way has no "real" effects. We have seen, however, that the neoclassical homogeneity postulates are based on extremely specific assumptions – the unit elasticity of price expectations – and that they have to be abandoned anyway if one wishes to get a consistent short-run theory, since in that case intertemporal substitution effects have to be taken into account. Hence, if one follows this approach, it would seem that money is not in general neutral in the short run.

That kind of reasoning is wrong because it neglects the information that is generated by the Government's policy itself, and its influence on the traders' expectations.

Let us assume that the once-and-for-all scalar change of initial money holdings is *publicly* announced by the Government. The parameter λ_1 is then observed by all traders, in addition to the current price system p_1. It must therefore be an argument of each trader's price expectations, which will be denoted accordingly $\psi_{at}(p_1,\lambda_1)$, for $t = 1, \ldots, n_a$. The problem that typical traders must solve at date 1 is then to maximize their utility function under their current and expected budget constraints (we drop the trader's index a for convenience):

$$p_1c_1 + m_1 = p_1e_1 + \lambda_1\bar{m}$$

$$\psi_t(p_1,\lambda_1)c_t + m_t = \psi_t(p_1,\lambda_1)e_t + m_{t-1}$$

by giving it partly to the "young" traders – i.e., to those who just came into the market, and who accordingly have no money initially – this injection is bound to be nonneutral, since it does not respect the initial distribution of money holdings.

for $t = 2, \ldots, n$. This is in fact (1.3.II), which is reformulated so as to take into account the information embodied in the Government's policy.

A trader's excess demand for current goods and his current demand for money depend now on the current price system p_1, on his initial money holding $\bar{m}$, and on the parameter λ_1. If we note them $z_a(p_1,\bar{m}_a,\lambda_1)$ and $m_a^d(p_1,\bar{m}_a,\lambda_1)$, the equations defining the short-run equilibrium price vectors that are associated with the policy parameter λ_1 are given by:

$$(1.7.\mathrm{C}') \quad \sum_a z_a(p_1,\bar{m}_a,\lambda_1) = 0$$

$$(1.7.\mathrm{D}') \quad \sum_a m_a^d(p_1,\bar{m}_a,\lambda_1) = \sum_a \lambda_1 \bar{m}_a$$

It becomes easy to see, once the problem has been properly reformulated in these terms, that a publicly announced once-and-for-all scalar change of all traders' initial money holdings is neutral – provided that the economic agents believe this neutrality proposition to be true. Let us assume that *the functions $\psi_{at}(p_1,\lambda_1)$ are homogenous of degree 1 with respect to (p_1,λ_1).* In other words, the traders think that moving from a situation in which $\lambda_1 = 1$ and in which the current price system is p_1, to a situation in which λ_1 differs from unity and current prices are $\lambda_1 p_1$, will generate a proportional variation of equilibrium prices in the future.[16] Under this assumption, in view of the absence of money illusion property stated in (1.1.3), the functions z_a and m_a^d appearing in the system (1.7.C'), (1.7.D'), considered as functions of p_1 and λ_1, are homogenous of degree 0 and 1 respectively, in regard to these variables. Then if p_1 is a solution of (1.7.C'), (1.7.D') when $\lambda_1 = 1$, $\lambda_1 p_1$ is a solution of the same system for every value of λ_1. Moreover, real equilibrium quantities are independent of, whereas nominal money balances are proportional to, the parameter λ_1.[17]

[16] This assumption relates expected prices to p_1 and λ_1. It is therefore compatible with price expectations that are not unit elastic with respect to current prices p_1 alone, λ_1 being fixed. The assumption is in particular compatible with the kind of conditions which were used in (1.4.1) to ensure the presence of a strong stabilizing intertemporal substitution effect for a given λ_1.

[17] It is not difficult – but a little tedious – to show that short-run equilibrium prices will be multiplied by λ_1 not only at date 1, as we just showed, but at all subsequent dates too, if the agents form their expectations according to the above principle at date 1 and at all later dates. This is left as an exercise to the reader (Hint: Make an argument by induction. A trader's expected prices at date 2 are functions of p_1, λ_1 and p_2, and will be assumed to be homogenous of degree 1 with respect to these variables). In this sense, the postulated homogeneity properties of price expectations with respect to λ_1 and observed prices are "rational" or more precisely, self-fulfilling.

Such a publicly announced one-shot money injection – or destruction – is then neutral. It goes without saying that this result has been obtained by following a route quite different from the neoclassical approach. For we relied on certain homogeneity properties of the functions z_a and m_a^d with respect to current prices p_1 and the Government's policy parameter λ_1, and not with respect to current prices and initial money holdings, as neoclassical theorists do.

