MICROECONOMICS II
Problem set 1
Universitat Pompeu Fabra – Winter 2006
Professor: Antonio Cabrales

1. Show that the game

	L	M	R
U	1, -2	-2, 1	0, 0
M	-2, 1	1, -2	0, 0
D	0, 0	0, 0	1, 1

   has a unique equilibrium. (Hint: Show that it has a unique pure-strategy equilibrium; then show that player 1, say, cannot put positive weight on both U and M; then show that player 1, say, cannot put positive weight on both U and D, but not on M, for instance.)

2. Each of n people chooses whether or not to become a political candidate, and if so which position to take. There is a continuum of citizens, each of whom has a favorite position; the distribution of favorite positions is given by a density function. A candidate attracts the votes of those citizens whose favorite positions are closer to his position than to the position of any other candidate; if k candidates choose the same position then each receives the fraction $\frac{1}{k}$ of the votes that position attracts. The winner of the competition is the candidate who receives the most votes. Each person prefers to be the unique winning candidate than to tie for first place, prefers to tie for first place than to stay out of the competition and prefers to stay out of the competition than to enter and lose.

   (a) Formulate this situation as a strategic game, find the set of Nash equilibria when $n = 2$.

   (b) Show that there is no Nash equilibrium when $n = 3$.

3. Consider the following game where player 1 chooses row, player 2 chooses column and player 3 chooses matrix.

L

	l	r
u	$\frac{1}{4}, \frac{1}{4}, \frac{1}{2}$	$\frac{1}{3}, 0, \frac{1}{3}$
d	$0, \frac{1}{3}, \frac{1}{3}$	$0, 0, \frac{1}{3}$

R

	l	r
u	$0, 0, \frac{1}{2}$	$0, \frac{1}{3}, \frac{1}{2}$
d	$\frac{1}{3}, 0, \frac{1}{2}$	$\frac{1}{3}, \frac{1}{3}, \frac{1}{3}$

   (a) What are the (pure and mixed) Nash equilibria of this game? Which of these survive the iterated deletion of weakly dominated stragies?

   (b) Repeat the last question when the preferences of player 3 are such that where we wrote $\frac{1}{3}$ for her payoff we now write $\frac{1}{2}$ and where we wrote $\frac{1}{2}$ for her payoff we now write $\frac{1}{3}$.

4. Suppose that you have a two-person game where the utility functions are such that for all pairs of strategy profiles $s, s' \in S$

$$u_1(s) \geq u_1(s') \iff u_2(s) \leq u_2(s')$$

Suppose that $\sigma' = (\sigma_1', \sigma_2')$ and $\sigma'' = (\sigma_1'', \sigma_2'')$ are Nash equilibria of this game.

(a) Show that for all $i = 1, 2$, $u_i(\sigma') = u_i(\sigma'')$.

(b) Show that the profiles (σ_1', σ_2'') and (σ_1'', σ_2') are also Nash equilibria.