The result shows that *the only sources of the nonneutrality of a once-and-for-all scalar change of all traders' initial money holdings are either that economic units do not believe in the neutrality of such a change, or that they are not fully informed about the Government's policy.*

In order to illustrate the latter point, it is most convenient to look at the simple case in which the traders know that the Government is implementing at date 1 a scalar change of their money balances $\bar{m}_a$, but in which the size of this policy, i.e., the parameter λ_1, is not publicly announced. In such a case, of course, all agents who initially own a positive cash balance observe immediately the value of λ_1, since they receive the lump-sum transfer $\Delta\bar{m}_a = (\lambda_1 - 1)\bar{m}_a$. But the traders who have no money at the outset of the period and who accordingly receive nothing, are in a much worse position – this will be the case in particular for the "young" or "newborn" traders in an overlapping generation model without bequests. These traders will have to guess both the value of λ_1 and the real characteristics of the economy at date 1 through the observation of the sole price system p_1. It may be the case that these traders cannot make such an inference because the price system does not convey enough information. Money will then be nonneutral. That kind of "informational" nonneutrality, however, seems to be based on very specific assumptions. It requires the presence in the economy of traders who are not informed of the size of the change of the money supply, whereas that kind of information could be made public quite easily. And perhaps more importantly, *the uninformed traders must initially have no money at all, because otherwise they would immediately observe the size of the Government's policy* λ_1.[18]

Remark (Preannounced changes of the money supply). It should be emphasized that the neutrality proposition that we have established concerns exclusively a publicly announced once-and-for-all scalar variation of the

[18] This is one of the basic sources of the nonneutrality of money in Lucas's influential paper (1972), as I understand it, which enabled him to establish the existence of a "Phillips' curve," i.e., of a positive correlation between equilibrium prices and output, in a Walrasian equilibrium setting.

traders' initial money balances in the current period, i.e., at date 1. By contrast, a scalar change of current money holdings will have generally "real" effects if the traders expect, or if the Government announces at date 1 that a variation of the money supply of a given size ΔM_θ will take place too, through lump-sum transfers, at a prespecified date θ in the future. Similarly, the announcement by the Government at date 1 that it will make a once-and-for-all change of the money supply at a given date θ in the future, through lump-sum transfers, will not generally be neutral. It is perhaps useful to spend some time discussing this issue more precisely, for it has been somewhat controversial recently. Moreover, it has important policy implications. For instance, it has been argued that deterministic variations of the timing of taxes have no effect on the equilibrium of the real sector whenever they are expected by private economic units or preannounced by the Government.[19] That sort of argument has been employed to revive under the "new classical macroeconomics" banner, an old proposition which some attribute to Ricardo, namely, that it is equivalent to finance a government's deficit by printing money, by levying taxes, or by issuing bonds, whenever the agents anticipate that interest payments on bonds will have to be covered by taxation later on. In all cases, it is argued, the induced variations of the money stock will be offset by a correlative movement of prices, and the equilibrium of the real sector will be unaltered.[20]

We are going to see that this viewpoint is wrong, and that expected or preannounced modifications of the time profile of the money stock through lump-sum transfers produce generally real effects if one takes into account the various *liquidity constraints* that the agents face.[21]

Let us consider our simple economy at date 1, and let us assume that the Government announces that it makes the outstanding money stock to move from M to $\lambda_1 M$ at that date by making lump-sum transfers to the traders living in the current period. Let us assume furthermore that the Government announces too that it will add to the existing money stock the quantity $\Delta M_t = \lambda_t M$ in every period t in the future, again through lump-sum transfers. The λ_t are arbitrary real numbers for $t \geqq 2$, with the only restriction, of course, that the stock of money at t, i.e., $\Sigma_1^t \lambda_\tau M$, is positive for all t. A once-and-for-all variation of the current money stock corresponds to $\lambda_1 \neq 1$, and $\lambda_t = 0$ for all $t \geqq 2$.

The only way to avoid distribution effects among the traders a who are living in the current period, as it will become clear shortly, is to

[19] See, e.g., Sargent and Wallace (1975).
[20] See Barro (1974).
[21] For similar arguments, see Tobin (1980, Chap. 3).

make the transfers $\Delta\bar{m}_{at}$ that they receive in their lifetimes proportional to their initial money holdings $\bar{m}_a$.[22] So we shall assume $\bar{m}_a + \Delta\bar{m}_{a1} = \lambda_1\bar{m}_a$ and $\Delta\bar{m}_{at} = \lambda_t\bar{m}_a$, for all a and t. The Government's policy is then fully described by the array $\lambda = (\lambda_1, \lambda_2, \ldots)$. Each trader's price expectations at date 1 are now functions of the current price system p_1 and of the "policy parameter" λ. They will be denoted $\psi_{at}(p_1, \lambda)$.

A typical trader's decision problem at date 1 is then to maximize his lifetime utility function under the current and expected budget constraints (we will drop the index a momentarily for convenience):

$$p_1 c_1 + m_1 = p_1 e_1 + \lambda_1 \bar{m}$$

$$p_t c_t + m_t = p_t e_t + m_{t-1} + \lambda_t \bar{m} \qquad t = 2, \ldots, n$$

where the expected prices p_t stand for $\psi_t(p_1, \lambda)$.

The trader's resulting excess demands for current goods and his demand for money at date 1 are then functions of the current prices p_1, of his initial money balance $\bar{m}$, and of the policy parameter $\lambda = (\lambda_1, \lambda_2, \ldots)$. If we note them, for each trader, $z_a(p_1, \bar{m}_a, \lambda)$ and $m_a^d(p_1, \bar{m}_a, \lambda)$, the equilibrium price vectors that are associated at date 1 with the policy parameter λ are given by the equations[23]:

(1.7.C″) $\sum_a a_a(p_1, \bar{m}_a, \lambda) = 0$

(1.7.D″) $\sum_a m_a^d(p_1, \bar{m}_a, \lambda) = \sum_a \lambda_1 \bar{m}_a$

Examination of a typical trader's decision problem shows that the functions z_a and m_a^d above are homogenous of degree 0 and 1 respectively, in regard to (p_1, λ), whenever the traders' expectation functions $\psi_{at}(p_1, \lambda)$ are themselves homogenous of degree 1 with respect to these variables. Scaling up or down all the λ_t values then generates a proportional variation of equilibrium prices p_1 in the system (1.7.C″), (1.7.D″), and leaves

[22] Hence, promising to make a lump-sum transfer $\Delta\bar{m}_{at} \neq 0$ to a trader who initially owns no money will yield a distribution effect if we start, say, from a situation where there are no changes of the money supply. That kind of nonneutrality will occur in particular in an overlapping generation model without bequests whenever the preannounced changes of the money supply ΔM_t, for $t \geqq 2$, are implemented by giving money to or by levying taxes from the traders who are "young" or just "newborn" at date 1.

[23] The short-run analysis that was developed in Sections 1.2–1.5 applies, strictly speaking, only to the case where $\lambda_t = 0$ for $t \geqq 2$. The reader will easily verify, however, that all the arguments presented there are still valid for an arbitrary λ, provided that the sums $\Sigma_1^t \lambda_\tau$ are positive. In fact, the existence theorem stated in (1.4.1) still applies in such a case, as the mathematically oriented reader will check without difficulty by looking at its proof in Appendix B.

unaltered all real equilibrium magnitudes at date 1. It can be shown, by recurrence, that the same neutrality proposition holds at later dates t too, whenever the traders' price expectations are homogenous of degree 1 with respect to λ and the prices $(p_1, \ldots, p_t)$ that are observed from the date of the announcement of the policy, up to the date t. *A publicly announced scalar variation of all the λ_t values is therefore neutral whenever the traders believe in its neutrality.* This result includes as a particular case the neutrality of a once-and-for-all scalar variation of current money holdings $\bar{m}_a$, which we established previously, since then $\lambda_t = 0$ for all $t \geqq 2$.

It is easily seen that, by contrast, a variation of the λ_t values that modifies the time profile of the money stock, i.e., one that alters some of the ratios λ_{t+1}/λ_t, will generally change the equilibrium of the real sector at date 1.

In order to see this point, it is most convenient to consider again the decision problem of a typical trader. By adding the t first budget constraints, one gets a liquidity constraint that expresses the fact that the money balance m_t cannot be negative:

$$\Sigma_1^t p_\tau(c_\tau - e_\tau) \leqq (\Sigma_1^t \lambda_\tau)\bar{m}$$

The trader's optimum consumption program is then obtained by maximizing his utility function under all these liquidity constraints for $t = 1, \ldots, n$.

Assume now that we start from an initial equilibrium situation at date 1, which corresponds to a given policy parameter λ, and that the λ_t values are changed in an arbitrary way. If one looks at a typical trader's liquidity constraint for $t = n$, i.e., at his lifetime intertemporal budget constraint, one sees that current and expected prices have to move proportionally to $\Sigma_1^n \lambda_\tau$, if $\bar{m}$ is positive, in order to keep the agent's consumption program unchanged. Suppose now that the trader's liquidity constraint for $t = \theta < n$ was initially binding. Then current and expected prices, up to date θ, have to move proportionally to $\Sigma_1^\theta \lambda_\tau$ too in order to avoid a variation of the trader's consumption program. These two requirements are not generally compatible, unless the λ_t values are all changed in the same proportion. In particular, if the initial situation corresponds to $\lambda_1 = 1$, $\lambda_t = 0$ for $t \geqq 2$, and if the policy move is to vary a single λ_t at a given date $\theta < t \leqq n$, such a preannounced change of the money supply will modify the traders' equilibrium consumptions at date 1.

Even if one ignores the possibility that the traders' liquidity constraints for $t < n$ may be initially binding, it remains that in order to keep the equilibrium of the real sector invariant at date 1, current and expected prices must move in proportion to $\Sigma_1^n \lambda_\tau$, for each trader. Again, this will be generally impossible to achieve unless the λ_t values move altogether proportionally, since the (lifetime) planning horizons n_a of the traders who are living at date 1 are typically different.

The conclusion of this brief qualitative analysis is that *preannounced deterministic variations of the money supply that alter the time profile of the money stock through lump-sum transfers may have real effects.* The conclusion should not be surprising after all, for such variations modify the rate of growth of the money supply at some date. This is a "real" change, and one should expect it to generate real effects if money has any meaningful role to play in the economy, by inducing at least momentarily a change of the rate of inflation (this is sometimes referred to in the literature as the "inflation tax"). The foregoing analysis shows that *the basic channel through which such deterministic changes of the rate of growth of the money supply influence the real sector in a Walrasian equilibrium setting, are the various "liquidity constraints" that the agents face.*[24]

Notes on the literature

Part of the analysis presented in this chapter is based on an earlier paper by Grandmont (1974), and on its extensions to overlapping generation models by Grandmont and Laroque (1973), and by Grandmont and Hildenbrand (1974).

The analysis was restricted here to point expectations and certain endowments in order to simplify the exposition. It can be generalized to the case in which price forecasts take the form of probability distributions, as in Grandmont (1974), and in which exogenous random shocks alter the traders' endowments, as in Grandmont and Hildenbrand (1974). The conditions used in these papers to get existence of a short-run equilibrium, when they are specialized to the case of point expectations, yield the assumptions stated in (1.4.1). These papers considered the simple case of all agents making plans at most one period ahead. They can be extended, without changing the economics of the arguments, to the case in which traders are planning for more than two periods and are uncertain about future prices and endowments. The only cost is to introduce more technical complexities. A short-run analysis of the same sort of model, which includes production, can be found in Sondermann (1974).

Hool (1976) studied the existence of a short-run monetary equilibrium in a model where traders are planning only one period ahead ($n_a = 2$ for

[24] For a further study of the effects of changes of the money supply by means of lump-sum transfers, the reader may consult the collective volume edited by Kareken and Wallace (1980) and the references contained there, and Hahn (1982).

all a) and in which money plays a role as a medium of exchange, by employing the methods described in this chapter. Hool shows that the assumptions of (1.4.1) can be weakened in that case, and that a short-run Walrasian monetary equilibrium does exist whenever there is a trader who owns initially a positive money balance, whose expected prices are bounded below by a positive vector ϵ for all current prices p_1, and such that the ratios of expected prices to current prices eventually become less than the trader's marginal rates of substitution between future and current consumption at the trader's endowment point when current prices p_1 go to infinity. These are precisely the sort of conditions used in our heuristic discussion of the short-run regulating mechanisms in Section 1.4 for the case of a single good (see in particular footnote 7, above). Hool's assumptions rule out unit elastic price expectations – contrary to what the author mistakenly claims – and are there to ensure the presence of a stabilizing and strong intertemporal substitution effect.

Grandmont and Younès (1972, 1973) used the methods of this chapter to analyze a model of outside money in a certain environment where the traders may face liquidity constraints on their transactions, within each period, that are due to the role of money as a medium of exchange, and where they have an infinite planning horizon. The authors study in addition, within that framework, Friedman's theory of the "optimum quantity theory of money" (1969, Chapter 1). They show in particular that deterministic changes of the rate of growth of the money supply, through lump-sum transfers, do affect real equilibrium magnitudes whenever the liquidity constraints that the agents face on their transactions within each period are binding (this effect is known in the literature as the "shoe-leather effect"). On this point, the reader may also consult Niehans (1978). For an extension of the model to the case of uncertainty, see Lucas (1980).

Suggested reading

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